

Spatial Variation of the Marginal Utility of Income and Unequal Treatment of Equals¹

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Received March 7, 1984; revised April 25, 1984

1. INTRODUCTION

In one of the most famous contributions to the "new" urban economics literature, Mirrlees [6] showed that a utilitarian planner, able to control the distribution of income (or endowments) across households, the consumption of land (or housing), and the distribution of population over space, would treat households with identical tastes unequally in the sense that utility levels would vary among them at the planner's optimum. Similar results were obtained by Dixit [2], who showed that even more inequality-averse social welfare functions than the utilitarian one would result in unequal treatment. He concluded that only the Rawlsian maximin welfare function would produce the equal treatment outcome.² This result has generated considerable attention (see, e.g., Mills and Mackinnon [5]) because of its apparently paradoxical nature.

The purpose of this note is to present a new way of thinking about, and making plausible, the unequal-treatment result.³ To develop the intuition, recall that in monocentric city models where equals are treated equally, that is, where households have equal endowments, a market equilibrium with free mobility is characterized by equal utilities for all (assuming identical preferences). As is well known, in the absence of congestion or other market failures, this market equilibrium is Pareto-efficient, producing the maximum feasible common level of utility. Imagine now a Mirrlees planner, coming across such an equal-utility equilibrium. Under what conditions would the planner not want to disturb the market equilibrium? Clearly, a necessary condition for equal treatment to be utilitarian-optimal is that the marginal utility of income be the same for all households in the market equilibrium. Otherwise, freezing the allocation of resources in every other respect, social

¹Helpful comments by Jan Brueckner are greatly appreciated.

²Actually, as Kanemoto [3] observes, other social welfare functions with kinks at equal-utility outcomes would result in equal treatment also.

³See also Arnott and Riley [1] and Levhari *et al.* [4] for discussions of this result.

welfare would be increased by a transfer from households with low marginal utilities of income to those with high.

This leads us to inquire about the variations in the marginal utility of income across households in the equal-treatment market equilibrium. We show below that if household preferences are such that the income elasticity of demand for space is zero, then the marginal utility of income is constant across households and the necessary conditions for optimality obtain at the equal-treatment market equilibrium. In fact, sufficiency can be established in this case, and we conclude that the market equilibrium is utilitarian-optimal, so that equal treatment and utilitarianism do not conflict. If the income elasticity of demand for space is positive (negative), however, the marginal utility of income rises (falls) with distance from the central business district (CBD), so that a utilitarian planner would wish to engage in redistribution toward households further out from (closer in to) the center of the city. This result, of course, reestablishes the conflict between equal treatment and utilitarianism found by Mirrlees, and provides a straightforward rationale for this conflict. It also explains the pattern of redistribution that takes place as one moves from the market equilibrium to the utilitarian planner's optimum.⁴

2. THE MODEL AND RESULT

We need only the simplest features of the standard monocentric city model. Let $p(x)$ and $I(x)$ denote the price of land and income net of transportation cost faced by households residing at distance x from the CBD. For simplicity, assume that all transportation costs are pecuniary, hence $I'(x) < 0$.⁵ Let q be a vector of other output and factor prices, quantities of public goods, and other parameters that do not vary spatially across households.

In a market equilibrium where households have identical gross (of transport cost) incomes and identical tastes, equal utilities must obtain at all locations. If u is the equilibrium utility level and $v(p(x), q, I(x))$ is the indirect utility function, $p(x)$ must satisfy the equilibrium condition

$$v(p(x), q, I(x)) = u \quad \text{for all } x. \quad (1)$$

⁴Kanemoto [3, pp. 29–30] shows that, at the utilitarian-optimum, utility rises with distance from the CBD provided that land is a normal good. This is precisely the implication of the results presented below, which show that a planner would redistribute in favor of more peripheral households. It should be noted that Wheaton [8] also shows that the marginal utility of income rises with distance from the city center if land is normal. Wheaton does not make the connection with the unequal-treatment feature of utilitarian and other optima, however.

⁵In models where transportation costs take the form of foregone leisure/work time, we also have $I'(x) < 0$. See Wildasin [9] for such a model.

From (1), it follows that

$$v_p p' + v_I I' = 0$$

or

$$p' = -\frac{v_I}{v_p} I', \quad (2)$$

where subscripts on v indicate partial differentiation, primes denote differentiation with respect to x , and arguments are suppressed for simplicity. Recall Roy's identity,

$$\frac{v_p(p(x), q, I(x))}{v_I(p(x), q, I(x))} \equiv -h(p(x), q, I(x)), \quad (3)$$

where $h(\cdot)$ is the demand for space at location x . Then (2) can be written

$$p' = \frac{I'}{h}. \quad (4)$$

Moreover, differentiation of Roy's identity with respect to I yields

$$\frac{v_{pI}}{v_I} - \frac{v_p}{v_I^2} v_{II} = \frac{v_{pI}}{v_I} + \frac{h v_{II}}{v_I} = -h_I \quad (5)$$

where h_I is the income derivative of demand for space.

Now consider how the marginal utility of income, v_I , varies with x :

$$\begin{aligned} v'_I &= v_{Ip} p' + v_{II} I' \\ &= (v_{Ip} + h v_{II}) \frac{I'}{h} \quad \text{by (4)} \\ &= -h_I \frac{v_I I'}{h} \quad \text{by (5)}. \end{aligned} \quad (6)$$

This proves the results noted in the introduction. If the income elasticity of demand for land is zero, it follows that the marginal utility of income is constant across locations. When $h_I > 0$, as is found empirically, the marginal utility of income rises with distance from the city center, and the equal-treatment equilibrium will be overturned by a utilitarian planner, in favor of more distant households. The converse holds with $h_I < 0$.

Another way to see this result is to note that, from the consumer's expenditure function $e(p, q, u)$, the cost of providing one more unit of utility is e_u . In general, this varies with the price p

$$e_{up} = \frac{\partial x}{\partial u}. \quad (7)$$

Thus, for example, if x is normal, an extra dollar given to an individual facing higher prices will produce less utility than one given to an individual facing low prices. Since prices vary spatially in an initial equal-treatment equilibrium, it follows that a planner could increase total utility by transferring from high-price to low-price individuals.

3. DIAGRAMMATIC PRESENTATION

The basic idea of the argument is easily illustrated with a diagram. In Fig. 1, we portray the consumption bundles of two identical households in a market equilibrium. Point A shows the consumption bundle for a household living far from the CBD and facing a budget constraint I , characterized by a low relative price for land (h) in terms of all-purpose good (c). Point B shows the equilibrium consumption for a close-in household. Both A and B are on the same indifference curve, corresponding to the equal-treatment characteristic of market equilibria, as in (1).

Now suppose we consider giving an increment of income ΔI to each household. If the income elasticity of demand for land is zero, indifference

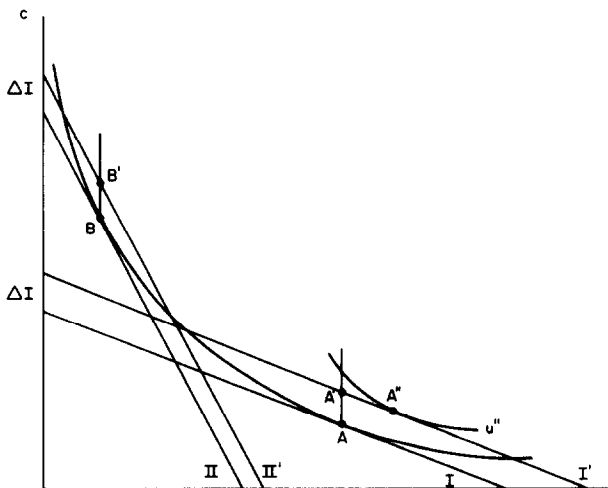


FIGURE 1.

curves will differ only by vertical displacements (the case of "parallel preferences"), and both households will move to new equilibria at A' and B' on the same higher indifference curve. This shows, in accordance with (6) and (7), that both households are equally efficient at converting income into utility, even though facing different prices. In this case, the sum of utilities cannot be increased by a transfer ΔI from one household to another, and so the egalitarian market equilibrium would not be disturbed by a utilitarian social welfare maximizer.

In contrast, if the income elasticity of demand for land is positive, the first household will move to a new equilibrium like A'' when income increases. As shown, the vertical distance between the new indifference curve u'' and the initial equilibrium point A is greater than ΔI . Thus, for a consumer facing a higher land price, a greater increase in income will be required to attain the indifference curve u'' , that is, the marginal utility of income is lower, as (7) indicates. A transfer of ΔI from the high-price to the low-price individual would then increase the sum of utilities.

4. CONCLUSION

We note, finally, that the unequal-treatment result arises from the fact that the marginal utility of income depends on prices, as well as income. Because prices and net income vary spatially, the marginal utility of income is certainly expected to vary as well, and the unequal treatment of a utilitarian planner follows immediately. Clearly, this unequal treatment property will appear whenever prices or income vary across otherwise identical people, e.g., in any spatial setting, including models of interregional or international trade, or when prices vary randomly across households, as in Stiglitz [7].

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