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Some rudimetary 'duopolity' theory

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Let there be a pair of jurisdictions whose choices of tax, expenditure, or regulatory policies affect each other. Their strategic interactions can be modelled in various ways. Candidate models of duopolitical equilibrium include Nash equilibria in taxes, expenditures, or regulatory standards. Examples of several of these equilibria are presented and compared. Choosing among them is a game-theoretic problem involving the determination of strategic variables subject to side constraints. The problem can be formulated as a two-stage game, in which governments commit to using certain strategic variables in the first stage, and then determine the values of these variables themselves in the second stage.

> 'The present essay will accordingly be devoted chiefly to an attempt to clarify issues rather than to give direct and explicit answers to questions. It seems to the writer that the greatest need is for effective warning against easy solutions for problems which are very hard, which means against any definitive solution to problems which have none, so that any "solution" found must be false (and many are found!). Compromising and temporizing are unpleasant words; but where one cannot see one must grope...'
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> F.H. Knight (1950, pp. 467–468).

1. Introduction

The economic analysis of many policy issues requires a model of the

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behavior of governments and the interactions among governments. When the number of governments involved is small, strategic behavior must be taken into account. By far the most common procedure for modelling such behavior is to assume that the governments attain Nash equilibria in some policy variable(s). A few examples will convey the flavor of this more or less standard approach:

(*i*) Kolstad and Wolak (1983, 1986) consider the taxation of coal by Wyoming and Montana, states in which much coal is produced and which can be expected to have some monopoly power in the coal market. The objective of the Kolstad and Wolak analysis is to model the equilibrium tax rates that these states will choose, taking into account their competitive relationship with each other. The favored equilibrium concept in the Kolstad and Wolak is that of a Nash equilibrium in tax rates.
(*ii*) A line of work going back at least to Johnson (1953–1954) models tariff-setting between countries. In the standard formulation, one supposes that there are two countries and two traded commodities; each country can influence the terms of trade in these commodities through its tariff policy. The problem is to describe the equilibrium tariff structure and identify the welfare properties of the equilibrium, including any potential gains from tariff negotiations. Typically it is assumed that the countries will find a Nash equilibrium in tariffs.
(*iii*) Suppose that there are two cities, states, or countries providing public goods which benefit not only their own residents but those in the other jurisdiction as well. How does one describe the equilibrium level of public good provision in each? The standard approach in the literature [see, e.g., Williams (1966), Pauly (1970), Boskin (1973), Boadway et al. (1990)] is to assume that the governments will achieve a Nash equilibrium in the level of public expenditure.
(*iv*) Suppose that capital is mobile between two countries, and that each uses a tax on capital to finance public goods; each must choose whether to provide credits for taxes paid by residents on capital income derived in the other country, or whether to use tax deductions. The problem is to describe the equilibrium tax rates that would hold under either of these regimes, and identify the welfare implications of equilibria under different regimes. Authors writing on this problem [e.g., Bond and Samuelson (1989)] typically assume that the governments will end up at a Nash equilibrium in tax rates, conditional on the crediting/ deduction regime.
(*v*) Suppose that there are two countries and two traded commodities; each country uses a commodity tax (such as a value added tax) on one of these commodities to finance its expenditures on a public good. The problem is to model the equilibrium commodity tax rates for these goods and to identify the welfare properties of the resulting equilibrium. In models of this type,

Mintz and Tulkens (1986) and de Crombrugghe and Tulkens (1990) hypothesize a Nash equilibrium in value-added tax rates.

(*vi*) Suppose that there are two regions, each with labor markets exhibiting wage ridigities that preclude full employment. Each region might use an employment subsidy in order to promote regional welfare. Suppose that there is a central government that provides public goods or transfers to these regions, and that central government revenues depend on the level of employment in each region. Regional employment subsidies can produce fiscal spillover benefits operating through the central government tax structure. Marchand and Pestieau (1987) analyze such a model and assume that regions achieve a Nash equilibrium in their subsidy rates.

In all of these examples. Nash non-cooperative equilibrium is used to model the strategic policy-setting behavior of governments. Inherent in the use of Nash equilibrium, however, is the identification of the strategic variable or variables used by the players. The examples illustrate that modelling practice varies widely in the literature in this respect. Many different strategic variables have been used by various authors, including tax rates, tariff rates, and public expenditure levels. These examples are obviously not all-inclusive, and one can easily imagine models in which the strategic variables might be monetary growth rates, the size of government deficits, or any other policy variables. This presents a dilemma. In any given applied problem, how is one to decide which strategic variables are the 'best' ones to use for the analysis of non-cooperative equilibrium behavior?

To see the nature of this problem, consider any model in which governments use taxes or tariffs on mobile factors or goods to finance public expenditures of some sort. Any such model must incorporate (explicitly or implicitly) government budget constraints, and must incorporate at least two policy variables: one tax and one expenditure. Governments must then choose two (or more) policy variables subject to a budget constraint. Standard practice in the literature has been to use the budget constraint to eliminate one policy variable, and to look for a Nash equilibrium in the remaining variable(s). This modelling approach requires one to answer the following questions. Which variable does one eliminate? Which variable is the 'strategic' variable? No systematic way of answering these questions has been formulated in the literature on strategic policy equilibrium. In different branches of literature, different traditions have arisen, but there are few explicit discussions of the choice of strategic variables and, indeed, little explicit recognition that the issue exists. A notable exception is provided by Kolstad and Wolak (1983), who explicitly consider the possibility that coal-producing states in the western U.S. might achieve a Nash equilibrium in output levels rather than tax rates; but argue (p. 445) that 'if tax rates are the actual decision variables, then strategies for setting taxes will be based on how competitors set taxes, not on the indirectly determined output levels of competitors'. The Kolstad–Wolak judgment about the importance of tax rates for western coal producing states may well be correct; this is an empirical question which will not be investigated here. Their remarks, however, are indicative of the lack of established criteria to guide researchers in the formulation of this aspect of their models. In a generic strategic policy equilibrium setting, it is far from obvious how on can determine what the 'actual decision variables' are. From a scientific viewpoint, it is clearly highly desirable to augment informed personal judgments (which of course will remain an invaluable part of the model-building enterprise) with some explicit methodology; otherwise, a crucial part of the modelling procedure remains in a black box.[1]

This paper examines several simple models in which these general questions take on a specific form. For the sake of simplicity, the discussion will focus on the case where there are only two governments (hence, 'duopolity'), and on the case where there are only two policy instruments to be chosen by each government. The first model, presented in section 2, considers the determination of tax and expenditure policies by a pair of jurisdictions that use taxation of mobile capital to finance their provision of some public goods. The second model, discussed in section 3, considers the determination of tax and expenditure policies by a pair of jurisdictions that are in a reciprocal externality relationship with each other.[2] In each model, there are different strategic policy equilibria, corresponding to different assumptions about strategic variables. It is shown how governments might try to establish certain types of Nash equilibria through the announcement of commitments to particular strategic variables. This leads to consideration of two-stage games, in which the first stage consists of announcements of strategic variables to be employed in the second stage.[3] In each of the models considered, it turns out that there is a dominant strategy in the first stage of this game. This provides at least a tentative basis for predictions about the nature and ultimate outcome of the second-stage policy-setting sub-game.

After examining specific cases in sections 2 and 3, the concluding section 4

[1]The concluding section of this paper returns briefly to the Kolstad–Wolak model. It is noted that the methodology presented in the body of the paper would suggest, in contrast to the Kolstad–Wolak argument, that quantities of coal and *not* tax rates on coal would be the strategic variables used by the western coal-producing states.

[2]A third model, analyzed in an earlier version of this paper [Wildasin (1989)], considers a problem in international trade, where two countries use tariffs to finance some public expenditures – see example (ii) above.

[3]This approach was pioneered by Singh and Vives (1984) in the context of duopoly theory. Cheng (1985) provides further development of this idea, and Cheng and Wong (1990) have begun to explore its application in a policy context.

returns to a more general perspective by presenting an abstract formulation of a general problem in modelling non-cooperative games, and showing that the models of the preceding sections, as well as standard oligopoly problems, are all special cases of this general problem. There are many more examples of this general type of problem that could be fruitfully examined using methods similar to those employed in the present analysis.

2. A model of fiscal competition

The basic structure of the following model has already been used on several occasions, so the exposition can be kept brief.[4] The essential features are as follows: First, homogeneous capital is fixed in supply to the economy as a whole, but is costlessly mobile across jurisdictions, so that the net return to capital is equalized across jurisdictions in equilibrium. Capital is used as the sole variable input in a production process in each jurisdiction that yields a homogeneous output. There is a fixed factor in each jurisdiction, such as labor or land. Each jurisdiction is endowed with a production function $f_i(k_i)$ that is a strictly increasing and concave function $(f_i'>0>f_i'')$ of the amount of capital employed in jurisdiction i. The homogeneous output is used as a numeraire.

Each jurisdiction is inhabited by many identical and immobile individuals, with preferences represented by a twice continuously differentiable and strictly quasi-concave utility function $u_i(x_i, z_i)$ defined over consumption of the numeraire good x_i and the public good z_i. Public goods are financed by taxation of capital; per unit taxes are assumed for concreteness. If t_i denotes the tax rate in jurisdiction i, the community i faces the budget constraint

$$z_i = t_i k_i. \tag{1}$$

Capital is allocated across jurisdictions so as to equalize net returns; hence, if ρ denotes the economy-wide net return to capital,[5]

$$f_i'(k_i) - t_i = \rho \quad \forall i \quad \text{such that} \quad k_i > 0. \tag{2}$$

In equilibrium, all capital must be located in some jurisidiction. Hence, letting $\bar{k}$ denote the aggregate capital stock,[6]

[4]The model is well-suited for presentations of the 'new view' of property tax incidence, pioneered by Mieszkowski (1972), and has been used in the tax incidence literature by such authors as Courant (1977) and Bradford (1978). It has appeared in the literature on fiscal competition in papers by Bucovetsky (1991), Wildasin (1988, 1989), Wilson (1991), and Bucovetsky and Wilson (1991).

[5]We assume free disposal of capital, so that $\rho \geqq 0$.

[6]This equilibrium condition is stated in equality form, which is correct if $\rho > 0$ in equilibrium. It must be modified in the usual way if there is excess supply of capital.

$$\bar{k}-\sum_i k_i=0. \tag{3}$$

Eqs. (2) and (3) provide a system of equations that can be used to solve for the equilibrium amount of capital in every jurisdiction and the equilibrium net return to capital, conditional on the structure of taxes. Letting $t \equiv (t_1, \ldots, t_n)$, let these equilibrium values of the endogenous variables be denoted by $k_i(t)$ and $\rho(t)$. This simple general equilibrium system has obvious properties: the amount of capital in each jurisdiction is a declining function of the tax rate there, and an increasing function of the tax rate in any other jurisdiction; the equilibrium net return to capital is a decreasing function of all tax rates.

Suppose for simplicity that households in jurisdiction i receive all of the returns accruing to the fixed factor, and that they own a share θ_i of the total capital stock. We also allow for absentee ownership of some capital, denoted by the share $\theta_0 \geqq 0$. Then the net income of households in i, which is spent on the private consumption good, is

$$x_i=f_i(k_i)-k_i f_i'(k_i)+\theta_i\rho\bar{k}. \tag{4}$$

Substituting $k_i(t)$ and $\rho(t)$ into (1) and (4), and then into the utility function, we obtain utility as a function $v_i(t)$ of the vector t of tax policies chosen by all jurisdictions. This function is crucial for the analysis, as it relates payoffs – utility levels – to policy instruments. Let us refer to it as the 'indirect' utility function (even though it is somewhat different from an ordinary indirect utility function because it subsumes the government budget constraint).

The problem now is to decide how to model the equilibrium policies of the jurisdictions. Several candidate equilibria will be compared here. The first candidate is a Nash equilibrium in tax rates. Formally, define[7]

T-equilibrium. A vector t^* is a *T-equilibrium* if, for all i, t_i^* maximizes $v_i(t^*/t_i)$ with respect to t_i.

The standard interpretation of the T-equilibrium is that each jurisdiction believes that its own policy choice will not influence the tax rates chosen by other jurisdictions; it therefore optimizes against these tax rates. In particular, an increase in t_i is expected not to result in a change in t_j. However, this means that k_j will increase, as capital flows out of jurisdiction i and into jurisdiction j to maintain equality of the net return to capital. But then, by (1), this means that z_j will increase as t_i rises. In short, the Nash assumption

[7]For any vector $\bar{x}=(\bar{x}_1,\ldots,\bar{x}_n)$, the notation $\bar{x}/x_i$ denotes $(\bar{x}_1,\ldots,\bar{x}_{i-1},x_i,\bar{x}_{i+1},\ldots,\bar{x}_n)$.

that the other jurisdictions keep their tax rates invariant to changes in t_i implies that their expenditure levels vary positively with t_i.

This suggests that one might wish to consider a different type of equilibrium: one in which each jurisdiction expects the other jurisdictions to keep their *expenditure levels* fixed. Given any vector of expenditure levels z, suppose that one can solve for the vector of tax rates $t(z)$ such that all of the government budget constraints (1) are simultaneously satisfied. (The conditions under which this can be done are discussed below.) Define:

Z-equilibrium. A vector z^* is a *Z-equilibrium* if, for all i, z_i^* maximizes $v_i(t[z^*/z_i])$ with respect to z_i.

If jurisdiction i expects jurisdiction j to keep z_j constant for any choice of z_i, this means that the tax increase that would be required to finance some additional expenditures in i would be associated with a *reduction* in t_j. This is because j must cut its tax rate to keep its expenditures constant as jurisdiction i raises its expenditure level and capital tax rate.[8]

As a matter of terminology, let us say that a jurisdiction has *T conjectures* if it expects other jurisdictions to use their tax rates as strategic variables, and *Z conjectures* if it believes the other jurisdictions are using expenditure levels as strategic variables.[9] To clarify the distinction between T and Z equilibria, it is helpful to consider some examples in which equilibrium tax rates, welfare levels, etc. can be explicitly calculated.

Examples. Let there be two jurisdictions, each with production functions that are quadratic in capital, and hence with linear marginal product of capital schedules:

$$f_i'(k_i) = a_i - b_i k_i, \qquad a_i > 0, \quad b_i > 0. \tag{5}$$

Two different specifications of preferences and capital ownership are considered:

Example 1. $u_i(x_i, z_i) = \phi_i^1(x_i, z_i) \equiv \sqrt{x_i} + z_i$; $\theta_0 = 1, \theta_1 = \theta_2 = 0$.

[8]This follows from the fact that changes in tax policy in one jurisdiction affect the tax base in another. Scotchmer (1986) considers Nash equilibria for property-value-maximizing jurisdictions (with mobile labor, rather than capital) that take other jurisdictions' expenditure levels as given. In one version of the model, public expenditures are financed by distortionless land taxes. In this case, a Nash equilibrium in expenditures is identical to a Nash equilibrium in land tax rates. A second version of the model allows for the possibility that communities may also set head tax rates. With mobile labor, this creates a new interdependency between the tax bases of different communities. In this case, it appears that the choice of strategic variables would affect the Nash equilibrium of the model.

[9]The concept of conjectural variations is not an essential part of the interpretation or analysis of the model, but the terminology is suggestive and therefore perhaps easy to remember.

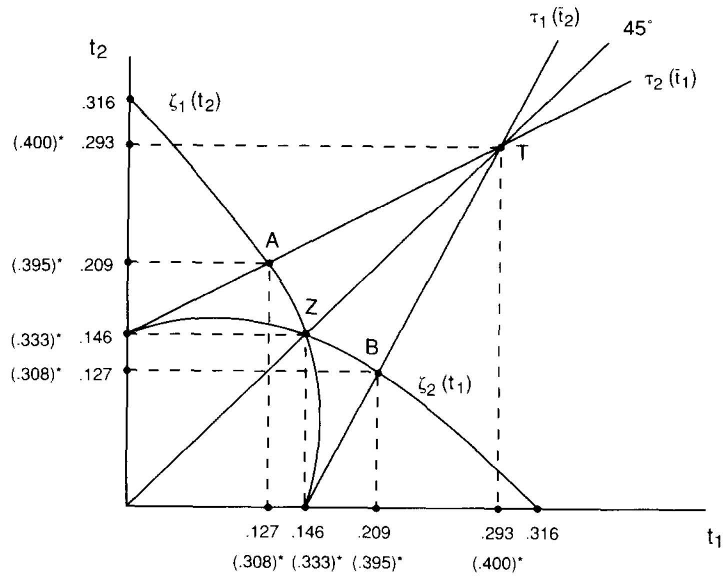

Fig. 1.

Note: Numbers marked with * correspond to Example 2 and are not drawn to scale.

Example 2. $u_i x_i, z_i) = \phi_i^2(x_i, z_i) \equiv x_i + \frac{1}{3}\log(z_i)$; $\theta_0 = 0, \theta_1 = \theta_2 = \frac{1}{2}$.

For the numerical calculations, assume that $b_i = \bar{k} = 1$. In Example 1, the value of a_i does not affect the computed outcomes given below; in the second example, we assume that $a_i = 1$.[10] Note that Examples 1 and 2 give us two quite different cases: both are quasi-linear preference structures, Example 1 implying a marginal propensity to consume of 0 for the private good and 1 for the public good, and conversely for Example 2.

For any given $\bar{t}_j$, one can determine that tax rate t_i that maximizes $v(t_i, \bar{t}_j)$. In both examples, this maximization problem has a unique solution $t_i = \tau_i(\bar{t}_j)$. For Example 1, with the assumed numerical values for the parameters, the function $\tau_i(\bar{t}_j)$ is plotted in fig. 1. This is jurisdiction i's reaction function when it has T conjectures. Symmetrically, $\tau_j(\bar{t}_i)$ is j's reaction function under T conjectures.

For any given $\bar{z}_j$, one can determine the expenditure level z_i that maximizes $v_i(t[z/\bar{z}_j])$. This solution, call it z_i^*, is unique in our example. For Example 1, with the assumed numerical values for the parameters, the

[10]Many other combinations of parameter values would be chosen without affecting the nature of the example. The derivations and calculations for these examples were performed using MACSYMA, a symbolic manipulation program developed at the MIT Laboratory for Computer Science and supported since 1982 by Symbolics, Inc. of Burlington, MA.

resulting tax rates $(t_i, t_j) = t(z_i^*, \bar{z}_j)$ can be plotted as the curves $t_i = \zeta_i(t_j)$, for $i = 1, 2$, in fig. 1, with higher values of t_j corresponding to higher values of $\bar{z}_j$. This is jurisdiction i's reaction function, in tax rate space, under Z conjectures.[11]

Fig. 1 identifies four possible Nash equilibria. These are illustrated by the tax rate pairs corresponding to the points T, Z, A, and B. T is the unique T-equilibrium and Z is the unique Z-equilibrium. A is the 'mixed' Nash equilibrium that results when 1 has Z conjectures and 2 has T conjectures, and B is the Nash equilibrium for the converse case.[12]

The intuition for the geometric configuration of the Nash equilibria follows from consideration of the elasticity of the tax base that a jurisdiction faces under T and Z conjectures. Under T conjectures, each jurisdiction expects some outflow of capital to occur if it increases its expenditures and tax rate while the other keeps its tax rate fixed. This tax base elasticity 'discourages' tax rate increases – it raises the welfare cost of obtaining public funds in the jurisdiction. Under Z conjectures, each jurisdiction i expects an even larger outflow of capital as it raises its expenditures and tax rate. This is because the other jurisdiction j *lowers* its tax rate in response to the tax increase in i (in order to keep z_j constant) and this magnifies the flow of capital from i to j. From the viewpoint of i, then, the tax base is more elastic (or the strategic situation vis-à-vis the other jurisdiction is 'more competitive') under Z conjectures. The utility-maximizing expenditure level and tax rate for i is therefore smaller under Z conjectures. Hence the curve ζ_1 lies to the left of the curve τ_1. The same argument explains why ζ_2 lies below τ_2. These properties of the reaction functions give rise to the pattern of Nash equilibria shown in fig. 1. While fig. 1 only shows the reaction functions drawn to scale for the Example 1, the diagram for Example 2 (not shown to save space) would be configured in essentially the same way. Thus, points in fig. 1 are identified numerically with two numbers. The numbers without asterisks correspond to Example 1, and to the actual scale of the diagram. The numbers with asterisks correspond to Example 2; these numerical values do not correspond to the scale of the diagram.

The multiple Nash equilibria portrayed in fig. 1 present a dilemma. If tax rates are the appropriate strategic variables, then the model has a unique Nash equilibrium at T. Similarly, there is a unique Nash equilibrium at Z if expenditure levels are the strategic variables. Thus, the assumed preferences

[11]It might seem more natural to display reaction functions under Z conjectures in (z_i, z_j) space. However, for comparison with reaction functions under T conjectures, it is convenient to map these Z-reaction curves in (t_i, t_j) space. Formally, let $\xi_j(z_i)$ be j's best choice of z_j given z_i. Then the curve $\zeta_j(t_i)$ in fig. 1. is the graph of the parameterized curve $(t_i[z_i, \xi_j(z_i)], t_j[z_i, \xi_j(z_i)])$. That is, it shows the tax rates for i and j when i sets z_i and j makes its best reply.

[12]These pure strategy equilibria should not be confused with equilibria in mixed strategies. 'Mixed' simply refers to the fact that the two governments are using different strategic variables.

and technology provide a model that is nicely behaved in these respects. But we face the problem of determining whether tax rates or expenditures would be the more appropriate strategic variables. How is this question to be decided?

Note that this problem has a direct parallel in oligopoly theory. Two popular models of duopoly are the Cournot and Bertrand models. A Cournot equilibrium is a Nash equilibrium in quantities, while a Bertrand equilibrium is a Nash equilibrium in prices. As in the present analysis, these two competing models present the problem of deciding which of two (or more) Nash equilibria offers the preferred model. To resolve this issue, Singh and Vives (1984) suppose that the duopolists engage in a two-stage game.[13] In the first stage, they commit themselves to a choice of strategic variables, either prices or quantities. In the second stage, a Nash equilibrium in these variables is attained. The choice of strategic variables at the first stage is thus taken in light of the payoffs that are ultimately realized at the second stage. Singh and Vives show that there can often be a dominant strategy at the first stage of this game, that is, one duopolist may find it profit-maximizing to commit to price or quantity at the first stage of the game independently of the commitment made by the other duopolist. Depending on the demand conditions in the model, it can be a dominant strategy at the first stage to commit to price at the strategic variable at the second stage, in which case the model predicts a Bertrand equilibrium, or, alternatively, a commitment to quantity as the strategic variable may be the dominant strategy at the first stage, in which case the model predicts a Cournot equilibrium.

The Singh–Vives approach can also be applied in the present context. Suppose that the two jurisdictions engage in a two-stage game. In the first stage, they commit themselves to a choice of strategic variable, either expenditures or taxes. In the second stage, a Nash equilibrium in these variables is determined. This approach has the quite attractive feature that the choice of strategic variable is made endogenous to the model, rather than being derived from some deus ex machina. Let us refer to such a two-stage game as the *two-stage fiscal competition game.*

From the viewpoint of jurisdiction i, a move in the first stage amounts to a determination of the reaction function of jurisdiction j for the second stage. If i commits to taxes as a strategic variable, it knows that j's second stage behavior is described by the reaction function τ_j, whereas if it commits to expenditures, j will act according to the reaction function ζ_j. Jurisdiction i might 'induce' j's reaction function in a variety of ways. For example, policymakers might adopt and communicate certain procedures targeting one policy instrument or the other in a way that signals commitment. A school board might be obliged to submit a certain millage (property tax) rate to the

[13] See also Cheng (1985) and Cheng and Wong (1990).

Table 1
Payoff matrix for two-stage fiscal competition game.[a]

	Jurisdiction 2	
Jurisdiction 1	Tax rate (t_2)	Public expenditure (z_2)
Tax rate (t_1)	(0.500, 0.500) (−0.361, −0.361)*	(0.452, 0.421) (−0.374, −0.393)*
Public expenditure (z_1)	(0.421, 0.452) (−0.393, −0.374)*	(0.428, 0.428) (−0.389, −0.389)*

[a]*Source*: Author's calculations. First coordinate in each pair is payoff to jurisdiction 1.
Asterisks denote payoffs in Example 2.

electorate for referendum approval, and then be required to work within the budget constraint implied by this tax rate. The budget procedures for the U.S. Federal government established under the Gramm–Rudman–Hollings bill and, more recently, the Budget Enforcement Act, could be interpreted as a commitment to adjust expenditures down to the level of revenue supported by the existing tax structure. (It must be admitted, however, the experience of the past five years does not indicate that this 'commitment' should be taken very seriously.) On the other hand, policymakers could make commitments to certain program objectives, for example in the form of so-called 'entitlement' programs, or through the setting of program standards (e.g., determining educational policy by setting norms for student–teacher ratios, exam scores, library standards, etc.), implying that the tax rate would have to adjust as required.

To model the first stage of the game, we form a 2×2 payoff matrix as shown in table 1. The entries in this table are the payoffs to each jurisdiction in each of the four possible second-stage Nash equilibria. Each position in the table has two pairs of numbers. The first (upper) pair shows the payoffs to 1 and 2 under the given strategies for Example 1; the second (lower) pair, marked with an asterisk, shows the payoffs under the same strategies for Example 2. The numbers in the table show, for both examples, that it is a dominant strategy in the first stage of the game for each jurisdiction to commit to the use of its tax rate as a strategic variable in the second stage of the game. Jurisdiction 1 is better off inducing the reaction function τ_2 rather than ζ_2, no matter whether 2 induces 1 to move along τ_1 or ζ_1, and conversely for jurisdiction 2. In these particular numerical examples, then, there is a non-trivial justification for choosing the T-equilibrium as the final predicted policy equilibrium for the two jurisdictions. Since the model is continuous in its parameters, this conclusion is valid at least for all parameter values in some neighborhood of the given numerical specifications.

These two examples can be generalized, at least partially. To do so in a way that minimizes technical complications, several simplifying assumptions will be imposed. First, continue to assume that the jurisdictions are symmetric, with identical preferences, endowments, and technologies. Noting that with just two jurisdictions the equilibrium allocation of capital can be written as $k_1(t_1-t_2)$, $k_2(t_1-t_2)$, with $k_i' = -k_2' < 0$, define the function $R_i(t_i, t_j) \equiv t_i k_i(t_1 - t_2)$. It will be assumed that R_i is monotonically increasing in t_i (for any given t_j) (a) for all values of t_i or (b) for all t_i in some closed interval (which must include 0) and that R_i reaches its global maximum in this interval.[14] Restricting attention henceforth to values of t_i in this interval, the equation $z_i = R_i(t_i, t_j)$ can be inverted to give $t_i = \gamma_i(z_i, t_j)$ such that $\partial\gamma_i/\partial z_i > 0 > \partial\gamma_i/\partial t_j$. It will be further assumed that the maximization problems $\max_{\langle t_i \rangle} v_i(t_i, t_j)$ given t_j and $\max_{\langle t_i \rangle} v_i(t_i, \gamma_j[z_j, t_i])$ given z_j have unique solutions, denoted $\tau_i(t_j)$ and $\psi_i(z_j)$, respectively. The uniqueness of these solutions implies continuity of the functions τ_i and ψ_i. The functions $(\psi_i(z_j), \gamma_j(z_j, \psi_i(z_j)))$ define a continuous parameterized curve $\zeta_i(z_j)$ in the (t_1, t_2) plane. It will finally be assumed that the curve τ_i intersects τ_j and ζ_i once and only once, and similarly for the curve ζ_j. By symmetry, this implies that τ_1 and τ_2 intersect at some point T on the 45° line, as do ζ_1 and ζ_2.[15] These key assumptions will be referred to as *symmetry*, *monotonicity of the revenue functions*, *uniqueness of best replies*, and *uniqueness of second-stage Nash equilibria*.

The effect of one jurisdiction's tax rate on welfare in the other is important in the following analysis. From (1) and (4),

$$\left.\frac{\partial z_i}{\partial t_j}\right|_{t_i} = t_i |k_i'| > 0, \tag{6a}$$

$$\left.\frac{\partial x_i}{\partial t_j}\right|_{t_i} = (k_i - \theta_i \bar{k}) |f_i'' k_i'| > 0 \quad \text{if} \quad k_i \geqq \theta_i \bar{k}. \tag{6b}$$

The first of these expressions shows that an increase in the tax rate in j must raise the tax base in i and thus (for a given t_i) the level of public expenditures. The second expression shows that the effect of t_j on after-tax income (or the level of private good consumption) in i is determined by terms-of-trade considerations. An increase in t_j lowers both the gross

[14] A sufficient (but not necessary) condition for this is that $f_i''' \leqq 0$. The quadratic production function satisfies this condition.

[15] Note that the Singh–Vives two-stage approach to selecting among strategic variables cannot be implemented in a simple way if there are multiple equilibria in one or more pairs of strategic variables, since payoffs differ at different second-stage equilibria and the choice of strategic variables at the first stage does not have a unique payoff. Thus, uniqueness of second stage equilibria – or, equivalently, a selection rule for choosing among multiple equilibria – is practically a necessary condition for the two-stage approach to be considered.

(purchase) price of capital and the net (selling) price of capital for jurisdiction i. If i is a capital importer, $k_i > \theta_i \bar{k}$ and private good consumption in i rises as t_j rises, whereas the opposite is true if i is a capital exporter. Since welfare is increasing in private and public good consumption, it follows that

$$\frac{\partial v_i(t_i, t_j)}{\partial t_j} > 0 \quad \text{if} \quad k_i(t_1 - t_2) \geqq \theta_i \bar{k}. \tag{7}$$

By symmetry, $k_i(t_1 - t_2) \geqq \frac{1}{2}\bar{k} \geqq \theta_i \bar{k}$ when $t_i \leqq t_j$. In particular, $\partial v_i(t_i, t_j)/\partial t_j > 0$ along the 45° line in the (t_1, t_2) plane.

Crucial information about the τ_i and ζ_i curves can be established using a simple revealed-preference arguments. Consider the point Z on the 45° line at which ζ_j and ζ_i intersect. Let z^* denote the common value of z_1 and z_2 at this point, and let the corresponding tax rates be denoted by $t_1 = t_2 = t^*$. By the definition of $\zeta_1, t_1 = t^*$ maximizes $v_1(t_1, \gamma_2[z_2^*, t_1])$. But for all $t_1 \leqq t^*$, monotonicity implies that $\gamma_2(z_2^*, t_1) > t^*$, and since v_1 is increasing in t_2 everywhere to the left of the 45° line, it follows that $v_1(t_1, t^*) < v_1(t_1, \gamma_2[z_2^*, t_1]) \leqq v_1(t^*, t^*)$ for all $t_1 \leqq t^*$. Since $t_1 = \tau_1(t^*)$ maximizes $v_1(t_1, t^*)$ (by definition of τ_1), it follows that $\tau_1(t^*) > t^*$. Thus, the τ_1 curve lies to the right of ζ_1 at $t_2 = t^*$. Exactly the same argument holds for any value of $t_2 > t^*$, establishing that τ_1 lies to the right of ζ_1 everywhere above the 45° line.

From the above considerations, it follows that the intersection of ζ_1 and ζ_2 at point Z must lie closer to the origin than the intersection of τ_1 and τ_2 at point T. Furthermore, the intersection of ζ_1 and τ_2 at point A must lie above the 45° line, and conversely for the intersection of ζ_2 and τ_1 at point B. Let the tax rate and expenditure levels at the T- and Z-equilibria be denoted (t^{**}, z^{**}) and (t^*, z^*), respectively. Let the tax rates at points A and B be denoted (t_i', z_i') and (t_i'', z_i''), respectively. See fig. 2.

It is necessary to determine some of the welfare properties of the four possible Nash equilibria of the second-stage game in order to evaluate the desirability of commitments to tax rates or expenditures as strategic variables. Consider first the welfare ranking of points Z and A from the viewpoint of jurisdiction 1. Since ζ_1 crosses the 45° line only once (by symmetry and the assumption of uniqueness of Z-equilibria), it must be the case that $z_2' > z^*$. Since $t_1 = t_2 = t^*$ at Z and $t_1' < t_2'$ (since A lies above the 45° line), $z_2' > z^*$ implies $t_2' > t^*$. By (7), $v_1(t_1, t_2)$ is increasing in t_2 on or above the 45° line, and hence $v_1(t^*, \gamma_2[z_2', t^*]) > v_1(t^*, \gamma_2[z^*, t^*])$. Furthermore (by the definition of ζ_1), $v_1(t_1', \gamma_2[z_2', t_1]) > v_1(t_1, \gamma_2[z^*, t_1])$ for all $t_1 \neq t_1'$. It follows that the welfare of jurisdiction 1 is higher at Z than at A. Symmetrically, the welfare of jurisdiction 2 is higher at B than at Z.

Next, consider jurisdiction 1's welfare ranking of the points B and T.

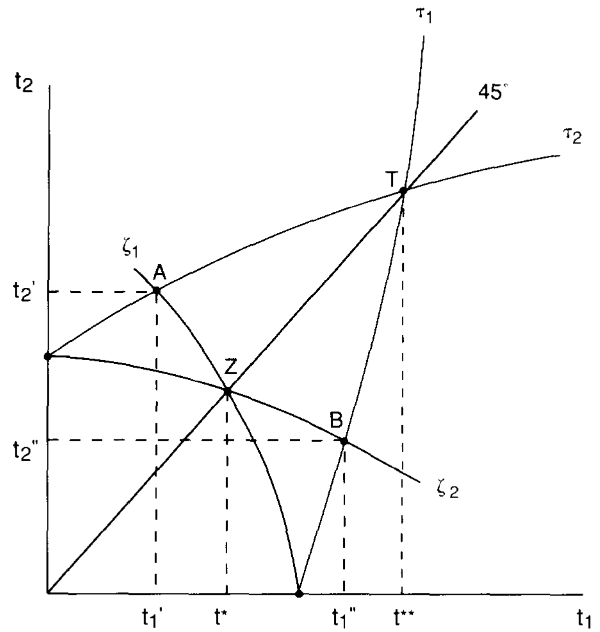

Fig 2.

Suppose first that all capital is owned by absenteees, as assumed in Example 1, so that $k_1(t_1-t_2) \geqq \theta_1\bar{k}=0$ for all tax rates. It then follows from (7) that $v_1(t_1,t_2)$ is increasing in t_2 along the curve τ_1 between points B and T, and thus $v_1(t^{**},t^{**})>v_1(t_1'',t_2'')$. Thus, jurisdiction 1 prefers the outcome at T to that at B when all capital is held by absentee owners. Symmetric reasoning shows that jurisdiction 2 prefers T to A. It now follows that tax rates are the dominant strategy in the first stage of the two-stage fiscal competition game when capital is owned by absentees and when the regularity conditions (symmetry, monotonicity, and uniqueness) are met.

Unfortunately, this simple argument does not apply to the case where capital is owned, wholly or in part, by the residents of the two jurisdictions. The reason is that jurisdiction 1 may be a capital exporter when $t_1>t_2$. For instance, when $\theta_1=\theta_2=1/2$ and $\theta_0=0$, $k_1(t_1,t_2) \leqq \theta_1\bar{k}$ for all $t_1 \geqq t_2$. In such a case, the effect of t_2 on welfare in jurisdiction 1 is ambiguous: while an increase in t_2 raises z_1 by increasing the tax base in jurisdiction 1 [eq. (6a)], it lowers the level of private good consumption [eq. (6b)]. The latter effect arises because an increased tax on capital in jurisdiction 2 lowers the economy-wide net return on capital, and jurisdiction 1, as a capital exporter, is harmed by this. This harm could conceivably outweigh the benefit that

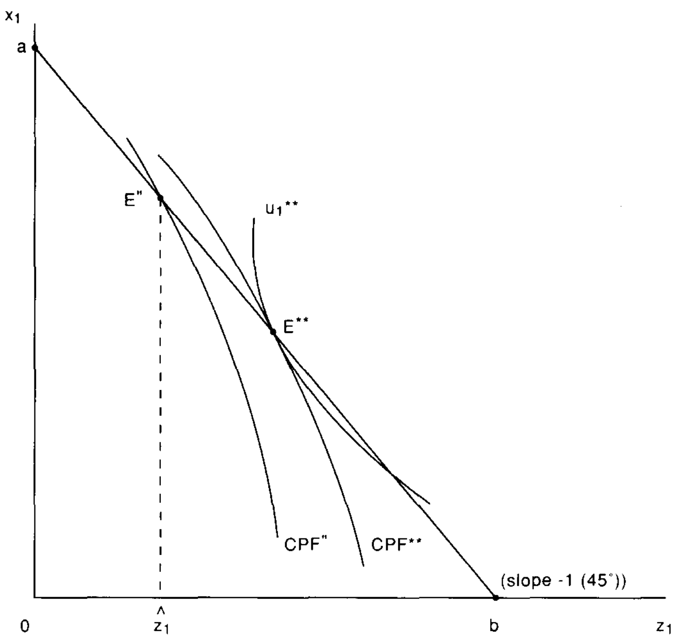

Fig 3.

jurisdiction 1 gets from the increase in its capital tax base, and thus the level of its public expenditure, that results from an increase in the tax rate in jurisdiction 2. It thus seems possible that jurisdiction 1 would prefer the outcome at B to that in the T-equilibrium at T.

To examine this issue in more detail, consider the case where, as in Example 2, all capital is owned by residents of the two jurisdictions (i.e., $\theta_0 = 0$) and where the production technology is quadratic, as specified in (5). The curve ab in fig. 3, with a slope of -1, shows the set of consumption opportunities that are available to the residents of jurisdiction 1 when capital is immobile.[16] Any symmetric equilibrium with mobile capital must also yield a consumption bundle along this line, since each jurisdiction obtains half of the capital stock in such an equilibrium. Wilson (1991) shows that a T-equilibrium results in a consumption bundle on this frontier. This consumption bundle is located at the tangency of an indifference curve with a 'consumption possibility frontier' showing the consumption allocations available to a single jurisdiction, given the tax rate imposed by the other

[16] ab is the graph of the equation $x_1 = f_1(\bar{k}/2) - z_1$. A figure of this type appears in Zodrow and Mieszkowski (1986). I am indebted to J. Wilson for several helpful discussions leading to the arguments in the next paragraphs, which also draw on Wilson (1991).

jurisdiction and given that the interjurisdictional allocation of capital adjusts to equalize net returns. In fig. 3, the T-equilibrium is illustrated by the tangency of the consumption possibility frontier CPF^{**} and the indifference curve u_1^{**} at the point E^{**}. The (absolute value of the) slope of CPF^{**} shows how much private good consumption jurisdiction 1 must sacrifice in order to obtain an additional unit of z_1, given that the public good must be financed by the tax on capital. This slope, denoted MC_1 because it shows the effective marginal cost of the public good to jurisdiction 1, is calculated by differentiating (1) and (4) with respect to t_1:

$$MC_1(t_1,t_2) = -\frac{\mathrm{d}x_1}{\mathrm{d}z_1} = -\frac{\partial x_1/\partial t_1}{\partial z_1/\partial t_1}$$

$$= \frac{k_1 f_1'' k_1' - \frac{\bar{k}}{2}(f_1'' k_1' - 1)}{k_1 + t_1 k_1'}$$

$$= \frac{k_1 + \bar{k}/2}{2k_1 - t_1/b}, \tag{8}$$

where the last equality uses the assumption of a quadratic production function as specified in (5).[17] Clearly, $MC_1(t_1,t_2) > 1$ for any $t_1 = t_2$, since $k_1 = \bar{k}/2$ implies that the numerator of MC_1 is just k_1 and $t_1 > 0 > k_1'$ implies that the denominator is less than k_1. Thus CPF^{**} in fig. 3, and the indifference curve u_1^{**} tangent to it, must cut ab from above at the point E^{**}.

Now consider the equilibrium corresponding to point B in fig. 2. The crucial question is whether this equilibrium yields a consumption bundle lying above u_1^{**} in fig. 3. To answer this question, consider the consumption possibility frontier CPF'' facing jurisdiction 1 when $t_2 = t_2''$. First, define $\hat{z} = t_2''\bar{k}/2$, the level of public expenditure that each jurisdiction obtains when both set their tax rates equal to t_2''. Since $t_2'' < t^{**}$, $\hat{z} < z^{**}$. This implies that CPF'' must cross ab at some point E'' to the left of E^{**}, and CPF^{**} must lie above the point E''. The equilibrium B in fig. 2 is obtained by jurisdiction 1 choosing a best reply to t_2'', that is, a utility-maximizing point on CPF''. Since we know that $t_1'' > t_2''$, this point must lie on CPF'' somewhere to the right of E'', that is, it must correspond to a value of $z_1'' > \hat{z}$.

To attain any $z_1 > \hat{z}$ requires a higher tax rate in jurisdiction 1 when $t_2 = t_2''$ than when $t_2 = t^{**}$, i.e., $\gamma_1(z_1, t_2'') > \gamma_1(z_1, t^{**})$. But, differentiating (8),

[17]Given (5), $f_i'' = -b$ and $k_i' = (f_1'' + f_2'')^{-1} = -(2b)^{-1}$.

$$\frac{\partial MC_1}{\partial t_1}=\frac{-k_1' t_1-2bk_1'\bar{k}+k_1+\dfrac{\bar{k}}{2}}{b\left(2k_1-\dfrac{t_1}{b}\right)^2}>0,$$

$$\frac{\partial MC_1}{\partial t_2}=\frac{k_1'\left(\dfrac{t_1}{b}+\bar{k}\right)}{\left(2k_1-\dfrac{t_1}{b}\right)^2}<0.$$

Thus, at any level of $z_1>\hat{z}$, CPF'' must be steeper than CPF^{**} because the value of t_1 along CPF'' must be higher than that along CPF^{**} and the value of t_2 is equal to t_2'' which is lower than t^{**}. Since CPF^{**} lies above CPF'' at $z_1=\hat{z}$ and since CPF'' is steeper than CPF^{**} for all $\hat{z}_1>\hat{z}$, CPF'' must lie below CPF^{**} at all points to the right of $\hat{z}$. But this means that the welfare of jurisdiction 1 must be lower when $t_2=t_2''$ than when $t_2=t^{**}$. Hence, jurisdiction 1 prefers the outcome at point T to that at point B. Similarly, jurisdiction 2 prefers the outcome at T to that at A.

Note the role played in this argument by the assumption of a quadratic production technology. It insures that the consumption possibility frontier for jurisdiction 1 must shift downward at all points below the curve ab as the tax rate in jurisdiction 2 rises. This means that jurisdiction 1 must be worse off when jurisdiction 2 lowers its tax rate, in effect establishing the inequality (7) for cases where jurisdiction 1 is a capital exporter rather than a capital importer. In terms of fig. 3, the only way that jurisdiction 1 could benefit from a reduction in the tax rate in jurisdiction 2 would be for the consumption possibility curve through E'' to cut above CPF^{**} to the right of z^{**} and then to cut above u_1^{**}. The quadratic production technology makes it impossible for this to occur.

These results can be summarized as follows:

Proposition 1. Assume symmetry, monotonicity of the revenue function, uniqueness of best replies, and uniqueness of second-stage Nash equilibria. Assume that either (a) all capital is owned by absentee owners, or (b) there is no absentee ownership and the production technology is quadratic. Then it is a dominant strategy for both jurisdictions to commit to tax rates as strategic variables in the first stage of the two-stage fiscal competition game.

Thus, Examples 1 and 2 have been generalized by replacing specific assumptions about preferences with general regularity conditions on preferences and technology sufficient to rule out essentially technical complications.

Of course, Proposition 1 is still not very general. Is it possible that tax rates might *not* be the dominant choice of strategic variables in the two-stage fiscal competition game for some reasonable specifications of the data of the model? This question remains open. In the symmetric case, it seems conceivable that terms-of-trade effects could be sufficiently strong to make the outcome at B preferred to T by jurisdiction 1, although of course we have shown that this cannot occur with a quadratic production technology. If this did happen, there would exist *two* (symmetric) Nash equilbria in the two-stage fiscal competition game. In each of these equilibria, one jurisdiction would commit to its public expenditure level as a strategic variable while the other would commit to its tax rate. (Given the regularity conditions, it could never happen that *both* jurisdictions would commit to their public expenditure levels as strategic variables in the first stage of the two-stage fiscal competition game.) If one relaxes the symmetry assumption, the range of theoretical possibilities expands greatly. As noted above, the underlying continuity of the model sufficies to demonstrate that there certainly are asymmetric specifications of preferences, endowments, and technologies close to those used in Examples 1 and 2 in which tax rates emerge as the dominant strategy in the first stage of the two-stage fiscal competition game. However, it is difficult to know how far these results can be generalized.

Perhaps this discussion estalishes a modest 'presumption' in favor of the hypothesis that jurisdictions actually do compete in tax rates rather than expenditure levels in a small-number setting. At least it is clear that this is a very non-pathological outcome within the Singh–Vives framework. The results are obviously not completely general, however. One can only claim that they illustrate the potential workability of the Singh–Vives approach to the difficult problem of selection of strategic variables in fiscal competition models. We now turn to another application in which the same analytical issues appear in a different guise.

3. A model of interjurisdictional spillovers

The model of section 2 focused on tax interactions between two jurisdictions. Now suppose instead that there are interactions on the expenditure side, in the form of spillover benefits from public goods. There are numerous examples of such situations. For instance, two jurisdictions (cities, states, or countries) might be located in the same airshed or watershed, so that environmental policies undertaken in one, such as water treatment, generate benefits in the other.

Spillovers are important because they may result in inefficiently small levels of public good provision. In the literature, two ways of dealing with this inefficiency have been discussed. The first is to shift the activity to a higher level of government, so that the benefits of the activity in question are internal to jurisdiction providing it. Failing this, intergovernmental matching grants can be introduced to provide corrective subsidies that internalize the externality along standard Pigovian lines [Oates (1972), Boadway (1980)]. This is an argument of great practical importance. It has been used to justify the structure of the AFDC program in the U.S., which is the primary cash redistribution program aimed at the poor, under which the Federal government may bear 50–80% of the cost of the welfare expenditures undertaken by states in the U.S.[18]

Proper analysis of these policy issues inevitably requires a model of the behavior of lower-level governments that provide public goods with spillover benefits. It is from such an equilibrium model that one derives the conclusion that there is underprovision of the public goods in equilibrium (a predictive statement). Moreover, the optimal corrective policy is sensitive to the properties of such an equilibrium model. In particular, the magnitude of any corrective subsidy depends on the magnitude of the departure from efficient public good provision that occurs in equilibrium. We shall discuss here a more-or-less standard small-number equilibrium model of spillovers, following previous authors such as Williams (1966). In contrast to previous work, however, the focus will be on the choice of strategic variables in such a model.

A simple model of the strategic interactions between two jurisdictions providing public goods with spillover externalities can be formulated as follows. As in section 2, suppose that welfare in each jurisdiction can be represented by a well-behaved utility function defined over consumption of a private good and a public good. Let x_i denote private good consumption in jurisdiction i, and let s_i denote public good consumption. (As examples, think of s_i as an index of water quality, safety, fire hazard, road congestion, etc.) Public good consumption in jurisdiction i is assumed to depend on the level of public expenditure undertaken both by itself, denoted z_i, and by the other jurisdiction, denoted z_j, as represented by the production function $s_i = f_i(z_i, z_j)$. Thus, the public expenditures (e.g., on water treatment, police, etc.) are inputs into a production process that yields the final public good. In order to minimize technical difficulties, assume that both goods are normal goods in both jurisdictions. Also assume that the production function for public goods is linear and that own-expenditures are (weakly) more productive than the expenditures of the other jurisdiction:

$$s_i = \alpha_{ii} z_i + \alpha_{ij} z_j, \qquad \alpha_{ii} \geqq \alpha_{ij}. \tag{9}$$

[18]For further disussion of such policies, see, e.g., Brown and Oates (1987), Wildasin (1991), and references therein.

While simple and tractable, this specification includes as special cases the situations where there are no spillovers ($\alpha_{ij}=0$) and perfect spillovers ($\alpha_{ij}=\alpha_{ii}$).

Suppose that each jurisdiction has some exogenously-given income y_i, and that it can finance its public expenditure through lump-sum taxation. The community then faces a consolidated private and public budget constraint

$$x_i + z_i = y_i. \tag{10}$$

Each jurisdiction seeks to maximize its welfare subject to (10). However, when there are spillovers, the welfare of each jurisdiction depends on decisions made by the other. This is seen most clearly by substituting the production function f_i and the budget constraint (10) into the utility function $u_i(x_i, s_i)$ to get what we will call the 'indirect utility function' $v_i(z_i, z_j) \equiv u_i(y_i - z_i, f_i[z_i, z_j])$.

In the literature, the most common approach to modelling the strategic interactions between jurisdictions in such a reciprocal externality relationship is to assume that they find a Nash equilibrium in expenditure levels, (z_i, z_j).[19] When jurisdiction i treats jurisdiction j's expenditures as given, we say that it has Z conjectures. Formally, we can define an equilibrium when both jurisdictions have Z conjectures as follows:

Z-equilibrium. A vector $z^* = (z_i^*, z_j^*)$ is a *Z-equilibrium* if, for both i, z_i^* maximizes $v_i(z_i, z_j^*)$ with respect to z_i.

As an alternative to Z-conjectures, one might assume that each jurisdiction chooses a level of public good consumption, s_i, taking as given the level of public good consumption s_j in the other jurisdiction. For example, each jurisdiction might set a water quality standard (or an index of crime, fire hazard, educational attainment, etc.) taking as given the water quality standard in the other jurisdiction. In the case of income redistribution by state governments, let s_i be interpreted as the welfare (i.e. utility) of the poor in each state. One might plausibly argue that the extent of each state's redistributive activity is based on the 'need' of the poor – i.e., some target standard of living for the poor. Such behavior would correspond to using s_i as the strategic variable. A priori, this behavior rule is no less plausible (in some cases, it is more plausible) than taking the expenditure level of the other jurisdiction as given. If jurisdiction i treats $s_j = \bar{s}_j$ as given, we say that i has S conjectures. This means that as i adjusts its expenditures z_i, z_j will

[19]See Williams (1966). Pauly (1970) and Boadway et al. (1989). For a thorough discussion of Nash and consistent conjectures equilibria, see Cornes and Sandler (1986).

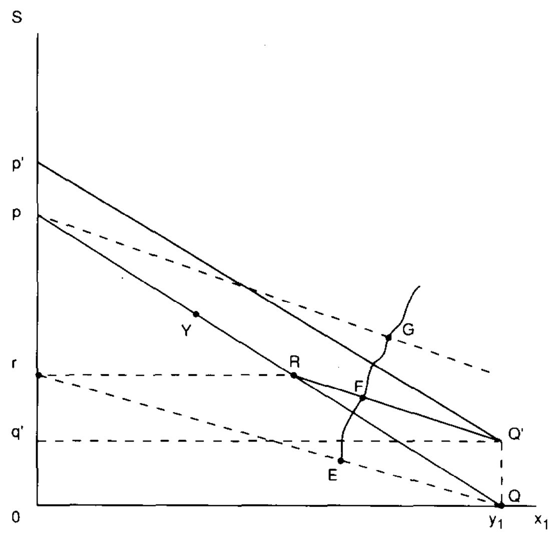

Fig 4.

adjust to maintain $\bar{s}_j$. Thus, we can solve (9) for $z_j = \psi_j(z_i, \bar{s}_j) \equiv (\bar{s}_j - \alpha_{ji} z_i)/\alpha_{jj}$, provided that this is non-negative. For $z_i \geqq \bar{s}_j/\alpha_{ji}$, we set $z_j = \psi_j(z_i \bar{s}_j) = 0$. The problem facing jurisdiction i under S conjectures is thus to choose z_i to maximize $v_i(z_i, \psi_j(z_i, \bar{s}_j))$. Hence, we can define

S-equilibrium. A vector $z^* = (z_i^* z_j^*)$ is an S-equilibrium if, for both i, z_i^* maximizes $v_i(z_i, \psi_j[z_i, z_j^*])$ with respect to z_i.

Let us now consider in more detail the problems facing jurisdiction i under Z and S conjectures. As in section 2, the crucial question is to determine the nature of the constraints facing the jurisdiction in each case.

Under Z-conjectures, the combinations of x_i and s_i available to the jurisdiction depend on the given level of $\bar{z}_j$. Using (9) to eliminate z_i in (10), the constraint facing jurisdiction 1 is that

$$\alpha_{11} x_1 + s_1 = \alpha_{11} y_1 + \alpha_{12} \bar{z}_2. \tag{11}$$

This constraint is plotted in fig. 4 for the case $\bar{z}_2 = 0$, as shown by the line PQ; it is also plotted for a positive value of $\bar{z}_2 = \bar{z}_2'$, as shown by the line $P'Q'$. These lines are parallel, and have a slope of $-\alpha_{11}$. Note that since an

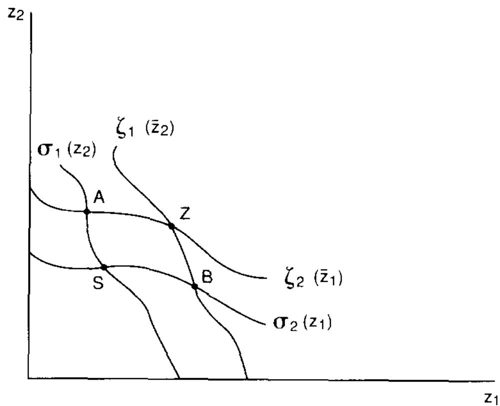

Fig 5.

increase in $\bar{z}_2$ shifts up the constraint facing 1, welfare for 1 must be increasing in $\bar{z}_2$. Given the linearity of the constraint under Z conjectures, the optimal choice of z_1 is unique for every $\bar{z}_2$. Let $z_1 = \zeta_1(\bar{z}_2)$ denote this optimal value. Given the normality assumption, $-1 < \zeta_1'(\bar{z}_2) < 0$. The reaction curve $\zeta_1(\bar{z}_2)$ is plotted in fig. 5, as is the reaction curve $\zeta_2(\bar{z}_1)$ for jurisdiction 2 under Z conjectures. The point Z in fig. 5 depicts the unique Z-equilibrium. For future reference, let point Y in fig. 4 represent 1's choice when $\bar{z}_2 = 0$, i.e., Y has coordinates $(y_1 - \zeta_1(0), \zeta_1(0))$.

Under S-conjectures, jurisdiction 1 treats $\bar{s}_2$ as constant; substituting for $z_2 = \psi_2(z_1, \bar{s}_2)$ into (9) yields

$$s_1 = \alpha_{11} z_1 + \max\left\{\frac{\alpha_{12}}{\alpha_{22}}(\bar{s}_2 - \alpha_{21} z_1), 0\right\}$$

$$= \max\left\{\left(\alpha_{11} - \frac{\alpha_{12}\alpha_{21}}{\alpha_{22}}\right) z_1 + \frac{\alpha_{12}}{\alpha_{22}} \bar{s}_2, \alpha_{11} z_1\right\}. \tag{12}$$

By (12), s_1 is a piecewise linear function of z_1. Since $\alpha_{11} \geqq \alpha_{12}$ and $\alpha_{22} \geqq \alpha_{21}$, s_1 is an increasing linear function of z_1 for $z_1 \in [0, \bar{s}_2/\alpha_{21})$, with slope $\alpha_{11} - \alpha_{12}\alpha_{21}/\alpha_{22} < \alpha_{11}$. For $z_1 \geqq \bar{s}_2/\alpha_{21}$, s_1 is linear in z_1 with slope α_{11}. Using (12) to eliminate z_1 in the constraint (10) yields

$$(\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21})x_1+\alpha_{22}s_1=(\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21})y_1+\alpha_{12}\bar{s}_2$$

$$\text{for} \quad \frac{\alpha_{12}}{\alpha_{22}}\bar{s}_2 \leqq s_1 \leqq \frac{\bar{s}_2}{\alpha_{21}}, \tag{13a}$$

$$\alpha_{11}x_1+s_1=\alpha_{11}y_1 \quad \text{for} \quad s_1 \geqq \frac{\bar{s}_2}{\alpha_{21}}. \tag{13b}$$

When $\bar{s}_2=0$, (13a) is irrelevant, (13b) is equivalent to (11), and the constraint collapses to the curve PQ in fig. 4, corresponding to $\bar{z}_2=0$. When $\bar{s}_2>0$, (13a) is operative for sufficiently low values of s_1. In fig. 4, suppose that $\bar{s}_2=\bar{s}'_2=\alpha_{22}\bar{z}'_2$. Then 1 faces the constraint (13a) for low values of s_1; this part of the constraint is represented by the linear segment $Q'R$. For higher values of s_1, (13b) is applicable. This part of the constraint is represented by the segment RP. For values of $\bar{s}_2$ greater than $\bar{s}'_2$, the segment $Q'R$ shifts up vertically, so that R approaches P. For $\bar{s}_2 \geqq \alpha_{21}\alpha_{11}y_1$, R lies above P and only (13a) is applicable.

The exact decisions made by 1 for various values of $\bar{s}_2$ obviously depend on 1's preferences. No matter what form these preferences take, however, there are some values of $\bar{s}_2$ sufficiently small (certainly in some neighborhood of $\bar{s}_2=0$) that the utility-maximizing choice for 1 lies on the segment of the feasible set corresponding to (13b). But Y is the unique utility-maximizing choice for 1 in this case. When 1 chooses Y, $s_2>\bar{s}_2$ and 2 chooses $\bar{z}_2=0$.

On the other hand, whatever 1's preferences, there some values of $\bar{s}_2$ sufficiently large (certainly for $\bar{s}_2>\alpha_{21}\alpha_{11}y_1$) that 1's optimal choice lies on the segment of the feasible consumption set corresponding to (13a). Let EFG be the 'income expansion path' for 1 corresponding to the 'price' $\alpha_{22}/(\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21})$ for s_1 in terms of x_1 – i.e., the locus of tangencies of 1's indifference curves with the constraint (13a) as $\bar{s}_2$ increases from 0. Given normality of both goods, EFG is upward-sloping. For sufficiently low values of $\bar{s}_2$, i.e., at and near E, the points on this locus are less preferred than Y. For sufficiently high values of $\bar{s}_2$, i.e., at, above, and in some neighborhood below G, the consumption bundles along this locus are preferred to Y. There exists some value of $\bar{s}_2$, say $\bar{s}'_2$ in fig. 4, such that 1 is indifferent between Y and the point F on the locus EFG.

Thus, for $\bar{s}_2 \in (0, \bar{s}'_2)$, 1 chooses point Y, so that $s_2>\bar{s}_2$ and $z_2=0$. At $\bar{s}_2=\bar{s}'_2$, 1 is indifferent between Y and F. For $\bar{s}_2>\bar{s}'_2$, chooses points along the path FG. In such cases, $s_2=\bar{s}_2$ and $z_2>0$. Moreover, since FG is upward-sloping, x_1 increases as $\bar{s}_2$ rises. By (10), z_1 therefore decreases. But if z_1 falls as $\bar{s}_2$ rises, z_2 must rise. We can therefore plot 1's reaction function under S

conjectures in fig. 5 as the curve $\sigma_1(z_2)$. One can show that $-1<\sigma_1'<0$.[20] Also, since $\sigma_1(0)$ corresponds to point F in fig. 4, whereas $\zeta_1(0)$ corresponds to Y, we have $\sigma_1(0)<\zeta_1(0)$. Similarly, let the curve $\sigma_2(z_1)$ in fig. 5 be the reaction function of jurisdiction 2, in (z_1, z_2) space, under S conjectures. Point S is the unique S-equilibrium. Points A and B correspond to 'mixed' Nash equilibria, where one jurisdiction has Z-conjectures and the other has S-conjectures.

There are 4 candidate Nash equilibria. Since A corresponds to a higher value of $\bar{s}_2$ than S does, the welfare of 1 is higher at A than at S. Similarly, since 1's welfare is increasing in $\bar{z}_2$ under Z conjectures, 1 is better off at Z than at B. Symmetric reasoning applies to jurisdiction 2: j prefers Z to A and B to S.

With this information, one can apply the Singh–Vives methodology. Imagine that the jurisdictions play a *two-stage interjurisdictional spillover game*. In the first stage, each commits either to expenditures or to the level of public good consumption as a strategic variable. In the second stage the jurisdictions choose their Nash equilibrium strategies in these variables. Then, in view of the welfare rankings for the points S, Z, A, and B, one has

Proposition 2. Let there be two jurisdictions with well-behaved preferences exhibiting normality in both private and public good consumption. Suppose that the public good production technology in each jurisdiction is characterized by

[20]By normality, using (13a),

$$0<\frac{\partial x_1}{\partial \bar{s}_2}<\frac{\alpha_{12}}{\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21}},$$

and hence $\partial z_1/\partial \bar{s}_2=-\partial x_1/\partial \bar{s}_2$ satisfies

$$-\frac{\alpha_{12}}{\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21}}<\frac{\partial z_1}{\partial \bar{s}_2}<0. \tag{F.1}$$

Since $\bar{s}_2$ is held fixed,

$$\alpha_{22}\frac{\partial z_2}{\partial \bar{s}_2}+\alpha_{21}\frac{\partial z_1}{\partial \bar{s}_2}=1. \tag{F.2}$$

Using (F.2), the slope of the curve $\sigma_1(z_2)$ is determined by

$$\frac{1}{\sigma_1'}=\frac{\partial z_2/\partial \bar{s}_2}{\partial z_1/\partial \bar{s}_2}=\frac{1}{\alpha_{22}}\left(\left(\frac{\partial z_1}{\partial \bar{s}_2}\right)^{-1}-\alpha_{21}\right).$$

By (F.1), this implies

$$-\frac{\alpha_{12}}{\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21}}<\left(\alpha_{21}+\frac{\alpha_{22}}{\sigma_1'}\right)^{-1}<0.$$

The right-hand inequality implies $\sigma_1'<0$, and the left-hand inequality implies $1/\sigma_1'<-(\alpha_{11}/\alpha_{12})<-1$, or $-1<\sigma_1'<0$, as required.

interjurisdictional spillovers satisfying (9). Then it is a dominant strategy for both jurisdictions to commit to public expenditures in the first stage of the two-stage interjurisdictional spillover game. The final policy equilibrium is a Nash equilibrium in public expenditure levels.

Note finally that the S and Z equilibria of this model can be very different from one another. In particular, suppose that spillovers are strong in the sense that public expenditures by the other jurisdiction are nearly as productive as own expenditures (i.e., $\alpha_{ij} \rightarrow \alpha_{ii}$, $\alpha_{ji} \rightarrow \alpha_{jj}$). Then it is easy to see that with symmetric jurisdictions, the point S in fig. 5 approaches the origin: the S equilibrium involves almost no public expenditure and almost no public good provision. In general, of course, the quantitative differences between Nash equilibria with different strategic variables depends on the specific data of the problem – tastes, technology, etc. However, it is clear from this simple example that these differences can be substantial at least in some cases, so the choice of strategic variables can really 'matter'.

4. Conclusion

The preceding sections have each highlighted a recurring issue in models of strategic policy determination. It may be useful to outline briefly the nature of the generic problem exemplified by the models examined in sections 2 and 3. To begin with, recall the specification of non-cooperative games as pioneered by Nash (1951). There are m players each of whom must choose a finite-dimensional strategy vector $s_i = (s_{i1}, \ldots, s_{in})$. Each player has a payoff $v_i(s)$ that depends on the strategy choices $s = (s_1, \ldots, s_m)$ of all players. Then s^* is a Nash equilibrium if s_i maximizes $v_i(s^*/s)$ with respect to s_i, for all i.

The games that we have been analyzing do not fit the standard Nash framework. In particular, they all have one crucial additional feature: they impose a side-constraint on each player's choice of strategy variables. This constraint takes the general form

$$\phi_i(s) = 0, \quad i = 1, \ldots, m. \tag{14}$$

Non-cooperative games with constraints like (14) will be called *constrained* non-cooperative games.

It is easy to see how the models that we have analyzed can be described as constrained non-cooperative games. In section 2, the side constraints ϕ_i were the government budget constraints (1) – which depended both on the own-tax rate and on the other jurisdiction's tax rate. In section 3, the functions ϕ_i were the production technologies for public goods (9), in which both own-expenditures and the expenditures of the other jurisdiction enter.

The same structure of side constraints arises in duopoly or oligopoly theory, although the problem is not usually formulated this way. The standard models assume that firms are able to choose both price and quantity, subject to the demand conditions that they face. In this case, the ϕ_i functions would be what are often called the residual demand functions, showing combinations of price and quantity that are available to one firm, given the price and quantity decisions of its rivals.[21]

In the presence of side conditions like (14), the standard Nash equilibrium concept can no longer be used, since in general not all players can simultaneously choose their s_i vectors and yet be sure that the feasibility constraints (14) are satisfied. Roughly speaking, the constraints (14) remove one degree of freedom for each player. Thus, in Cournot duopoly, each firm uses its residual demand curve to compute its product price as a function of the output choices of all m firms: in effect, (14) is used to solve implicitly for m prices as functions of m quantities. Using the market conditions to 'eliminate' prices leaves m quantities to be chosen by the firms, with no side constraints. At this point, of course, one can look for the 'unconstrained' Nash equilibrium in quantities – which is the usual Cournot equilibrium. By contrast, in Bertrand duopoly, each firm uses its residual demand curve to compute its output as a function of the price choices of all m firms: (14) is used to solve implicitly for m quantities as functions of m prices. Using the market conditions to 'eliminate' quantities leaves m prices to be chosen by the firms, with no side constraints. The Nash equilibrium of this 'unconstrained' problem is the usual Bertrand equilibrium. The problem is that the basic model, as originally specified, does not provide any justification for one or the other of these approaches. Seen from this perspective, the Singh–Vives approach is to embed the constrained non-cooperative game into a two-stage framework, and to look for subgame perfect equilibria of this unconstrained game. The literature of oligopoly theory is replete with alternative resolutions of this basic problem. No doubt there are many alternative approaches to

[21]In standard duopoly models, the difference between Cournot and Bertrand equilibria disappears when the number of firms gets very large. As discussed in Wildasin (1988), the difference between T- and Z-equilibria in a model like that in section 2 disappears when the number of jurisdictions gets large, and that is true rather generally in non-cooperative game models with side constraints like (14). This is one way of seeing that the issue of choice of strategic variable is a crucial feature of models of small-number interactions; the issue becomes irrelevant precisely when the number of players is 'large'.

strategic policy equilibrium problems as well, and these deserve to be explored in future research.[22]

It is hardly necessary to emphasize that we have only considered a small selection of specific models that might be analyzed using the two-stage approach. The general method appears to be surprisingly easy to apply. The analysis already conducted by Kolstad and Wolak (1983) offers one further example. They assume that each of two states is attempting to maximize the revenue collected from a severance tax on coal. They calculate the Nash equilibrium tax rates both under the assumption that tax rates are the strategic variables, and under the assumption that output levels are the strategic variables. They find that tax rates, and tax revenues, are higher in the latter case. Indeed, it is easy to see in their model that the states would choose output levels as strategic variables in a two-stage game.[23] This might cast some doubt on the original Kolstad–Wolak argument that tax rates are the strategic variables. Alternatively, if one feels that tax rates are the right choice, it might lead one to reformulate the underlying model in such a way that tax rates emerge endogenously as the choice actually made by the governments in the two-stage game.

Problems of intergovernmental policy coordination are important and increasingly so. Policy issues connected with economic integration in Europe, international trade and capital flows, welfare reform, regional and state economic development and environmental policy all present examples of situations in which small-number policy interactions among governments may be crucial. Many policy questions cannot properly be discussed without workable models of non-cooperative behavior among governments. If past experience is any guide, models of Nash equilibrium will play a large and perhaps predominant role in the study of these questions. Of necessity, these models will have to specify the strategic variables chosen by the governments

[22]The analysis here has examined Nash equilibria in one-shot games in models where the strategy vectors of the two players are two-dimensional [i.e., $s_i=(s_{i1},s_{i2})$]. While not essential for the purposes of this paper, it is possible to give a conjectural variations interpretation to the choice of strategic variables. If agent j treats s_{i1} as given, (14) implies that

$$\frac{\partial s_{i2}}{\partial s_{jk}}=-\frac{\partial \phi_i/\partial s_{jk}}{\partial \phi_i/\partial s_{i2}},\quad k=1,2.$$

That is, a *zero* conjectural variation with respect to one of the other players' strategic variables necessarily implies a *non-zero* conjectural variation with respect to the other. Thus, in section 2, Z-conjectures can be interpreted to mean that a tax increase in one jurisdiction will induce a tax cut in the other, but it is equivalent to say that T-conjectures means that an expenditure increase in one jurisdiction will induce an expenditure increase in the other. Thus, the choice of strategic variable for a Nash equilibrium is isomorphic to an assumption about conjectural variations. (H. Tulkens first suggested such an interpretation to me.) To discuss conjectural variations is to invite consideration of a dynamic game structure which, however, takes one beyond the simple static problems examined here.

[23]Details of this argument are omitted for brevity's sake. The intuition is very much like that given in sections 2 and 3.

being analyzed. Prior experience with models of this type, especially in oligopoly theory where they have been most thoroughly studied, has established that models with different strategic variables lead to very different outcomes and have very different properties, in general. The specification of strategic variables is thus a central element, not a peripheral one, in the model-building process. Unfortunately, there seems to be little guidance in the literature as to how this aspect of model-building can best proceed.

One methodology for dealing with this issue might be termed the 'common law' approach. Traditions have evolved in various applied areas – tariff rates have traditionally been used as strategic variables in trade policy models, tax rates are used in fiscal competition models, public expenditure levels are used in models of spillovers, and so on. Faute de mieux, one might simply rely on scientific precedent for deciding among competing models. Alternatively, one can ask whether there is some incentive, within the basic modeling framework, for players to try to commit to one or another strategic variable. If certain strategic variables would be chosen in the Singh–Vives two-stage game setting, this provides some justification *within the model* for a particular strategic formulation. Formulations based on strategy variables that do not meet this test are less attractive. They might still be the appropriate ones to use, but it would be desirable to explain why. In the process of doing so, one may be led to elucidate further some important aspect of the issues under investigation and, perhaps, modify the underlying model accordingly.

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