Journal of Public Economics 25 (1984) 103–125. North-Holland

THE WELFARE EFFECTS OF INTERGOVERNMENTAL GRANTS IN AN ECONOMY WITH DISTORTIONARY LOCAL TAXES

A simple general equilibrium analysis

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Received September 1983, revised version received March 1984

This paper examines the welfare impact of intergovernmental transfers when recipient governments use distortionary taxes. Both lump-sum and matching grants are investigated. A 'distortionary factor' depending on local demand and supply elasticities and the local tax rate on the taxed commodity determines the real income change to a jurisdiction per dollar transferred. Matching grants are dominated by lump-sum grants when these can be set optimally for each recipient. If grant policy must be uniform, positive (or negative) matching rates are desired if equity-adjusted distortionary factors are positively (negatively) correlated with local public service levels.

1. Introduction

Intergovernmental grants are a major feature of federal systems in many countries, but relatively little is presently known about the nature or magnitude of their welfare consequences. In part, this is because it is difficult to predict the way that grants will influence recipient government tax and expenditure policy,[1] since such policies are determined in a sometimes intricate political process that is not very well understood.[2] Furthermore, even if recipient government responses to grant policy could be accurately predicted, the resulting effects on the allocation of resources and economic welfare are many and complex: local governments determine the levels of provision of certain public goods, may use distortionary taxes to raise

*The present version of this paper was written during a visit to the Department of Economics, Queens University; I thank the Department for its hospitality. I would also like to thank Sam Wilson and two anonymous referees for helpful comments on an earlier version. The support of National Science Foundation grants SOC 78-05195 and SOC 79-20648 is gratefully acknowledged.

[1]Let us ignore debt finance for simplicity.

[2]As examples of empirically-oriented studies that attempt to develop predictive models of recipient government response to lump-sum and matching grants, see, for example, Mieszkowski and Oakland (1979), Break (1980, ch. 3), and references therein. Also, the analysis of Romer and Rosenthal (1980) and Filimon *et al.* (1982) [building on Romer and Rosenthal (1979)] shows that recipient government response to grants is likely to be sensitive to the precise institutional mechanism by which expenditures are determined.

revenues, and may pursue policies which alter the spatial pattern of allocation of both human and non-human resources.[3] Thus, when recipient government policies respond to a change in grant policy, the overall pattern of resource allocation will be disturbed in many ways which are relevant for the determination of the final impact on economic welfare of the grant policy change.

The goal of this paper is to examine in detail one of the many ways that grants influence economic welfare. Since recipient governments use distortionary taxes, whether on income, sales, property, or other goods, a change in grant policy will cause the equilibrium in the tax-distorted markets to change as a result of grant-induced changes in recipient government tax rates (as well as for other reasons). Are these effects of grants on distorted markets likely to be important for welfare?

In section 2 we present a static general equilibrium model of an economy with local governments, each of which provides a single public good to its residents, and with a single central government that uses lump-sum tax revenues to pay out grants to cities. Each city uses a tax on a good which is variable in supply, called 'housing', together with grant funds, to finance its expenditures. For the sake of simplicity, this is the only market imperfection that we allow to enter the model, so that the implications of distortionary local taxes can be revealed most clearly.[4]

We consider two different hypotheses about the determination of local public expenditure levels. The first is that local public goods are provided in exogenously fixed amounts. This implies that any increase in transfers to a given locality is passed on to residents in the form of a housing tax rebate. The second hypothesis is that local public good levels are determined by a political process controlled by self-interested and well-informed voters. Equipped with this behavioral theory of local government decision-making, it is possible to study the welfare effects of intergovenmental grants when local expenditure levels are endogenous.

Section 3 applies this model to the evaluation of two kinds of intergovernmental transfers: lump-sum grants and matching grants.[5] In section 3.1 we examine how a small change in grant policy affects the welfare of a single locality's residents. A 'distortionary factor' that measures the marginal excess burden of local taxes is defined and used for this purpose. Section 3.2 turns to the overall evaluation of changes in grant policy. Holding total transfers

[3] For discussions of some of the welfare effects of local government policy, see, for example, Arnott and Stiglitz (1979), Arnott and Grieson (1981), Starrett (1980a, 1980b), Wildasin (1980), Gordon (1983), and references therein.

[4] For some discussion of grant policy with inefficient locational choice, see Boadway and Flatters (1982) and Wildasin (1983). Sheshinski (1977) also examines intergovernmental grants with distortionary recipient taxes, but focuses more on positive rather than normative issues, and gives less attention to interjurisdictional differentials in tax distortions than we do.

[5] The analysis of a tax-effort-based grant formula would be similar to (but more complex than) that for a matching grant.

to all jurisdictions constant, changes in the structure of grant policy that transfer resources from communities where (equity-adjusted) distortionary factors are low to communities where they are high are welfare-enhancing. It is shown that matching grants are welfare-reducing if an ideal system of lump-sum grants, with grants varying by locality, is feasible. However, if the grant structure must be uniform, at least small positive (negative) matching rates will be desirable if distortionary factors and local public expenditure levels are positively (negatively) correlated across localities. Section 3.3 reports illustrative empirical estimates of the distortionary factor in order to assess the possible relevance of local tax distortions for policy evaluation and to determine the sensitivity of the distortionary factor to various parameters.

In section 4 we show how the same basic approach can be applied where both recipient and donor governments use (wage) income taxes, the new complication here being the central government's use of a distortionary tax. Relevant amendments to the distortionary factor and illustrative calculations are presented. In this section we also examine changes in the scale of the grant program, where central government tax increases are used to finance a larger total transfer to recipients. It is shown (subject to some qualifications) that this will be welfare-enhancing if (equity-adjusted) distortionary factors are negatively correlated with jurisdictional tax base. Finally, section 5 concludes with some discussion of directions for further research.

2. A model of politico-economic equilibrium

Let us assume that there are A localities (say cities), $\alpha, \beta, = 1, \ldots, A$, each with exogenously fixed spatial boundaries and each thus containing a fixed amount of land $\bar{l}_\alpha$. There are two types of private goods in the economy other than land: an all-purpose good and housing. The all-purpose good is directly consumable by households, is used along with land to produce housing in each city, and is used as the sole input in the provision of local public goods.

2.1. The housing and land sectors

Housing in each city is produced by profit-maximizing competitive firms according to a constant returns to scale technology using all-purpose good and land as inputs. Land has no use other than as housing input and is inelastically supplied. Letting $y_{f\alpha}$ and $x_{f\alpha}$ be housing production and all-purpose input, we have $y_{f\alpha} = \Phi_{f\alpha}(x_{f\alpha}, \bar{l}_\alpha) = \phi_{f\alpha}(x_{f\alpha})$, where $\Phi_{f\alpha}$ is the underlying constant returns production function and $\phi_{f\alpha}$, incorporating the land constraint, exhibits decreasing returns:

$$\phi'_{f\alpha} > 0 > \phi''_\alpha.$$

If we let p_α denote the price of housing in city α and take the all-purpose good as a numeraire, we can write the condition for profit-maximizing choice of $x_{f\alpha}$ as $p_\alpha \phi'_{f\alpha} = 1$, which determines $x_{f\alpha}$, and hence $y_{f\alpha}$, as a function of the output price p_α. We will denote the derivative of this supply function as $y'_{f\alpha}$ and its elasticity $\varepsilon_{f\alpha}$, both positive given $\phi''_{f\alpha} < 0$. The zero-profit equilibrium condition determines equilibrium land rents, denoted $\pi_{f\alpha}$ for city α, such that $\pi_{f\alpha} = p_\alpha y_{f\alpha} - x_{f\alpha}$. By the envelope theorem, $\partial \pi_{f\alpha} / \partial p_\alpha = y_{f\alpha}$.

2.2. Households

Each city α contains only one resident (or many identical residents), consuming a bundle $(x_{h\alpha}, y_{h\alpha}, z_\alpha)$ of all-purpose good, housing, and local public good. The household has an endowment $\bar{x}_{h\alpha}$ of the all-purpose good, and owns the land in its own city, collecting the land rents of $\pi_{f\alpha}$. Land itself is not directly subject to taxation, but taxes are assessed on housing at a rate t_α per unit.[6] The household must also pay a federal head tax of τ. It therefore faces a budget constraint

$$x_{h\alpha} + (p_\alpha + t_\alpha) y_{h\alpha} = \bar{x}_{h\alpha} + \pi_{f\alpha} - \tau \tag{1}$$

and, taking z_α as given, chooses $(x_{h\alpha}, y_{h\alpha})$ to maximize utility, yielding an indirect utility function $v_{h\alpha}(p_\alpha + t_\alpha, z_\alpha, \bar{x}_{h\alpha} + \pi_{f\alpha} - \tau)$, and demand functions $x_{h\alpha}(\cdot)$ and $y_{h\alpha}(\cdot)$ with the same arguments. We denote the derivatives of $v_{h\alpha}$ and $y_{h\alpha}$ by subscripts p, z, and I.

2.3. The public sector

There are two levels of govenment in the economy, localities and the federal government. The only role of the latter is to collect head taxes and distribute them to localities either in lump-sum or matching form. Formally, if L_α is the grant received by city α, we have

$$L_\alpha = L_{\alpha 0} + L_{\alpha 1} z_1, \tag{2}$$

where $L_{\alpha 0}$ is the lump-sum transfer to city α and $L_{\alpha 1}$ is the transfer per unit of public good implied by the matching rate for city α. (The grant formula might be subject to side constraints requiring that $L_{\alpha 0}$ or $L_{\alpha 1}$ be the same for all cities — that is, the program may be specified to be uniform across cities.)

[6]A land tax would be non-distortionary, and the central issue to be investigated in this paper would disappear if we permitted one. For an analysis of grant policy with non-distortionary local taxes, see Wildasin (1983). The predominant local tax in the United States, the property tax, is in the nature of the housing tax discussed here rather than a tax on land per se.

The federal government's budget constraint is

$$\Sigma_a L_\alpha - A\tau = 0. \tag{3}$$

τ, and thus the size of the federal budget (and the total amount of grants), is exogenously fixed in this and the next section.

Local governments provide local public goods, using $C_\alpha(z_\alpha)$ of the all-purpose good as an input to produce z_α. The localities finance expenditures from taxes on housing and from grants, so as to satisfy the budget constraint

$$t_\alpha y_{h\alpha} + L_\alpha - C_\alpha = 0. \tag{4}$$

2.4. *Market equilibrium*

Conditional on federal and local government policy, an equilibrium housing price is determined in each locality such that

$$y_{h\alpha} - y_{f\alpha} = 0. \tag{5}$$

Eqs. (4) and (5) provide $2A$ equations which can be used to solve for the t_α's and p_α's in terms of the parameters of the system: the z_α's, $L_{\alpha 0}$'s and $L_{\alpha 1}$'s.[7] The partial derivatives of t_α with respect to $L_{\alpha 0}$ and $L_{\alpha 1}$ are needed later and are computed in appendix A.

2.5. *Public sector equilibrium*

As one special case, we assume that the levels of local public good provision are exogenously fixed. In this case, grants can only affect local taxes, and the relationship between intergovernmental transfers and distortionary local taxes is thus exposed in its purest form. However, the stated purpose of many intergovernmental transfers is precisely to induce changes in local government expenditure. Furthermore, the economic distinction between lump-sum and matching grants disappears once local public spending is assumed to be fixed, and we cannot hope to say anything about the choice between them under this assumption.

Thus, we also consider the case where voters in any city α know the system (4) and (5), and choose z_α to maximize utility $v_{h\alpha}(\cdot)$ accordingly. If we assume a regular maximum, the locally-optimal level of z_α must satisfy $\mathrm{d}v_{h\alpha}/\mathrm{d}z_\alpha = 0 > \mathrm{d}^2 v_{h\alpha}/\mathrm{d}z_\alpha^2$, where these are *total* derivatives. Now, using the fact that $v_{h\alpha z} v_{h\alpha I}^{-1} = MRS_{h\alpha}$ is the marginal rate of substitution between the public

[7]When the t_α's and p_α's satisfy (4) and (5) for all α and when the federal government and household budget constraints (3) and (1) are satisfied, the economy-wide all-purpose good market is also in equilibrium.

and all-purpose goods, and using Roy's identity $-v_{h\alpha p}\, v_{h\alpha I}^{-1} = y_{h\alpha}$, the first-order necessary condition implies:

$$v_{h\alpha I}^{-1}\frac{\mathrm{d}v_{h\alpha}}{\mathrm{d}z_\alpha} = MRS_{h\alpha} + \frac{v_{h\alpha p}}{v_{h\alpha I}}\frac{\partial(p_\alpha + t_\alpha)}{\partial z_\alpha} + \frac{\partial \pi_{f\alpha}}{\partial p_\alpha}\frac{\partial p_\alpha}{\partial z_\alpha}$$

$$= MRS_{h\alpha} - y_{h\alpha}\frac{\partial t_\alpha}{\partial z_\alpha} = 0. \tag{6}$$

Although a detailed calculation is not necessary for our purposes, $\partial t_\alpha/\partial z_\alpha$ can be computed from (4) and (5). It is interesting to observe that the expression $y_{h\alpha}(\partial t_\alpha/\partial z_\alpha)$ does not equal C'_α, and (6) therefore does not reduce to the standard Samuelson formula. Both the presence of matching grants ($L_{\alpha 1} \neq 0$) and the fact that the local housing tax is distortionary complicate the optimal public good formula.[8]

3. Evaluation of grant structure

With the above model, it is now straightforward to see how a change in grant policy affects welfare. As noted, we assume that the size of the federal budget is fixed, so that the total amount of grants paid out in either lump-sum or matching form, the *scale* of the program, must be fixed. We can, however, consider variations in the amounts paid to individual cities, or in the mix between lump-sum and matching grants; that is, we can evaluate different *structures*.[9]

3.1. *Real income change for an individual locality*

We first study the impact on households in city α of a change in one of the grant parameters facing them, $L_{\alpha i}$. This affects welfare through p_α, t_α, and

[8]It may be of independent interest to note that (6) can be expressed more explicitly as

$$MRS_{h\alpha} = D_\alpha\left[(C'_\alpha - L_{\alpha 1}) - \frac{t_\alpha}{p_\alpha} y_{h\alpha z}\varepsilon_{f\alpha}\left(\frac{\varepsilon^*_{h\alpha}}{p_\alpha + t_\alpha} - \frac{\varepsilon_{f\alpha}}{p_\alpha}\right)^{-1}\right],$$

where $\varepsilon^*_{h\alpha}$ is the compensated demand elasticity for housing and where D_α is defined in (9) below. Formulae like this one have not appeared in the literature so far because other studies have assumed either that the tax structure is optimal [e.g. Stiglitz and Dasgupta (1971), Atkinson and Stern (1974)] or that producer prices are fixed [e.g. Browning (1976), Wildasin (1979), (1984)]. Here the local tax structure is not optimal (since land rents are not taxed), while producer prices, namely the prices of housing and land, are variable. As a consequence, the elasticity of supply must enter the formula.

[9]Focusing on structure with the scale held fixed is of interest both because it is an important policy problem in its own right and because it is analytically useful to be able to ignore the effects of changes in federal government taxation. In section 4 the effects of changes in program scale will be examined in the simple case where both donor and recipient governments use the same tax base.

$\pi_{f\alpha}$, all of which depend on $L_{\alpha i}$, and also through z_α which depends on $L_{\alpha i}$ in the case where z_α is chosen optimally (from city α's perspective). Let $\partial z_\alpha/\partial L_{\alpha i}$ show how z_α depends on $L_{\alpha i}$ in general, with the understanding that $\partial z_\alpha/\partial L_{\alpha i} \equiv 0$ in the case where z_α is exogenous. Then, totally differentiating $v_{h\alpha}$ with respect to $L_{\alpha i}$, the household's real income change is

$$v_{h\alpha I}^{-1}\frac{\mathrm{d}v_{h\alpha}}{\mathrm{d}L_{\alpha i}}=\left[MRS_{h\alpha}-y_{h\alpha}\frac{\partial t_\alpha}{\partial z_\alpha}\right]\frac{\partial z_\alpha}{\partial L_{\alpha i}}-y_{h\alpha}\frac{\partial t_\alpha}{\partial L_{\alpha i}}=-y_{h\alpha}\frac{\partial t_\alpha}{\partial L_{\alpha i}}. \tag{7}$$

The last equality follows either from $\partial z_\alpha/\partial L_{\alpha i}=0$ (z_α exogenous) or from (6).

We now define the partial derivative of L_α with respect to $L_{\alpha i}$, holding z_α constant, as $\partial L_\alpha/\partial L_{\alpha i}=1$ (if $i=0$) or $=z_\alpha$ (if $i=1$). The *total* derivative is $\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha i}=\partial L_\alpha/\partial L_{\alpha i}+L_{\alpha 1}(\partial z_\alpha/\partial L_{\alpha i})$, which takes into account any grant-induced change in local public expenditures. Then, as shown in appendix A, (7) can be written as

$$v_{h\alpha I}^{-1}\frac{\mathrm{d}v_{h\alpha}}{\mathrm{d}L_{\alpha i}}=D_\alpha\frac{\partial L_\alpha}{\partial L_{\alpha i}}=D_\alpha\left(\frac{\mathrm{d}L_\alpha}{\mathrm{d}L_{\alpha i}}-L_{\alpha 1}\frac{\partial z_\alpha}{\partial L_{\alpha i}}\right), \tag{8}$$

where

$$D_\alpha=\left[1-\frac{(t_\alpha/p_\alpha)\varepsilon_{h\alpha}\varepsilon_{f\alpha}}{\varepsilon_{h\alpha}^{*}-(1+t_\alpha/p_\alpha)\varepsilon_{f\alpha}}\right]^{-1}, \tag{9}$$

in which $\varepsilon_{h\alpha}$ and $\varepsilon_{h\alpha}^{*}$ are the uncompensated and compensated price elasticities of demand for housing, respectively.

Eq. (8) is a central result which is most easily interpreted in the special case where z_α is exogenously fixed. $\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha i}$ is the total change in grant funds received per unit change in $L_{\alpha i}$, so that dividing (8) through by $\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha i}$ shows that D_α is the increase in the real income of the household in city α per grant dollar transferred. Appendix A shows that $D_\alpha \geq 1$. In particular, if housing is either perfectly inelastically demanded ($\varepsilon_{h\alpha}=0$) or supplied ($\varepsilon_{f\alpha}=0$), $D_\alpha=1$: a dollar transferred to the local public sector is equivalent in its welfare impact to a dollar transferred in lump-sum form directly to the household. If, however, demand and supply are both not perfectly fixed, $D_\alpha>1$: with a distortionary local tax, each dollar collected imposes a real cost of more than one dollar on the private sector; hence, each dollar rebated through tax cuts generates a real income gain of more than one dollar. In fact, $D_\alpha-1$ is just the marginal excess burden of the local tax. Notice that D_α will be higher, the greater the tax wedge t_α between the demand and supply prices for housing, and the greater the elasticities of supply and demand for housing. We henceforth refer to D_α as a 'distortionary factor'.

Now turning to the more interesting case where z_α is endogenous, note that the interpretation just given goes through unchanged provided that $L_{\alpha 1}$

$=0$. The reason is that z_α has been optimized, and with no matching grant wedge between the marginal benefit and cost of the local public good, a grant-induced change in z_α has no first-order welfare effect.[10] If $L_{\alpha 1} \neq 0$, the interpretation of (8) must be modified to take into account the welfare effect of a change in the grant-distorted level of the local public good.

3.2. *Overall evaluation of grant policy*

By (8), for any given policy change, we can in principle determine the associated change in real income for each locality. To go beyond this and make statements about the overall impact of grants requires some implicit or explicit basis for comparing real income changes for various households. We shall be explicit, and postulate a social welfare function $W(v_{h1}, \ldots, v_{hA})$ defined over the vector of utility levels. We can now evaluate feasible grant policy changes, i.e. those satisfying (3).

The simplest case to consider is that in which grant policy can be *individualized*, that is, in which the grant structure parameters $L_{\alpha i}$ can vary by city, and in which the level of local public expenditure in each city is exogenously fixed. In this case, it is trivial that matching grants are redundant instruments, and the only policy problem is to decide whether to increase $L_{\alpha 0}$, while reducing $L_{\beta 0}$ for some $\beta \neq \alpha$. Letting W_α denote $\partial W / \partial v_{h\alpha}$ and using (8), this policy change is desirable or not according as

$$W_\alpha \frac{\mathrm{d}v_{h\alpha}}{\mathrm{d}L_{\alpha 0}} - W_\beta \frac{\mathrm{d}v_{h\beta}}{\mathrm{d}L_{\beta 0}} = W_\alpha v_{h\alpha I} D_\alpha - W_\beta v_{h\beta I} D_\beta \gtrless 0. \tag{10}$$

If grant policy were chosen optimally, of course, W would be stationary so that the expression in (10) would equal zero. More generally, a marginal welfare improvement occurs if the *equity-weighted* distortionary factor for city α, $W_\alpha v_{h\alpha I} D_\alpha$, is greater than that for city β. Intuitively, D_α is the real income change for households in α per dollar transferred to city α, and $W_\alpha v_{h\alpha I} D_\alpha$ is its social valuation; grant funds should be redistributed toward cities for which this social benefit is high, and away from cities for which it is low. In the special case where a dollar's real income change is equally socially valuable for all cities ($W_\alpha v_{h\alpha I} = W_\beta v_{h\beta I}$, all α, β), only the unadjusted distortionary factors (the D_α's) are involved in the evaluation of grant policy, and an optimum, in this special case, would require distortionary factors to be equalized across cities.

The analysis of individualized grant policy is also of interest when local public expenditures are determined in a locally-optimizing manner. In this

[10]With a complicated political model, there would be no presumption that local public expenditure levels would be set in a locally-optimal fashion. In this case, the welfare effects of a marginal change in z_α would appear in the bracketed term in the penultimate equality in (7).

case, lump-sum and matching grants can meaningfully be compared. Suppose we consider a change in the grant structure for city α alone, with grants to every other city held fixed; in particular, consider an increase in lump-sum transfers to city α, offset by a reduction in the matching rate such that the total amount of grants paid to city α, L_α, remains constant. The utility of the household in α (the only household whose welfare is affected by this policy change) will increase or decrease as

$$\frac{D_\alpha\,\partial L_\alpha/\partial L_{\alpha 0}}{\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha 0}}-\frac{D_\alpha\,\partial L_\alpha/\partial L_{\alpha 1}}{\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha 1}}$$

$$=D_\alpha\left[\frac{\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha 0}-L_{\alpha 1}(\partial z_\alpha/\partial L_{\alpha 0})}{\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha 0}}-\frac{\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha 1}-L_{\alpha 1}(\partial z_\alpha/\partial L_{\alpha 1})}{\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha 1}}\right]$$

$$=D_\alpha L_{\alpha 1}\left[\frac{\partial z_\alpha/\partial L_{\alpha 1}}{\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha 1}}-\frac{\partial z_\alpha/\partial L_{\alpha 0}}{\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha 0}}\right]\gtrless 0. \tag{11}$$

Since a dollar increase in the total transfer to city α brought about by an increase in the matching rate increases local public expenditure by more than a dollar increase in the total transfer achieved by an increase in the lump-sum grant,[11] it follows that welfare decreases with a shift toward matching grants if $L_{\alpha 1}>0$. Notice that welfare is stationary when $L_{\alpha 1}=0$, and increasing in $L_{\alpha 1}$ if $L_{\alpha 1}<0$. In short, when $L_{\alpha 1}\neq 0$, it is incrementally welfare-enhancing to move the matching rate toward zero. This is a strong but plausible conclusion: when individualized grants are possible, matching provisions have no place in the optimal grant structure.

If we now assume that all matching rates are zero ($L_{\alpha 1}=0$), it is apparent from (11) that the equity-adjusted distortionary factors are the crucial determinants of the welfare effects of feasible changes in the distribution of lump-sum transfers among localities. The endogeneity of local spending does not alter the criterion presented in (10) for the case where the z_α's are fixed.

Finally, we consider non-individualized or *uniform* grant structures such that $L_{\alpha 0}=L_0$ and $L_{\alpha 1}=L_1$ for all α. With only two grant parameters, there is only one policy issue: should (say) L_1 be increased and L_0 reduced? This will be welfare-increasing or not as

$$\frac{\Sigma_\alpha W_\alpha\,\mathrm{d}v_{h\alpha}/\mathrm{d}L_1}{\Sigma_\beta\,\mathrm{d}L_\beta/\mathrm{d}L_1}-\frac{\Sigma_\alpha W_\alpha\,\mathrm{d}v_{h\alpha}/\mathrm{d}L_0}{\Sigma_\beta\,\mathrm{d}L_\beta/\mathrm{d}L_0}=\Sigma_\alpha W_\alpha v_{h\alpha I}D_\alpha\left[\frac{z_\alpha}{\Sigma_\beta\,\mathrm{d}L_\beta/\mathrm{d}L_1}-\frac{1}{\Sigma_\beta\,\mathrm{d}L_\beta/\mathrm{d}L_0}\right]\gtrless 0, \tag{12}$$

[11]This can be demonstrated using (6) and the properties of the system (4) and (5). Details are given in appendix B.

using (8). Clearly, both equity judgements and tax distortions are important here, but little more can be said in the general case. Suppose, however, we imagine an initial situation with no matching grants, i.e. $L_1 = 0$, and examine whether welfare is locally stationary, increasing, or decreasing in L_1 at this point. Then, using $\mathrm{d}L_\alpha/\mathrm{d}L_{\alpha i} = \partial L_\alpha/\partial L_{\alpha i}$, multiplying (12) through by $\Sigma_\beta z_\beta (>0)$, and defining $\bar{z}$, $\bar{\Omega}$, and $\bar{D}$ as the averages of the z_α, $W_\alpha v_{h\alpha I} D_\alpha$, and D_α terms, respectively, we obtain that welfare rises with L_i if:[12]

$$\Sigma_\alpha (W_\alpha v_{h\alpha I} D_\alpha - \bar{\Omega})(z_\alpha - \bar{z}) = \mathrm{cov}\,(W_\alpha v_{h\alpha I} D_\alpha, z_\alpha) > 0, \tag{13}$$

or, in the special case where the welfare weights $W_\alpha v_{h\alpha I}$ are all equal:

$$\Sigma_\alpha (D_\alpha - \bar{D})(z_\alpha - \bar{z}) = \mathrm{cov}\,(D_\alpha, z_\alpha) > 0. \tag{14}$$

Thus, if welfare weights can be ignored, at least a small positive (negative) matching rate would be desirable if distortionary factors and levels of local public good provision are positively (negatively) correlated.

Intuitively, under a uniform grant policy a non-zero matching rate may promote the transfer of revenue toward cities that have relatively high equity-adjusted distortionary factors, depending on the correlation between these factors and the 'handle' on which matching grants can operate, the z_α's. Of course, the sign and size of the covariances in (13) and (14) are based on empirical magnitudes and welfare judgements, and cannot be determined here. However, we may consider several possible cases where these covariances can be signed.

(i) Suppose that all cities are identical except insofar as public expenditure levels vary because of differences in tastes for local public goods. Then, if the matching rate is zero and lump-sum grants are uniform, cities with higher expenditure levels can be expected to have higher local tax rates, which, by (9), tends to make D_α larger: the z_α's and D_α's would then be positively correlated.

(ii) Suppose localities are alike in all respects except population, and the local public goods are either (a) purely public or (b) highly congestible. In case (a), cities with larger populations will face lower tax-prices for the public good, and, if C'_α is constant, will have lower (higher) tax rates and hence distortionary factors if the demand for the public good is inelastic (elastic). In the inelastic case, D_α and z_α will be inversely correlated, favoring a matching 'tax', i.e. a grant with a negative matching rate, and conversely in the elastic case. In case (b), if costs of public service provision are directly proportional to population, D_α and z_α would be the same for all cities, and so zero

[12]These results bear a striking formal resemblance to formulae characterizing an optimal commodity tax structure in a many-person economy. See, for example, Atkinson and Stiglitz (1980, lecture 12), and references therein.

matching rates would be desired. If costs rise more rapidly than population, larger cities would have higher tax rates and lower z_α's, again favoring a negative matching rate. (Note that z_α is interpreted as the *service level* enjoyed by residents, not the amount of expenditure in the city.)

(iii) Suppose localities are alike in all respects except initial endowments, $\bar{x}_{h\alpha}$. If local public goods and housing are normal goods, both z_α and $y_{h\alpha}$ will tend to be higher in higher-income communities. Depending on their relative income elasticities, it would be possible for D_α and z_α to be either negatively or positively correlated. Of course, if the social welfare function attaches a higher welfare weight to lower-income localities, the equity-adjusted distortionary factors will be increased for poor relative to rich jurisdictions. If z_α is not very income-elastic, D_α and z_α will be inversely correlated. Equity considerations would presumably magnify this negative correlation, and matching taxes would be called for. If, on the other hand, z_α is highly income elastic, and housing is relatively inelastic, D_α and z_α will be positively correlated, suggesting a positive matching rate. Equity considerations would dampen or conceivably overturn this result, however.

These examples show that we cannot say a priori whether the existence of local tax distortions favor matching grants or not. They all support the general theoretical conclusion, however, that tax distortions can be relevant for the evaluation of grant policy. We now consider the possible size and variation of the distortionary factor that might be observed empirically.

3.3. *Illustrative empirical estimates*

In order to see whether local property tax distortions might be sufficiently empirically important to take into account in practice, we now present some illustrative calculations of the distortionary factor based on U.S. data. Since it is the *variation* in the value of the distortionary factor across cities which is important for grant policy, our objective is to get a feeling for the range of values that this factor might take.

Of course, D_α can vary across cities for several reasons. Perhaps most importantly, effective property tax rates vary quite widely across U.S. cities. We consider two rates, a low of 1 percent and a high of 5 percent. Assuming an annual rent to house value ratio of 10 percent, these correspond to 10 percent and 50 percent taxes on annual rental value.[13]

One must have some estimate of the demand and supply elasticities for housing in order to compute D_α. For housing supply elasticities, estimates

[13]In cities with populations of 100000 and over in 1976, the median effective tax rate was about 2 percent, with roughly 10 percent of single family homes facing levies at a rate of 1 percent or less and about 10 percent facing rates in excess of 3 percent [U.S. Department of Commerce (1978, table 5, p. 27)]. During this period, about 40 percent of local government expenditure was financed by transfers from the federal and state governments. If such transfers had not existed, rates as high as 5 percent might have been observed for significant numbers of cities.

range from around 0.5 [De Leeuw and Ekanem (1971)], to 2 [Grieson (1973, 1974)] to over 3 [Smith (1976)]; we shall consider values of 0.5, 2, and 3. Ordinary demand elasticities range from about -0.15 to over -1 [see Mayo (1981, table 1)]; the Polinsky and Elwood (1979) estimate of -0.7 is probably somewhere near the median. We shall consider values of -0.3 and -1. Finally, to compute the compensated demand elasticity for housing, we use the formula $\varepsilon^*_{h\alpha} = \varepsilon_{h\alpha} + \sigma\eta_{h\alpha}$, where σ is the share of income expended on housing and $\eta_{h\alpha}$ is the income elasticity of demand for housing. We take $\sigma = 0.3$, and consider values of the income elasticity of 0.3 and 1. This brackets the most widely reported estimates, which center on about 0.5–0.7 [again, see Mayo (1981, table 1)].

Table 1 presents estimates of the distortionary factor for these parameter values, showing considerable variation, from 1.02 to 1.41. The most interesting comparisons, for our purposes, are between rows 1 and 4, 2 and 5, and 3 and 6, since such comparisons hold preferences and technology fixed and allow tax rates to vary. When the demand elasticities are low, as in the first two columns, D_α varies by 6–8 percentage points as the tax rate rises from 10 to 50 percent. It is interesting to observe from the first two columns of table 1 that the supply elasticity of housing is virtually insignificant in determining the value of D_α when the demand elasticity is low.

Table 1
Estimates of the distortionary factor D for alternative parameter values.[a]

		$\varepsilon_{h\alpha} = -0.3$		$\varepsilon_{h\alpha} = -1.0$	
		$\eta_{h\alpha} = 0.3$	$\eta_{h\alpha} = 1.0$	$\eta_{h\alpha} = 0.3$	$\eta_{h\alpha} = 1.0$
$\frac{t_\alpha}{p_\alpha} = 0.1$	$\varepsilon_{f\alpha} = 0.5$	1.02	1.03	1.04	1.04
	$\varepsilon_{f\alpha} = 2.0$	1.02	1.03	1.07	1.07
	$\varepsilon_{f\alpha} = 3.0$	1.03	1.03	1.08	1.08
$\frac{t_\alpha}{p_\alpha} = 0.5$	$\varepsilon_{f\alpha} = 0.5$	1.08	1.11	1.18	1.21
	$\varepsilon_{f\alpha} = 2.0$	1.10	1.11	1.34	1.37
	$\varepsilon_{f\alpha} = 3.0$	1.11	1.11	1.38	1.41

[a]*Source*: Author's computation, as explained in text.

Now consider the last two columns in the table. As expected, the size of the distortionary factor is greater in this case, to an extent that depends positively and significantly on the tax rate and the housing supply elasticity. The range in D_α between low and high tax cities, for given tastes and technology, is also now much greater. As the tax rate rises from 10 to 50 percent, D_α rises by at least 14 percentage points (from 1.04 to 1.18), and by as much as 33 percentage points (1.08 to 1.41). If observed in practice, these variations would certainly be important in grant design.

In general, table 1 reveals that tax rates and demand and supply elasticities can all be important in determining the value of the distortionary factor, at least in some cases, and accurate estimates of each would be important if one sought to determine the D_a's for policy purposes. Least important for this exercise is the determination of the income elasticity of demand for housing.

4. The case of recipient and donor government income taxation

In this section we outline an application of our earlier method of analysis to the case where recipient governments use labor income taxes to finance their expenditures. This could loosely describe states in the U.S. context, or provinces in the Canadian.[14] In such an application, however, we must acknowledge that income is also the source of federal government taxes, and therefore must allow for both distortionary recipient and distortionary donor taxes.

First we sketch the model. Suppose an all-purpose good is produced in each locality (state) via a CRS technology using local labor and land as inputs. Residents own all local land. Letting $y_{f\alpha}$ denote local output, $x_{f\alpha}$ the amount of labor hired, w_α the local wage (in terms of all-purpose good), and $\pi_{f\alpha}$ local land rents, we have the zero profit condition $\pi_{f\alpha}=y_{f\alpha}-w_\alpha x_{f\alpha}$. If the local production function is $y_{f\alpha}=\phi_{f\alpha}(x_{f\alpha})$ (subsuming land), $x_{f\alpha}$ is chosen such that $\phi'_{f\alpha}=w_\alpha$, and hence $\mathrm{d}x_\alpha/\mathrm{d}w_\alpha=(\phi''_{f\alpha})^{-1}<0$. Of course, $\partial\pi_{f\alpha}/\partial w_\alpha=-x_{f\alpha}$.

Assume households have budget constraints

$$y_{h\alpha}=(w_\alpha-\tau_\alpha-\tau)x_{h\alpha}+\pi_{f\alpha}, \tag{15}$$

where $y_{h\alpha}$ is all purpose good consumption, $x_{h\alpha}$ is labor supplied, and τ_α and τ are state and federal income taxes (expressed in per-unit terms). From the underlying preferences and (15) one derives the indirect utility function $v_{h\alpha}(w_\alpha-\tau_\alpha-\tau, z_\alpha, \pi_{f\alpha})$, and a supply function for labor with the same arguments. (Subscripts denote derivatives.)

The local labor market clears when

$$x_{f\alpha}-x_{h\alpha}=0, \tag{16.1}$$

[14]Actually, the model above is too simple to apply to the analysis of federal transfers to states. States tax income from both labor and capital as well as sales (not to mention several other revenue sources). One should therefore develop a model with several taxed goods. Moreover, at least when discussing capital, allowance should be made for mobility of the taxed good. This goes beyond the scope of the present discussion, however.

and the local budget is balanced when

$$\tau_\alpha x_{h\alpha}-(C_\alpha-L_\alpha)=0, \tag{16.2}$$

while the federal budget requires

$$\tau\Sigma_\alpha x_{h\alpha}-\Sigma_\alpha L_\alpha=0. \tag{17}$$

From (16) one can solve for (τ_α, w_α) in terms of $(z_\alpha, L_\alpha, \tau)$. It is then possible to compute the effect of a change in grant policy on utility. Whether local public expenditures are fixed optimally or not, (8) describes the real income change resulting from a change in $L_{\alpha i}$. However, the relevant distortionary factor is now (see appendix C for this and other derivations)

$$D_\alpha=\left[1+\frac{(\tau_\alpha/w_\alpha)E_{h\alpha}E_{f\alpha}}{E^*_{h\alpha}-[1-(\tau_\alpha+\tau)/w_\alpha]E_{f\alpha}}\right]^{-1}, \tag{18}$$

where $E_{h\alpha}$ and $E^*_{h\alpha}$ are the ordinary and compensated supply elasticities for labor, and $E_{f\alpha}$ is the demand elasticity. This is essentially equivalent to (9), except that the central government tax rate now enters the expression.

We cannot, however, use (18) directly to evaluate, say, the effect of lump-sum transfers to one city or another. The reason is that transfers will affect equilibrium labor supplies and thus central governemnt tax revenue. If feasible policy choices are defined as those that satisfy (17), these effects must be taken into account. For simplicity, let us consider the case where all matching rates are zero, and where induced changes in z_α do not affect labor supplies. (Alternatively, let the z_α's be exogenously given.) To evaluate an increase in $L_{\alpha 0}$ accompanied by an offsetting decrease in $L_{\beta 0}$, use (17) to show $\partial L_{\beta 0}/\partial L_{\alpha 0}=-(1-\tau\,\mathrm{d}x_{h\alpha}/\mathrm{d}L_{\alpha 0})/(1-\tau\,\mathrm{d}x_{h\beta}/\mathrm{d}L_{\beta 0})$. If for simplicity we assume $W_\alpha v_{h\alpha I}^{-1}=W_\beta v_{h\beta I}^{-1}$ and $1-\tau\,\partial x_{h\alpha}/\partial L_{\alpha 0}>0$, welfare rises or falls according as $D_\alpha(1-\tau\,\mathrm{d}x_{h\alpha}/\partial L_{\alpha 0})^{-1}\gtrless D_\beta(1-\tau\,\mathrm{d}x_{h\beta}/\mathrm{d}L_{\beta 0})^{-1}$. That is, grant policy should be evaluated by comparing the real income change in each jurisdiction per dollar's worth of net central government resources expended. Some manipulations show that

$$D^*_\alpha\equiv\frac{D_\alpha}{1-\tau(\mathrm{d}x_{h\alpha}/\mathrm{d}L_{\alpha 0})}=\left[1+\frac{\dfrac{\tau_\alpha+\tau}{w_\alpha}E_{h\alpha}E_{f\alpha}}{E^*_{h\alpha}-\left(1-\dfrac{\tau_\alpha+\tau}{w_\alpha}\right)E_{f\alpha}}\right]^{-1}. \tag{19}$$

To illustrate possible values for D^*_α, note first that $D^*_\alpha=1$ if the ordinary factor supply elasticity is zero. In the present application, where the factor is labor, this elasticity may indeed be very small. However, for some population subgroups it is as high as 1, and the compensated elasticity certainly may be significantly positive. [See, for example, Hausman (1981).] Let us therefore consider values of $E_{h\alpha}=0.25$ and 0.5, and $E^*_{h\alpha}=1$.

If labor's income share is 0.75 and local production is Cobb–Douglas, $E_{f\alpha}=4$.[15] We also consider recipient government tax rates of 0.1 and 0.2, and central government tax rates of 0 and 0.3. These correspond to marginal rates, and of course would reflect sales and payroll taxes as well as income taxes per se. Table 2 shows, as expected, that D^*_α depends sensitively on the ordinary factor supply elasticity. (As noted, D^*_α is 1 if this elasticity is zero.) In the case where $\tau/w_\alpha=0$ (i.e. where the central government either uses lump-sum taxes or the taxes are at least initially zero), a doubling of the recipient tax rate causes an increase of D^*_α of only 3–6 percentage points. With distortionless central government taxes, then, wide interjurisdictional variations in tax rates have little effect on D^*_α. With $\tau_\alpha/w_\alpha=0.3$, this result is altered. For $E_{h\alpha}=0.25$, a doubling of the recipient tax rate leads to a 7 point increase in D^*_α; with $E_{h\alpha}=0.5$, the increase is 19 points. Thus, incorporating central government distortionary taxes has an important effect on the interjurisdictional variation in D^*_α, and therefore on the possible welfare gains from intergovernmental transfers.

Table 2

Estimates of the distortionary factor D^*_α for alternative parameter values.[a]

	$E_{h\alpha}=0.25$		$E_{h\alpha}=0.5$	
	$\frac{\tau_\alpha}{w_\alpha}=0.1$	$\frac{\tau_\alpha}{w_\alpha}=0.2$	$\frac{\tau_\alpha}{w_\alpha}=0.1$	$\frac{\tau_\alpha}{w_\alpha}=0.2$
$\frac{\tau}{w_\alpha}=0$	1.02	1.05	1.05	1.11
$\frac{\tau}{w_\alpha}=0.3$	1.13	1.20	1.31	1.50

[a]*Source*: Author's computation, as explained in text.

Finally, let us consider a balanced-budget change in the *scale* of the transfer program, confining attention to lump-sum grants for simplicity. By (17), the changes $L'_{\alpha 0}$ in lump-sum transfers must satisfy

$$X_h+\tau\Sigma_\alpha\frac{\mathrm{d}x_{h\alpha}}{\mathrm{d}\tau}=\Sigma_\alpha\left[1-\frac{\tau\,\mathrm{d}x_{h\alpha}}{\mathrm{d}L_{\alpha 0}}\right]L'_{\alpha 0}, \tag{20}$$

where $X_h=\Sigma_\alpha x_{h\alpha}$ and where $\mathrm{d}x_{h\alpha}/\mathrm{d}\tau$ denotes $x^*_{h\alpha w}(\partial w_\alpha/\partial\tau)-x_{h\alpha w}(\partial\tau_\alpha/\partial\tau+1)$.

[15] If $y_{f\alpha}=x^\gamma_{f\alpha}\cdot l^{1-\gamma}_\alpha$, profit maximization implies $\gamma x^{\gamma-1}_{h\alpha}l^{1-\gamma}_\alpha=w_\alpha$. Taking logs and differentiating, $E_{f\alpha}=1/(\gamma-1)=-4$ if $\gamma=3/4$.

The welfare change is

$$\frac{\mathrm{d}W}{\mathrm{d}\tau} = -\Sigma_\alpha W_\alpha v_{h\alpha I} x_{h\alpha}\left(\frac{\partial \tau_\alpha}{\partial \tau} + 1 + \frac{\partial \tau_\alpha}{\partial L_{\alpha 0}} L'_{\alpha 0}\right)$$

$$= \Sigma_\alpha W_\alpha v_{h\alpha I} D_\alpha (L'_{\alpha 0} - x_{h\alpha}), \tag{21}$$

the value of which of course depends on $L'_{\alpha 0}$, that is, on how funds are distributed. We examine three cases.

(i) $L'_{\alpha 0} = x_{h\alpha}$. In this case, where grants are distributed in proportion to tax base, the central government labor tax just substitutes for the local tax, leaving the combined rate, and the real equilibrium of the economy, unchanged. (It is easily checked that this grant policy change is feasible.) The welfare effect is therefore zero.

(ii) $L'_{\alpha 0} = L'_0$: *a uniform increment to all recipients.* This case is easier to analyze under the assumption of an incipient central government tax/transfer program, i.e. with τ initially zero. Then (20) can be solved for $L'_0 = \bar{x}_h = X_h/A$ and we find that

$$\frac{\mathrm{d}W}{\mathrm{d}\tau} \gtrless 0, \quad \text{as } \operatorname{cov}(W_\alpha v_{h\alpha I} D_\alpha, x_{h\alpha}) \lessgtr 0. \tag{22}$$

The intuition is clear. The grant is equal to the average tax base, while the locality's tax contribution to the program is its own tax base. The net transfer it receives is $L'_0 - x_{h\alpha} = \bar{x}_h - x_{h\alpha}$. If net transfers accrue to localities with high equity-adjusted distortionary factors [a negative covariance in (22)], welfare will increase.

If $\tau > 0$ initially, matters are complicated because the per-locality transfer may be greater or less than $\bar{x}_h$ due to feedback effects on central government tax revenues; to the extent that transfers are larger, the program increment obviously will be more attractive. The feedback effects are ambiguous, however, as (20) reveals: a higher τ tends to reduce labor supply (assuming $E_{h\alpha} > 0$), but grants result in lower τ_α's and hence larger labor supplies. One can show that[16]

$$\frac{\mathrm{d}W}{\mathrm{d}\tau} \gtrless 0, \quad \text{as } \operatorname{cov}(W_\alpha v_{h\alpha I} D_\alpha, x_{h\alpha}) - \frac{\bar{\Omega} \operatorname{cov}(x_{h\alpha}, B_\alpha)}{1 + \bar{B}} \lessgtr 0, \tag{23}$$

[16]To derive (23), use (21), express $L'_{\alpha 0}$ as $\bar{x}_h + (L_0 - \bar{x}_h)$, and use (20) to write $L'_0 - \bar{x}_h$ as $\tau\Sigma^\alpha[\mathrm{d}x_{h\alpha}/\mathrm{d}\tau_\alpha - \bar{x}_h \mathrm{d}x_{h\alpha}/\mathrm{d}L_{\alpha 0}]/[1 - \tau(\mathrm{d}x_{h\alpha}/\mathrm{d}L_{\alpha 0})]$. Writing out the total derivatives of $x_{h\alpha}$ and using the system (16) for the derivatives of τ_α and w_α with respect to τ and $L_{\alpha 0}$ yields (23).

where $\bar{\Omega}$ is the mean of the $W_\alpha v_{h\alpha I} D_\alpha$'s,

$$B_\alpha = \frac{\tau}{w_\alpha}\left[\frac{\tau_\alpha}{w_\alpha} + \frac{E^*_{h\alpha} - \left(1 - \frac{\tau_\alpha + \tau}{w_\alpha}\right)E_{f\alpha}}{E_{h\alpha}E_{f\alpha}}\right]^{-1} = \frac{\tau/w_\alpha}{\tau_\alpha/w_\alpha}(1 - D_\alpha) < 0, \tag{24}$$

and $\bar{B} = \Sigma_\alpha B_\alpha / A$. The second term in (23) reflects the ambiguous implications of $\tau > 0$ for the average transfer.

As in the previous section, a priori arguments cannot determine the signs of the covariances in (23). We may observe, however, that if all recipients have identical demand and supply elasticities for labor, and if recipients with smaller tax bases have higher own tax rates, D_α and $x_{h\alpha}$ will be negatively correlated, tending to favor the program. In this case B_α can be shown to vary inversely with the own tax rate, $\mathrm{cov}(x_{h\alpha}, B_\alpha) > 0$, also favoring the program.[17] In effect, the program transfers from base-rich to base-poor jurisdictions, permitting reductions in distortions in the most distorted markets.

(iii) *$L'_{\alpha 0}$ arbitrary, but grant structure has previously been optimized.* Assume that lump-sum grants have initially been chosen so that $W_\alpha v_{h\alpha I} D^*_\alpha = \mu$, all α. Then an incremental dollar should generate the same welfare gain, no matter which government receives it. By (20) and (21), this condition implies

$$\frac{dW}{d\tau} = \mu\left(X_h - \tau\frac{\Sigma_\alpha dx_{h\alpha}}{d\tau}\right) - \mu\Sigma_\alpha x_{h\alpha}\left(1 - \tau\frac{dx_{h\alpha}}{dL_{\alpha 0}}\right) = 0, \tag{25}$$

the second equality obtaining after explicitly writing out the total derivatives using the system (16).

In other words, the welfare gains from transfer programs of this sort derive entirely from interjurisdictional redistribution, not from the replacement of recipient by donor revenues per se. Once the optimal grant structure is achieved (which might in fact involve negative transfers to some jurisdictions), the scale of the program becomes irrelevant.

5. Extensions and further applications

Intergovernmental transfers influence the allocation of resources, and economic welfare, in many ways. The foregoing discussion has examined one dimension of this general issue that has been virtually neglected in the literature to date, namely the interactions between grant policy and distortionary local taxes. The theoretical analysis and illustrative calculations show this to be an important element in the overall evaluation of grant policy.

[17] To show that B_α varies inversely with τ_α/w_α, one differentiates holding all elasticities and τ/w_α fixed.

We have focused on very simple grant policies, involving only lump-sum and matching elements. This is useful because matching grants are actually quite widely used, and it is of interest to consider how they interact with local tax distortions. In practice, however, grants can also be conditioned on population, income, tax effort, fiscal capacity, and so on. It would be useful to enrich the model to allow for these sorts of policies. Indeed, our discussion of examples (ii) and (iii) in section 3.2 is suggestive in this regard. For instance, if (equity-weighted) distortionary factors happen to be positively correlated with population (perhaps because of strong congestion effects), then a policy that provides larger grants to more populous jurisdictions might obviate the need for matching grants. Alternatively, if low-income jurisdictions have higher equity-weighted distortions, one could condition grants on income. (Of course, from the perspective developed here, we have already discussed the best of all possible policies: individualized lump-sum grants would result in the complete equalization of all equity-adjusted distortionary factors.) A proper extension to different grant structures will be non-trivial, however. In a serious discussion of population-based grants, for instance, it seems essential to allow for interjurisdictional migration. This will necessitate a joint analysis of local tax distortions and inefficiencies in locational choice, and must be left for future research.[18]

Appendix A

Totally differentiating (4) and (5), we have:

$$\begin{bmatrix} y_{h\alpha}+t_\alpha y_{h\alpha p} & t_\alpha(y_{h\alpha p}+y_{h\alpha}y_{h\alpha I}) \\ y_{h\alpha p} & y_{h\alpha p}+y_{h\alpha}y_{h\alpha I}-y'_{f\alpha} \end{bmatrix} \begin{bmatrix} \mathrm{d}t_\alpha \\ \mathrm{d}p_\alpha \end{bmatrix} = \begin{bmatrix} -\mathrm{d}L_{\alpha 0}-z_\alpha \mathrm{d}L_{\alpha 1} \\ 0 \end{bmatrix}. \tag{A.1}$$

Let the matrix on the left be denoted B. Its determinant is easily computed after substituting from the Slutsky equation:

$$y^*_{h\alpha p}=y_{h\alpha p}+y_{h\alpha}y_{h\alpha I}, \tag{A.2}$$

where $y^*_{h\alpha p}$ is the derivative of the *compensated* demand for housing with respect to its price. We have:

$$|B|=y_{h\alpha}(y^*_{h\alpha p}-y'_{f\alpha})-t_\alpha y_{h\alpha p}y'_{f\alpha}, \tag{A.3}$$

which, since $y^*_{h\alpha p}<0<y'_{f\alpha}$, is definitely negative for t_α sufficiently small. In fact, $|B|<0$ in all situations of interest here because $|B|>0$ means that an

[18]See footnote 4 above. An earlier version of this paper developed a model with distortionary local taxes and mobile households. The present analysis, by considering only immobile households, is much more transparent.

increase in t_α *lowers* local tax revenue $t_\alpha y_{h\alpha}$, whereas $|B|<0$ means the opposite.[19] Voters in each city would never find it optimal to choose a level of local public expenditure in which the former occurs, and will always be in the non-pathological range of the 'Laffer curve'.

Assuming, then, that $|B|<0$, we can solve for $\partial t_\alpha/\partial L_{\alpha i}$. We have:

$$-y_{h\alpha}\frac{\partial t_\alpha}{\partial L_{\alpha i}}=y_{h\alpha}(y^*_{h\alpha p}-y'_{f\alpha})(\partial L_\alpha/\partial L_{\alpha i})|B|^{-1}$$

$$=\frac{\partial L_\alpha/\partial L_{\alpha i}}{1-\dfrac{t_\alpha y_{h\alpha p}y'_{f\alpha}}{y_{h\alpha}(y^*_{h\alpha p}-y'_{f\alpha})}}. \tag{A.4}$$

If we now define the elasticities $\varepsilon_{h\alpha}=y_{h\alpha}^{-1}(p_\alpha+t_\alpha)y_{h\alpha p}$, $\varepsilon^*_{h\alpha}=y_{h\alpha}^{-1}(p_\alpha+t_\alpha)y^*_{h\alpha p}$ and $\varepsilon_{f\alpha}=y_{f\alpha}^{-1}p_\alpha y'_{f\alpha}$, we can write:

$$D_\alpha=\left[1-\frac{t_\alpha\dfrac{\varepsilon_{h\alpha}}{p_\alpha+t_\alpha}\dfrac{\varepsilon_{f\alpha}}{p_\alpha}}{\dfrac{\varepsilon^*_{h\alpha}}{p_\alpha+t_\alpha}-\dfrac{\varepsilon_{f\alpha}}{p_\alpha}}\right]^{-1}, \tag{A.5}$$

from which (8) and (9) of the text follow. Note that $|B|<0$ implies $D_\alpha>0$.

Appendix B: Stimulative effects of lump-sum vs. matching grants

To see whether matching grants stimulate spending more than lump-sum grants, use the basic first-order condition for z_α to solve implicitly for z_α in

[19]To see this, consider the effect of a change in t_α on local revenues $t_\alpha y_{h\alpha}$ with z_α held fixed. Since the question is whether a higher tax rate brings in more tax revenue we must allow for the effect of t_α on p_α, the equilibrium housing price. Thus, we solve (5) for p_α in terms of t_α, and compute $\partial p_\alpha/\partial t_\alpha=-y_{h\alpha p}(y^*_{h\alpha p}-y'_{f\alpha})^{-1}$. Then

$$\frac{dt_\alpha y_{h\alpha}}{dt_\alpha}=y_{h\alpha}+t_\alpha\left[(y_{h\alpha p}+y_{h\alpha}y_{h\alpha})\frac{\partial p_\alpha}{\partial t_\alpha}+y_{h\alpha p}\right]$$

$$=y_{h\alpha}+t_\alpha\left[\frac{y^*_{h\alpha p}(-y_{h\alpha p})}{y^*_{h\alpha p}-y'_{f\alpha}}+y_{h\alpha p}\right]$$

$$=\frac{1}{y^*_{h\alpha p}-y'_{f\alpha}}[y_{h\alpha}(y^*_{h\alpha p}-y'_{f\alpha})-t_\alpha y_{h\alpha p}y'_{f\alpha}]$$

$$=\frac{1}{y^*_{h\alpha p}-y'_{f\alpha}}|B|,$$

so that tax revenues rise with t_α iff $|B|<0$.

terms of the $L_{\alpha i}$'s. From $\mathrm{d}v_{h\alpha}/\mathrm{d}z_\alpha = 0$ we have

$$\frac{\partial z_\alpha}{\partial L_{\alpha i}} = -\frac{\partial^2 v_{h\alpha}/\partial z_\alpha\, \partial L_{\alpha i}}{\partial^2 v_{h\alpha}/\partial z_\alpha^2}.$$

Since the denominator is negative when z_α maximizes $v_{h\alpha}$, $z_\alpha^{-1}\, \partial z_\alpha/\partial L_{\alpha 1} - \partial z_\alpha/\partial L_{\alpha 0}$ will be positive iff $z_\alpha^{-1}\, \partial^2 v_{h\alpha}/\partial z_\alpha\, \partial L_{\alpha 1} > \partial^2 v_{h\alpha}/\partial z_\alpha\, \partial L_{\alpha 0}$. Now

$$\frac{\partial v_{h\alpha}}{\partial z_\alpha} = v_{h\alpha z} + v_{h\alpha p}\frac{\partial p_\alpha}{\partial z_\alpha} + v_{h\alpha p}\frac{\partial t_\alpha}{\partial z_\alpha} + v_{h\alpha I}\frac{\partial \pi_\alpha}{\partial p_\alpha}\frac{\partial p_\alpha}{\partial z_\alpha} = v_{h\alpha z} + v_{h\alpha p}\frac{\partial t_\alpha}{\partial z_\alpha}.$$

$v_{h\alpha z}$ and $v_{h\alpha p}$ depend on p_α and t_α and thus on $L_{\alpha 0}$ and $L_{\alpha 1}$. However, it is evident from (A.1) that these vary equally *per dollar's worth* of aid in lump-sum or matching form. The effects of \$1 increases in lump-sum and matching aid on $v_{h\alpha z}$ and $v_{h\alpha p}$ will therefore be the same and can be ignored. Thus, what matters is the sign of $\partial^2 t_\alpha/\partial z_\alpha \partial L_{\alpha i}$. In fact, we can show that $z_\alpha^{-1}\, \partial^2 t_\alpha/\partial z_\alpha \partial L_{\alpha 1} < \partial^2 t_\alpha/\partial z_\alpha \partial L_{\alpha 0}$. To show this, note that

$$b_{11}\frac{\partial t_\alpha}{\partial L_{\alpha i}} + b_{12}\frac{\partial p_\alpha}{\partial L_{\alpha i}} + \frac{\mathrm{d}L_\alpha}{\mathrm{d}L_{\alpha i}} = 0$$

and

$$b_{21}\frac{\partial t_\alpha}{\partial L_{\alpha i}} + b_{22}\frac{\partial p_\alpha}{\partial L_{\alpha i}} = 0,$$

where the b_{ij}'s are elements of B defined in (A.1). From this we have that:

$$\frac{\mathrm{d}b_{11}}{\mathrm{d}z_\alpha}\frac{\partial t_\alpha}{\partial L_{\alpha i}} + \frac{\mathrm{d}b_{12}}{\mathrm{d}z_\alpha}\frac{\partial p_\alpha}{\partial L_{\alpha i}} + b_{11}\frac{\partial^2 t_\alpha}{\partial L_{\alpha i}\, \partial z_\alpha} + b_{12}\frac{\partial^2 p_\alpha}{\partial L_{\alpha i}\, \partial z_\alpha} + \frac{\mathrm{d}^2 L_\alpha}{\mathrm{d}L_{\alpha i}\, \mathrm{d}z_\alpha} = 0$$

and

$$\frac{\mathrm{d}b_{21}}{\mathrm{d}z_\alpha}\frac{\partial t_\alpha}{\partial L_{\alpha i}} + \frac{\mathrm{d}b_{22}}{\mathrm{d}z_\alpha}\frac{\partial p_\alpha}{\partial L_{\alpha i}} + b_{21}\frac{\partial^2 t_\alpha}{\partial L_{\alpha i}\, \partial z_\alpha} + b_{22}\frac{\partial^2 p_\alpha}{\partial L_{\alpha i}\, \partial z_\alpha} = 0.$$

For equal dollar grant changes, $\mathrm{d}L_{\alpha 0} = z^{-1}\mathrm{d}L_{\alpha 1}$, and $z_\alpha^{-1}\, \partial t_\alpha/\partial L_{\alpha 1} = \partial t_\alpha/\partial L_{\alpha 0}$, $z_\alpha^{-1}\, \partial p_\alpha/\partial L_{\alpha 1} = \partial p_\alpha/\partial L_{\alpha 0}$. Hence, the $\mathrm{d}b_{ij}$ terms cancel and we have:

$$b_{11}\overbrace{\left(\frac{\partial^2 t_\alpha}{\partial L_{\alpha 0}\, \partial z_\alpha} - z_\alpha^{-1}\frac{\partial^2 t_\alpha}{\partial L_{\alpha 1}\, \partial z_\alpha}\right)}^{a_1} + b_{12}\overbrace{\left(\frac{\partial^2 p_\alpha}{\partial L_{\alpha 0}\, \partial z_\alpha} - z_\alpha^{-1}\frac{\partial^2 p_\alpha}{\partial L_{\alpha 1}\, \partial z_\alpha}\right)}^{a_2} = z_\alpha^{-1}$$

and

$$b_{21}(a_1)+b_{22}(a_2)=0.$$

Solving for a_1:

$$a_1=\frac{\begin{vmatrix} z_\alpha^{-1} & b_{12} \\ 0 & b_{22} \end{vmatrix}}{|B|}=\frac{z_\alpha^{-1}b_{22}}{|B|}>0.$$

Hence,

$$\frac{\partial^2 t_\alpha}{\partial L_{\alpha 0}\,\partial z_\alpha}>z_\alpha^{-1}\frac{\partial^2 t_\alpha}{\partial L_{\alpha 1}\partial z_\alpha}. \quad \square$$

The upshot is that

$$z_\alpha^{-1}\frac{\partial z_\alpha}{\partial L_{\alpha 1}}>\frac{\partial z_\alpha}{\partial L_{\alpha 0}}.$$

Appendix C

This appendix explains the derivation of several results in section 4 of the main text. First, differentiate the system (16):

$$\begin{bmatrix} x_{h\alpha w} & x'_{f\alpha}-x^*_{h\alpha w} \\ x_{h\alpha}-\tau_\alpha x_{h\alpha w} & \tau_\alpha x^*_{h\alpha w} \end{bmatrix}\begin{bmatrix} \mathrm{d}\tau_\alpha \\ \mathrm{d}w_\alpha \end{bmatrix}=\begin{bmatrix} 0 & -x_{h\alpha w} \\ -1 & \tau_\alpha x_{h\alpha w} \end{bmatrix}\begin{bmatrix} \mathrm{d}L_{\alpha 0} \\ \mathrm{d}\tau \end{bmatrix}.$$

Call the left-hand matrix M_α. Then

$$|M_\alpha|=(x^*_{h\alpha w}-x'_{f\alpha})x_{h\alpha}+x'_{f\alpha}x_{h\alpha w}\tau_\alpha.$$

$|M_\alpha|>0$ if an increase in τ_α increases local tax revenue. We have:

$$\frac{\partial \tau_\alpha}{\partial \tau}=\frac{-\tau_\alpha x_{h\alpha w}x'_{f\alpha}}{|M_\alpha|},$$

$$\frac{\partial \tau_\alpha}{\partial L_{\alpha 0}}=\frac{(x'_{f\alpha}-x^*_{h\alpha w})}{|M_\alpha|},$$

$$\frac{\partial w_\alpha}{\partial \tau}=\frac{x_{h\alpha}x_{h\alpha w}}{|M_\alpha|},$$

$$\frac{\partial w_\alpha}{\partial L_{\alpha 0}}=\frac{-x_{h\alpha w}}{|M_\alpha|}.$$

Eqs. (18) and (19) for D_α and D_α^* follow from the above expressions for $\partial\tau_\alpha/\partial L_{\alpha 0}$ and $\partial w_\alpha/\partial L_{\alpha 0}$. Similarly, (21) follows using the expression for $\partial\tau_\alpha/\partial\tau$.

To show (23), note that (20) implies:

$$X_h+\tau\left[\Sigma_\alpha x^*_{h\alpha w}\frac{\partial w_\alpha}{\partial\tau}-x_{h\alpha w}\left(\frac{\partial\tau_\alpha}{\partial\tau}+1\right)\right]=\Sigma_\alpha\left[1-\tau\left(x^*_{h\alpha w}\frac{\partial w_\alpha}{\partial L_{\alpha 0}}-x_{h\alpha w}\frac{\partial\tau_\alpha}{\partial L_{\alpha 0}}\right)\right]L'_{\alpha 0}$$

or

$$X_h+\tau\Sigma_\alpha\frac{x_{h\alpha w}x'_{f\alpha}x_{h\alpha}}{|M_\alpha|}=\Sigma_\alpha\left(1+\frac{\tau x_{h\alpha w}x'_{f\alpha}}{|M_\alpha|}\right)L'_{\alpha 0},$$

where

$$\frac{\tau x_{h\alpha w}x'_{f\alpha}}{|M_\alpha|}=\frac{\tau}{w_\alpha}\left[\frac{\tau_\alpha}{w_\alpha}+\frac{(x^*_{h\alpha w}-x'_{f\alpha})x_{h\alpha}}{w_\alpha x_{h\alpha w}x'_{f\alpha}}\right]^{-1}=B_\alpha.$$

Hence, if $L'_{\alpha 0}=L'_0$:

$$L'_0=\frac{X_h+\Sigma_\alpha x_{h\alpha}B_\alpha}{\Sigma_\alpha(1+B_\alpha)}.$$

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