The q Theory of Investment with Many Capital Goods

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The "q" theory of investment, which relates investment to the ratio of market to replacement value of capital, has attracted considerable attention in a recent series of papers. For instance, a q variable has been used as an independent variable in empirical investment equations estimated by George von Furstenberg (1977), von Furstenberg et al. (1980), Burton Malkiel et al. (1979), and others. These papers, however, leave somewhat unclear the theoretical rationale for using q as an investment determinant. This has motivated work by Hiroshi Yoshikawa (1980), Lawrence Summers (1981), Michael Salinger and Summers (1981), and Fumio Hayashi (1982), who show that under certain conditions the rate of investment of a share-value-maximizing firm is indeed a function of q. This is an important result because it shows that investment equations with q an independent variable are not *ad hoc* constructions. Rather, they are grounded in a theory of the firm with an appealing behavioral hypothesis, viz, value maximization.

An important assumption underlying this research is that capital can be treated as a homogeneous good. Of course, this is an extremely common assumption in the analysis of investment, and is not particularly more bothersome in the q theory context than elsewhere. Nonetheless, there are certain situations where it may be desirable or even essential to be able to study investment disaggregated by type of capital good. Thus, the purpose of this paper is to examine whether and how the q theory can be extended to this more general case.[1]

Intuitively, one would expect some difficulty with the q theory in the many-capital-good context, as already noted by James Tobin and William Brainard (1977, p. 243) and by Salinger and Summers (p. 12). One way to see why is to recall the Tobin-Brainard distinction between "marginal" and "average" q. As Hayashi writes,

> The "q" theory...is not operational as long as q is not observable. Remember that q, which we call marginal q, is the ratio of the market value of an *additional* unit of capital to its replacement cost. What we can observe is *average* q, namely the ratio of the market value of *existing* capital to its replacement cost. [p. 214]
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> If we knew marginal q, then econometric implementation of the "q" theory would be quite straightforward. Unfortunately, however, marginal q is not directly observable. What we can (in principle) observe is average q....

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[1]Two examples will illustrate the significance of the extension to the many-capital goods case (which Salinger and Summers identify as one of the "two most important area[s] for further investigation," p. 47). First, relative price shocks or other changes in the economy may reduce the value of existing capital, may make new investments in some sectors of the economy unattractive, and may make some new investments very attractive, causing previously aligned marginal and average qs to diverge. The 1973 change in energy prices, cited by Salinger and Summers (p. 12), is the classic example. Numerous studies (see, for example, Martin Baily, 1981) suggest that this shock has effectively rendered a substantial amount of existing capital obsolete, simulanteously creating important incentives for new investment.

Second, consider the new "15-5-3" tax rules for depreciation of different categories of capital goods (cars and light trucks, other vehicles and equipment, structures, etc.). Whether one is interested in the effects of this policy change on investment disaggregated by these categories or just the effect on aggregate investment, there is little reason to believe that this policy change is adequately described by a change in average q, or by any other single variable. Note that the inadequacy of a single capital good model in this case is inherent in the complexity of the policy change one wishes to analyze. Short of ignoring the policy question itself, there is no way to finesse this difficulty by assuming a single capital good.

> [P]eople are busying themselves regressing investment on average q. Researchers should feel uneasy about doing this, unless they are sure that average q and marginal q are practically the same thing. The...[main theorem for the case of a competitive firm] states that marginal q and average q are essentially the same.... [p. 218]

In other words, marginal q is the fundamental determinant of investment because it shows how much increase in market value accompanies a dollar's worth of investment, while the actual stock market value of the firm reflects the profitability of existing total capital. Hayashi's main contribution is to establish the connection between the two. But with many capital goods, can a similar link be forged? A seemingly likely difficulty is that at any particular moment, a firm that employs a variety of capital goods will find that some kinds of investments may be highly attractive, others less so, and still others may be not worth undertaking at all. If so, there will be a variety of marginal qs, one for each capital good. Obviously, they cannot all be equal to the same "observable" average q. The link between average q and the investment-determining marginal qs may therefore be broken.

To explore this intuition formally, Section I presents a model similar to those of Hayashi, Summers, and Salinger-Summers, but which allows for an arbitrary number of capital goods. Section II briefly recapitulates the main q theory results for the special case of a single capital good. Section III turns to the main issues and, as expected, confirms the above intuition: in general, total investment in many capital goods cannot be expressed as a monotonic function of q. Necessary and sufficient conditions for this to be possible, involving mainly restrictions on adjustment cost functions, are presented, as well as particular examples of such functions.

Section III does not only contain this bad news, however. It also presents an important positive result. I derive a relationship between q and the *vector* of investment in the various capital goods, which shows how q is uniquely a function of this vector through the structure of the adjustment cost function(s). This means that regression equations with the q variable on the *left*-hand side and the vector of investment on the *right*-hand side can be used to estimate the underlying technology of investment as reflected in the parameters of the adjustment cost function(s). Since the relationship between the vector of investment and q is many-to-one rather than one-to-one, it cannot generally be inverted to express investment as a function of q. (This indeed is the bad news noted above.) Thus, we cannot use estimates from such an equation to predict total investment, conditional on q. Despite this limitation, however, the q theory approach still proves of value if one's interest is to determine structural parameters governing the investment process.

I. The Model

For the sake of simplicity in exposition, I assume the complete absence of personal or corporate taxes, and of debt finance. The results generalize to a model which includes those of Hayashi and Summers as special cases, however.[2]

The first step in constructing the model is to integrate the firm's share value equilibrium condition

$$(1) \qquad (\rho+\pi_g)V=\dot{V}+\mathrm{Div},$$

from $t=0$ to $t=\infty$, invoking a boundedness condition, to solve for the equilibrium value of shares outstanding at $t=0$:

$$(2) \qquad V(0)=\int_0^\infty \mu\,\mathrm{Div}\,dt,$$

where

$$(3) \qquad \mu(t)=\exp\left(-\int_0^t(\rho+\pi_g)\,ds\right).$$

Here and below all variables should be considered time dependent unless otherwise

[2]A version of this paper containing this extension is available from the author on request.

stated, although time arguments will be suppressed except where they are essential. We have V = aggregate nominal value of shares outstanding; ρ = exogenously given real rate of return on equity; π_g = exogenously given rate of change of general price level; Div = nominal dividends paid out, and a dot above a variable denotes a time derivative.

At any moment of time, the firm receives a flow of cash from sales of output, and expends cash on hiring variable factors and on investment, including adjustment costs. Any remaining cash flow is paid out as dividends, which are assumed always to be positive (or at least the nonnegativity constraint is ignored). I assume that output is a linear homogeneous function $F(K, L)$ defined over the vector $K=(K_1,\dots,K_m)^T$ of stocks of m capital goods and the vector $L=(L_1,\dots,L_n)^T$ of flows of n variable factors. The price of output is p_0, $w=(w_1,\dots,w_n)$ is the vector of variable input prices, and $p=(p_1,\dots,p_m)$ is the vector of capital goods prices, all in nominal terms. The stock of capital good j evolves according to

$$(4) \qquad \dot{K}_j = I_j - \delta_j K_j,$$

where I_j is gross investment and δ_j is the (not necessarily time invariant) proportional rate of depreciation. The firm incurs adjustment costs when undertaking investment, which is assumed to take the form either of lost output or of waste of the good being invested. Specifically, let $\Gamma(I, K)$ be a linear homogeneous adjustment cost function showing units of lost *output* when the vector of investment is $I=(I_1,\dots,I_m)$ and the stock of capital is K, and let $\Phi_j(I_j, K_j)$ be a linear homogeneous adjustment cost function showing units of *good j* lost from investment I_j given capital K_j. Both kinds of adjustment costs appear in the literature; presumably one or the other would be more appropriate in particular cases. Actually, still more general forms are conceivable, but consideration of them would not change the results of this paper. Here I shall carry both kinds of adjustment costs through most of the formal analysis, focusing later on one or the other special case as convenient. In addition to linear homogeneity, the adjustment-cost functions are assumed to be twice continuously differentiable; the $m\times m$ matrix $\partial^2\Gamma/\partial I_j \partial I_{j'}$ is assumed positive definite; $\partial^2\Phi_j/\partial I_j^2$ is assumed positive; $\Phi_j(0, K_j)=\Gamma(0,\dots,0; K)=0$; and $\Phi_j(I_j, K_j)\ge 0 \le \Gamma(I, K)$, for all I, K.

Given these assumptions, we can write

$$(5) \quad \text{Div} = \big(p_0[F(K,L)-\Gamma(I,K)] - wL\big) - \Sigma_j\big[p_j I_j + p_j \Phi_j(I_j, K_j)\big],$$

or, substituting into (2),

$$(6) \quad V(0) = \int_0^\infty \big\{ p_0[F(K,L)-\Gamma(I,K)] - wL - \Sigma_j\big[p_j I_j + p_j\Phi_j(I_j,K_j)\big]\big\}\mu\, dt.$$

The firm's problem is to choose paths for L and I to maximize (6) subject to (4). Associating auxilary variables λ_j with (4), we have the following necessary conditions for an optimum, letting subscripts on F, Γ, and Φ_j denote partial derivatives:

$$(7) \qquad p_0 F_{L_k} - w_k = 0, \text{ all } k, t;$$

$$(8) \quad \lambda_j - \big[p_0\Gamma_{I_j} + p_j\big(1+\Phi_{I_j}\big)\big]\mu = 0, \text{ all } j, t;$$

$$(9) \quad -\dot{\lambda}_j = \big[p_0\big(F_{K_j} - \Gamma_{K_j}\big) - p_j\Phi_{K_j}\big]\mu - \lambda_j\delta_j, \quad \text{all } j, t;$$

$$(10) \qquad \lim_{t\to\infty} \lambda_j(t)K_j(t) = 0, \text{ all } j.$$

II. The Single Capital Good Case

Let us first review the principal results of the q theory for the special case $m=1$. We begin by interpreting (8) as an equation determining the optimal I (dropping the capital good subscript) as a function of the shadow price of capital, λ, and other variables. The problem with this equation is that it cannot be estimated as it stands, since λ is unobservable. However, Hayashi proves that

$$(11) \qquad \lambda(0)K(0) = V(0),$$

by a method of proof explained in the next section. This equation says that the shadow value of the initial capital stock is equal to the current stock value of the firm. What is crucial about (11) is that it allows one to solve for $\lambda(0)$ and thus to rewrite (8) in a form more suitable for empirical application.

Specifically, by the homogeneity of Γ, note that $\Gamma(I,K) = K\Gamma(I/K,1) \equiv K\gamma(I/K)$, so that $\Gamma_1 = \gamma'$ is the lost output per unit increase of investment. Similarly, one may define $\phi(I/K) \equiv K\Phi(I/K,1)$. Then, recalling that p is the price of the capital good, $(p_0\gamma' + p\phi')/p$ is the adjustment cost, in dollars, per dollar's worth of investment. Using (11) in (8), we have an equation for I/K:

$$(12)\quad [p_0\gamma'(I/K) + p\phi'(I/K)]/p = V/pK - 1 \equiv Q$$

or

$$(13)\quad I/K = h(Q),$$

where $h(\cdot) = [(p_0\gamma'(\cdot) + p\phi'(\cdot))/p]^{-1}$ and where Q is Tobin's q minus 1.

The existence of the inverse defining $h(\cdot)$ in (13) is assured by the convexity of γ and ϕ. Thus, the rate of investment—or (multiplying both sides of (13) by K) the level of investment—is an increasing function of Q. Depending on the form of the adjustment costs (i.e., whether they involve lost output, with $\Phi \equiv 0$, or wasted investment goods, with $\Gamma \equiv 0$), we can use (13) in a regression of I/K on Q to identify parameters characterizing the adjustment cost technology, as in Summers and Hayashi.

Notice that the crucial contribution of the q theory in the single capital good case is that it permits one to replace the unobservable shadow value λ by V/K, from (11), which allows one to write the first-order condition for investment, (8), in a form that can be estimated.[3]

III. The General Case

A. *Unobservability of the Shadow Values of Capital*

Let us return to the general many capital good case. From (8) one sees that each I_j depends on the own-shadow price λ_j and, through the function Γ_{I_j}, on the amount of investment in other capital goods, as well as on various parameters. In fact, (8) can now be seen as a system in which the investment levels $\{I_j\}$ are all jointly determined by shadow values for all capital goods and by other parameters. Of course, this system cannot be used as it stands to estimate a system of investment equations because the λ_js are not observed. The natural question is whether the λ_js can be eliminated, as in the single good case, by using the market's forward-looking valuation of capital as reflected in the stock value.

We can proceed to answer this question by applying Hayashi's essential insight in the many-capital good case, and write, by (10),

$$(14)\quad \lambda_j(0)K_j(0) = \lambda_j(0)K_j(0) - \lim_{t\to\infty}\lambda_j(t)K_j(t) = -\int_0^{\infty}(\dot{\lambda}_j K_j + \lambda_j \dot{K}_j)\,dt.$$

Using (7), (8) and (9), we have

$$(15)\quad -\dot{\lambda}_j K_j - \lambda_j \dot{K}_j = \Big[p_0\big(F_{K_j}K_j - \Gamma_{K_j}K_j - \Gamma_{I_j}I_j\big) + \Sigma_k\big(p_0 F_{L_K} - w_k\big)L_k - p_j I_j - p_j\big(\Phi_{I_j}I_j + \Phi_{K_j}K_j\big)\Big]\mu.$$

[3] Hayashi and Summers stress the "observability" of average q (see the quotation in the opening discussion attributed to Hayashi, for example), and indeed (11) does appear to confirm that an unobservable shadow value can be expressed in terms of currently observable variables. Unfortunately, this is not possible in a world with taxes. As discussed in the unpublished version of this paper, a "tax-adjusted" Q of the sort used by Hayashi and Summers cannot be estimated without knowing the expected time path of all personal and corporate tax policy parameters (with the exception of the investment tax credit), and expected future tax policy is not directly observed. To get around this problem, Hayashi, Summers, and Salinger-Summers assume that agents held static expectations on tax policy parameters in every sample period, and on this basis they are able to compute Q-type variables which are then used in their regressions; unfortunately their estimates are thus dependent on the maintained hypothesis of static expectations. It is regrettable that the true observability of Q that obtains in the simple taxless world of this paper does not carry over to a world with taxes, for if a tax-adjusted Q really were observable it would be possible (in a single capital good world) to estimate policy-invariant parameters of an investment equation.

At this point the derivation of the tax-adjusted Q breaks down. In the single capital-good case, homogeneity of F and Γ (application of Euler's theorem) allows one to use (6) and (14) to produce (11). By contrast, with many capital goods, integrating and summing over j (using (6) and (14)) yields

$$(16) \qquad \Sigma_j \lambda_j(0) K_j(0) = V(0).$$

This equation, the many capital-good analogue of (11), has the same intuitive interpretation: the shadow value of the initial capital stock is equal to the current stock market value of the firm. However, there is also a fundamental difference between the two cases: whereas (11) allows one to estimate the single shadow value of capital from $V(0)$ and $K(0)$, (16) reveals that $V(0)$ is an initial capital-weighted sum of the multiple shadow prices, λ_j, and cannot be used to identify any *one* shadow price. In short, the many marginal qs, represented by the λ_js, cannot each be estimated by the one average q, represented by $V(0)$, confirming the intuitively obvious claim made in the introduction.

B. *Identification of Technological Parameters*

The above limitation certainly does not mean that a q-type variable cannot be useful for empirical work, however. Notice from (8) that (at $t=0$)

$$(17) \qquad \left(p_0 \Gamma_{I_j} + p_j \Phi_{I_j}\right) K_j = \lambda_j K_j - p_j K_j.$$

Summing over j using (16), and defining the weights $\omega_j = p_j K_j / pK$, one has

$$(18) \qquad \Sigma_j \left(\left(p_0 \Gamma_{I_j} + p_j \Phi_{I_j} \right) / p_j \right) \omega_j = V/pK - 1 = Q.$$

Note the similarities and differences between (12) and (18). The latter is clearly a natural generalization of the former, with a q-type variable on the right and a marginal adjustment cost term on the left. In the case of (12), the left-hand side is simply the marginal adjustment cost per dollar's worth of investment in the single capital good. In (18), the corresponding expression is a weighted sum of the marginal adjustment cost per dollar of investment for each of the m capital goods. The weights are simply the value share of each capital good in the firm's initial capital stock.

If one had observations on individual I_js and K_js, (18) could be used as the basis of an econometric model in which Q is regressed on the left-hand side variables, which are functions of the vector (I, K). Given a time-series for all relevant variables, one could estimate the parameters of the adjustment cost function(s), in either of the two specifications we have permitted (i.e., "lost output" or "lost capital").

As an example, one might suppose that adjustment costs take the form of wasted capital, and that each Φ_j is a quadratic:

$$\Phi_j(I_j, K_j) = (\alpha_j/2)(I_j/K_j)^2, \text{ all } j.$$

Then, by (18), one has

$$(19) \qquad Q = \Sigma_j \alpha_j \omega_j \left(I_j / K_j^2 \right)$$

which, if formulated as a linear regression model, would yield estimates of the parameters α_j. It would of course be possible (and desirable) to consider many other specifications as well.

C. *Investment Equations*

As we have just seen, the q theory can be used to formulate equations for the estimation of adjustment cost parameters with many capital goods as well as with a single capital good. The parallel between the two cases is incomplete, however, because while (12) can be inverted to give the rate of investment as a function of Q, this is not possible in the many-good case. That is, the value of Q does not, in general, uniquely determine either the *rate* of investment in value terms, pI/pK, or the *level* of investment, pI. This conclusion is obvious from (18), since it is perfectly clear that the left-hand side is not, in general, equal to a monotonic function of pI/pK, as would be required for the derivation of the investment rate as a function of Q.

To see this negative conclusion in a different way, let us suppose that such an investment equation *can* be written, and ask what restrictions on the model are implied thereby. It is convenient to start with the case where adjustment costs take the form of wasted capital (i.e., $\Gamma \equiv 0$), since it is easier and more complete results are available. I hypothesize that

$$pI/pK = h(Q), \tag{20}$$

where $h' > 0$, that is, the investment rate depends only on Q. Using (18), this is equivalent to

$$pI/pK = h\left(\Sigma_j \phi_j'\left(I_j/K_j\right)\omega_j\right) \tag{21}$$

where I define $\phi_j(I_j/K_j) = \Phi_j(I_j/K_j, 1)$.

In general, any given total investment rate pI/pK can be the result of widely different levels of investment I_j in individual capital goods, depending on output prices, capital goods prices, variable factor prices, and initial capital stock. If (21) is to obtain in all of these situations, then it should hold identically in I, K, and p. Thus, differentiating with respect to I_j, we have (after cancellation of p_j/pK)

$$1 = h'(\cdot)\phi_j''(\cdot), \text{ all } j. \tag{22}$$

This implies that $\phi_j''(\cdot) = \phi_{j'}''(\cdot)$ for all j, j' and for all I_j, $I_{j'}$, which means that $\phi_j''(\cdot) = \alpha$, a constant independent of j, for all j. Using the fact that $\phi_j(0) = 0$ and $\phi_j(\cdot) \geq 0$, this condition can be integrated to yield

$$\Phi_j\left(I_j/K_j\right) = \frac{\alpha}{2}\left(I_j/K_j\right)^2, \text{ for all } j; \tag{23}$$

or

$$\Phi_j\left(I_j, K_j\right) = \alpha I_j^2/2K_j, \text{ all } j. \tag{23'}$$

In other words, it is possible to write an investment equation like (20), such that the rate of total investment depends uniquely on Q, only if the adjustment cost functions are quadratic and identical. By the same token, it easily follows from (23) and (18) that $Q = \alpha pI/pK$ in this case, so that the identical quadratic specification is sufficient, as well as necessary, for an investment equation like (20) to hold. Note that this implies that the function h is just the adjustment cost parameter, α.

This demonstration can be repeated, essentially intact, for the case where *total* investment pI is expressed as a function $h(Q \cdot pK)$, Q being corrected by pK in order to provide the necessary homogeneity property. Beginning, then, with the analogue of (21), namely,

$$pI = h\left(\Sigma_j \Phi_{I_j} p_j K_j\right), \tag{21'}$$

one derives (22) just as before. Thus:

PROPOSITION 1: *Suppose adjustment costs take the form of wasted capital goods. Then the total investment rate (or total investment) can be expressed as a function of Q (or QpK) if and only if each adjustment cost function is identical and quadratic. In addition, the total investment rate (or total investment) must be linear in Q (or in QpK).*

Now let us consider the case where adjustment costs take the form of lost output. We again suppose (20) holds for all values of I, K, and p, and use (18) to express this relation as

$$pI/pK - h\left(\Sigma_j p_0 \Gamma_{I_j}\omega_j/p_j\right) = 0. \tag{24}$$

First, differentiating (24) with respect to p_j, yields

$$p_{j'}I_{j'} = \left[h(\hat{Q}) - h'(\hat{Q})\hat{Q}\right] p_{j'}K_{j'}. \tag{25}$$

Summing over j' and using (20), we find that

$$h(Q) = h(Q) - h'(Q)Q, \tag{26}$$

which cannot hold, given $h' > 0$. Thus, (20) cannot possibly be valid unless all capital good prices are fixed. To draw out the remaining implications of (20), I now make this assumption:

Then (20) *holds if and only if* (24) *holds identically in* (I, K). *By the implicit function theorem, this can occur only if the derivative of*

$pI/pK - h(\Sigma_j p_0(\partial\Gamma/\partial I_j)K_j\omega_j/p_j)$ with respect to every I_j and K_j, is zero. For if this condition is not met, it would be possible to solve for, say, I_1 in terms of $(I_2,\dots,I_m, K)$, which is to say that (24) *would be valid only for some unique value of I_1. Clearly this condition that all $2m$ derivatives of the left-hand side of* (24) *be zero will not, in general, be met.*

The economic and mathematical content of this restriction are given in

PROPOSITION 2: *Suppose adjustment costs take the form of wasted output. Then the total investment rate can be written as a function of Q if and only if* (a) *the relative prices of all capital goods are fixed, and* (b) *the weighted average marginal adjustment cost per dollar's worth of investment in each capital good (where the weights are each good's value share in the initial capital stock) must depend only on the rate of total investment independently of the composition of investment and of the initial capital stock. Condition* (b) *holds if and only if $\Gamma(I,K)$ satisfies*

$$(27)\quad \Sigma_j p_0\Gamma_{I_jI_{j'}}\omega_j/p_jp_{j'} = \frac{\Sigma_j \dfrac{p_0\Gamma_{I_j}\omega_j}{p_jpK} - \dfrac{p_0\Gamma_{I_{j''}}}{p_{j''}pK} - \Sigma_j\dfrac{p_0\Gamma_{I_jK_{j''}}\omega_j}{p_jp_{j''}}}{pI/pK}$$

for all $j', j'' = 1,\dots,m$ and for all (I,K).

The verbal statement of condition (b) in this result follows directly from the interpretation of Q given after equation (18). The formal statement of (b) in equation (27) follows from differentiation of (24) with respect to each I_j, K_j, setting every derivative equal to zero and eliminating h'.

From its verbal statement alone it is evident that not all adjustment cost functions meet condition (b). Similarly, (27) is a system of $2m-1$ equations, none of which would necessarily be satisfied by a general linear homogeneous adjustment cost function. Indeed, the natural question to ask is whether *any* adjustment cost function can satisfy condition (b). Only for the class of additively separable functions have I been able to discover a definitive answer to this question:

COROLLARY: *Suppose that adjustment costs take the form of lost output, that relative capital goods prices are fixed, and that the adjustment cost function is additively separable in I_j, K_j pairs: i.e., $\Gamma(I,K) = \Sigma_j\Gamma_j(I_j,K_j)$. Then condition* (b) *of Proposition 2 is met if and only if each Γ_j is a quadratic in I_j/K_j such that the schedule $p_0\Gamma_j/p_j$ of adjustment costs per dollar's worth of investment is identical for all capital goods j, and $h(Q)$ is linear. Formally,* (b) *holds if and only if there is some constant $\alpha > 0$ such that*

$$(28a)\quad \Gamma_j(I_j,K_j) = (\alpha p_j/2p_0)(I_j/K_j)^2$$

$$(28b)\quad h(Q) = Q/\alpha.$$

The proof of this result is easily patterned after the demonstration of Proposition 1, once it is realized that for fixed p_0 and p, the additively separable $\Gamma = \Sigma_j\Gamma_j$ is formally equivalent to the additively separable Φ_j functions considered in the "wasted capital" case. (It is easily checked that (28a) and (28b) satisfy (27), and conversely that (27) and additive separability imply (28a) and (28b).)

Thus there is definitely one case, albeit a quite special one, in which the total investment rate will be uniquely determined by tax-adjusted Q. Other similar special cases could presumably arise for Γ functions that are not additively separable. Any such Γ must, however, satisfy the $2m-1$ restrictions (27). The unambiguous conclusion must be, as stated earlier, that total investment in many capital goods is not, in general, uniquely determined by a q-type variable.

REFERENCES

Baily, Martin Neil, "Productivity and the Services of Capital and Labor," *Brookings Papers*, *1*:1981, 1–50.

Hayashi, Fumio, "Tobin's Marginal q and Average q: A Neoclassical Interpretation," *Econometrica*, January 1982, *50*, 213–24.

Malkiel, Burton G., von Furstenberg, George M. and Watson, Harry S., "Expectations, Tobin's q, and Industry Investment," *Journal of Finance*, May 1979, *34*, 549–61.

Salinger, Michael A. and Summers, Lawrence H., "Tax Reform and Corporate Investment: A Micro-econometric Simulation Study," Working Paper No. 757, National Bureau of Economic Research, 1981.

Summers, Lawrence H., "Taxation and Corporate Investment: A q-Theory Approach," *Brookings Papers*, *1*:1981, 67–127.

Tobin, James and Brainard, William C., "Asset Markets and the Cost of Capital," in Bela Belassa and Richard Nelson, eds., *Economic Progress, Private Values and Public Policy*, Amsterdam: North-Holland, 1977, 235–62.

von Furstenberg, George M., "Corporate Investment: Does Market Valuation Matter in the Aggregate?," *Brookings Papers*, *2*:1977, 347–97.

______, **Malkiel, Burton G. and Watson, Harry S.,** "The Distribution of Investment Between Industries: A Microeconomic Application of the 'q' Ratio," in G. M. von Furstenberg, ed., *Capital, Efficiency, and Growth*, Cambridge: Ballinger, 1980, 395–460.

Yoshikawa, Hiroshi, "On the 'q' Theory of Investment," *American Economic Review*, September 1980, *70*, 739–43.