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PUBLIC GOOD PROVISION WITH OPTIMAL AND NON-OPTIMAL COMMODITY TAXATION
The Single-Consumer Case

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The criterion for welfare-enhancing changes in the level of public good provision is studied in a model with optimal, and more general, commodity tax structures. The welfare rule is sensitive to the precise nature of interactions between public spending and (ordinary or compensated) demands for taxed goods.

1. Introduction

In 1974, Atkinson and Stern discussed the question of optimal public good provision when the government has recourse only to distortionary taxation. [1] Pigou (1947) had argued that distortionary taxes increase the marginal cost of public goods because of the indirect damage caused by additional taxation. Atkinson and Stern conclude that this view is in part correct but that Pigou ignored a 'revenue effect' that may work against and even reverse the Pigou conclusion. In this note I first sketch some comparative-statics results on consumer behavior with public goods, and then show that the analysis of the Pigou–Atkinson–Stern problem is sensitive to the precise relationship between public good provision and private good demand.

2. Consumer behavior with public goods

Let a consumer derive utility from private goods [2] $(x_0, x_1, ..., x_n)^T = x$ and from a single public good e, according to the well-behaved direct utility function $u(x, e)$.

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[1] See also Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971), and especially Drèze and Marchand (1976). There is some overlap between the latter work and this paper, insofar as Drèze–Marchand distinguish between compensated and ordinary demand derivatives with respect to a public good. Their basic concerns are, however, rather different than those here.

[2] T denotes transposition.

Taking e and consumer prices $q = (q_0, q_1, ..., q_n)$ as parametrically given, the consumer chooses x to maximize u subject to

$$qx = I = 0, \tag{1}$$

where, in the absence of profits and lump-sum transfers, lump-sum income I is zero. This generates demand functions $x(q, I, e)$ and an indirect utility function $v(q, I, e)$. Take $q_0 = 1$ as numeraire, and let $u_i = \partial u/\partial x_i$, $MRS = (\partial u/\partial e)/u_0$, $v_q = (\partial v/\partial q_1, ..., \partial v/\partial q_n)$, $v_1 = \partial v/\partial I$, $v_e = \partial v/\partial e$.

Call $\partial x/\partial e$ the derivative of the ordinary demand function with respect to e. We can distinguish from this a compensated derivative by asking how x changes with e if I is simultaneously varied to keep utility constant. For a compensated change, we have

$$\frac{dv}{de} = v_e + v_I \left.\frac{\partial I}{\partial e}\right|_{\bar{u}} = 0. \tag{2}$$

But, since $v_e/v_I = MRS$, [3] this means that $(\partial I/\partial e)_{\bar{u}} = -MRS$. Now define the compensated demand derivative according to

$$\left.\frac{\partial x}{\partial e}\right|_{\bar{u}} = \frac{\partial x}{\partial e} + \frac{\partial x}{\partial I}\left.\frac{\partial I}{\partial e}\right|_{\bar{u}} = \frac{\partial x}{\partial e} - MRS\frac{\partial x}{\partial I} . \tag{3}$$

Thus the ordinary derivatives can be broken into substitution and income effects. Note that we can derive an aggregation restriction

$$q\frac{\partial x}{\partial e} = q\left.\frac{\partial x}{\partial e}\right|_{\bar{u}} + MRS\, q\frac{\partial x}{\partial I} = q\left.\frac{\partial x}{\partial e}\right|_{\bar{u}} + MRS = 0, \tag{4}$$

so that not all of the substitution terms can be zero (assuming, of course, that $MRS > 0$). It is quite possible to have $\partial x_i/\partial e = 0$ for all i, however, and any n of the $n + 1$ $(\partial x/\partial e)_{\bar{u}}$ terms may be zero. Of course, $(\partial x_i/\partial e)_{\bar{u}} < 0$ for at least one i.

3. The welfare effects of a change in public expenditure

3.1. With optimal distortionary taxation

Atkinson and Stern suppose that the government is choosing a tax structure and a level of e to maximize utility [4] subject to the government's production and bud-

[3] $v_e/v_I = (\Sigma_i u_i(\partial x_i/\partial e) + u_e)/(\Sigma_i u_i\, \partial x_i/\partial I)$. Using $q_i = u_i/u_0$ and differentiating (1) with respect to e and I establishes $v_e/v_I = u_e/u_0$.

[4] This paper stays within the single-consumer case, so the objective function is that consumer's utility. The extension to many identical consumers is trivial. The extension to many non-identical consumers is non-trivial but is suppressed in order to bring other points out more easily. See Drèze and Marchand (1976) and Wildasin (1978) for some discussion.

get constraint. They ask what conditions characterize an optimum and find [their eq. (3)] [5]

$$MRT = \frac{v_I}{\lambda} MRS + t\,\frac{\partial x}{\partial e}\,, \tag{5}$$

where *MRT* is the marginal rate of transformation between the private and public good, λ is a Lagrange multiplier, and $t_0 = 0$ as a normalization. In particular, (5) implies that the familiar *MRS* = *MRT* rule does not, in general, characterize an optimum. They further show that

$$\frac{v_I}{\lambda} = 1 - t\,\frac{\partial x}{\partial I} + \Sigma_i t_i\,\frac{S_{ik}}{x_k}\,, \tag{6}$$

where $S_{ik} = (\partial x_i/\partial q_k)_{\bar{u}}$ is a Slutsky substitution term. They assume $\partial x/\partial e = 0$ and note that $\gamma = \Sigma_i t_i\, S_{ik}/x_k \leqslant 0$. It follows that v_I/λ is reduced by the third term in (6), the 'distortionary effect', while v_I/λ may be increased by the 'revenue effect', the second term. In particular, if there is only one taxed factor (labor) that is inferior (leisure a normal good), the revenue effect will work to increase v_I/λ. Thus, although the distortionary effect is consistent with the Pigou argument (since $v_I/\lambda < 1$ implies that *MRS* overstates the benefits – or, alternatively, *MRT* understates the costs – of the public good), the revenue effect can work against and even overturn his conclusion.

To see why this conclusion is sensitive to the seemingly innocuous assumption that $\partial x/\partial e = 0$, substitute from (3) and (6) into (5) to get

$$MRT = (1 + \gamma)MRS + t\,\left.\frac{\partial x}{\partial e}\right|_{\bar{u}}\,. \tag{7}$$

One possible specification of consumer behavior would be $(\partial x_i/\partial e)_{\bar{u}} = 0$ for $i = 1, \ldots, n$; in this case, we have *only* the distortionary effect and no revenue effect, and *MRS* must definitely exceed *MRT* at an optimum, as Pigou would have it. Needless to say, Pigou hardly envisioned this analysis in support of his conclusion.

3.2. With arbitrary distortionary taxation

Now suppose that taxes and expenditure may not be set optimally. Assume constant producer prices for simplicity, so that the consumer prices $q_i = p_i + t_i$ can change only as a result of tax rate changes. The problem is to evaluate the welfare effect of a marginal increase in e which is carried out subject to governmental production and budget-balance constraints. Suppose for simplicity that only the numeraire good is used as an input for public production; let $z(e) \geqslant 0$ be the amount of input required as a function of e. Then the government's budget con-

[5] There are obvious notational changes in (5).

straint is

$$tx(q, I, e) = z(e). \tag{8}$$

Thus in general t must vary to maintain equality in (8) as e varies. Let t depend on a parameter Θ, with $t' = (\mathrm{d}t_1/\mathrm{d}\Theta, ..., \mathrm{d}t_n/\mathrm{d}\Theta)$ describing how tax rates change with Θ. For now, t' is arbitrary. Let Θ depend on e in such a way that (8) is satisfied identically in e. Then

$$\left[t'x + t\frac{\partial x}{\partial q}(t')^T\right]\Theta' + t\frac{\partial x}{\partial e} = MRT, \tag{9}$$

from which one can solve for Θ' [assuming that the change in tax rates t' by itself would cause tax revenues to change – a weak non-degeneracy assumption on the function $t(\Theta)$].

The change in the household's utility from a marginal increase in e financed in this way, divided by v_I, is (using Roy's formula)

$$\frac{1}{v_I}\frac{\mathrm{d}v}{\mathrm{d}e} = \frac{1}{v_I}v_q(t')^T\Theta' + \frac{v_e}{v_I} = -\frac{t'x(MRT - t\,\partial x/\partial e)}{t'x + t(\partial x/\partial q)(t')^T} + MRS\,. \tag{10}$$

Using the Slutsky equation and defining $\gamma_k = \Sigma_i t_i S_{ik}/x_k$ which, if the tax structure is non-optimal, need not be independent of k, we have

$$t'x\bigg/\left(t'x + t\frac{\partial x}{\partial q}(t')^T\right) = \left(1 + \frac{\Sigma_k t'_k x_k \gamma_k}{t'x} - t\frac{\partial x}{\partial I}\right)^{-1}. \tag{11}$$

The expression in (11), which equals unity at zero rates of taxation, can be presumed positive. One concludes that $v_I^{-1}(\mathrm{d}v/\mathrm{d}e) \gtreqless 0$ as

$$MRS\left(1 + \frac{\Sigma_k t'_k x_k \gamma_k}{t'x} - t\frac{\partial x}{\partial I}\right) - MRT + t\frac{\partial x}{\partial e} \gtreqless 0. \tag{12a}$$

or, using (3), as

$$MRS\left(1 + \frac{\Sigma_k t'_k x_k \gamma_k}{t'x}\right) - MRT + t\left.\frac{\partial x}{\partial e}\right|_{\bar{u}} \gtreqless 0. \tag{12b}$$

The analysis culminating in eqs. (12a) and (12b) is more general than that of Atkinson and Stern (1974) [and Drèze and Marchand (1976)] in two respects. First, we can evaluate the effect on utility of a marginal change in e at any level of e, not just characterize the optimum at which conditions (12) are satisfied as equalities. The formulae do not look different on this account, aside from the inequality signs, but it is good to know that (possibly sizeable) deviations from an optimal e do not alter the *form* of the relationship between *MRS* and *MRT*. Second, and more importantly, the derivation here does not assume an optimal tax structure. It can, of course, include optimal taxes as a special case, in which event $\gamma_k = \gamma$ all k. The term multiplying *MRS* in (12a) under optimal taxation is just (6), and on the assumption that $\partial x/\partial e = 0$ we get the Atkinson–Stern results: *MRS* may over- or understate the marginal benefit depending on the relative strength of the distortionary and revenue effects. If we alternatively assume optimal taxation and $t(\partial x/\partial e)_{\bar{u}} = 0$, from (12b) we get the results following after (7): *MRS* definitely overstates the marginal benefit of the public good (or *MRT* understates the marginal cost).

More generally, one cannot get unambiguous results from (12) under arbitrary marginal tax structures. All of the terms in (12a) and (12b) are, in principle, observable, however, so that the theory coupled with the appropriate empirical information gives a fully determinate result.

One special case of interest is that in which only one commodity – say commodity 1 – is subject to taxation (totally and at the margin). Then we have $\Sigma_k t'_k x_k \gamma_k / t'x = \gamma_1 = t_1 S_{11}/x_1 = t_1^2 S_{11}/R$, where $R = t_1 x_1$ = government revenue > 0. Thus $\gamma_1 < 0$. Considering (12a), if we follow Atkinson and Stern (1974) and assume $\partial x_1/\partial e = 0$, we get a breakdown of the adjustment to the *MRS* term into distortionary and revenue components, as before. If x_1 is labor, and the labor supply curve is backward-bending, the revenue effect outweighs the distortionary effect, *MRS* under- rather than overstates the marginal benefit, and the Pigou conclusion is overturned. [This interesting and not implausible example was noted in Atkinson and Stern (1974).]

Still remaining in the model with only one taxed good, let us replace the assumption that $\partial x_1/\partial e = 0$ with the assumption that $(\partial x_1/\partial e)_{\bar{u}} = 0$. From (12b) it is clear that *MRS* now unambiguously overstates the marginal benefit of the public good and the Pigou conclusion is supported.

4. Conclusion

Public finance economists and textbooks in Pigou's time and even now commonly rely on partial equilibrium analyses depicted with familiar diagrams to explicate the welfare effects of distortionary taxes [see Browning (1976)]. When using such tools it is very helpful to assume only one taxed good. Moreover, it is by now well-established that excess-burden triangles are appropriately measured with reference to compensated, not ordinary, demand curves. Since the private–public good interaction that would shift the (compensated) demand curve for the taxed good is typically ignored by way of simplification in these partial equilibrium diagrams, it is not unreasonable to argue that the last case discussed above – only one taxed good, with $(\partial x_1/\partial e)_{\bar{u}} = 0$ – is really the one Pigou would have in mind as a typical or paradigm situation. If this is so, Pigou and other analysts employing this approach are correct, taken on their own ground.

These relatively unimportant historical/pedagogic questions aside, this paper has rather disconcerting implications for cost–benefit analysis in a distorted economy. The proper way of taking the effects of distortionary taxes into account in evaluating public expenditure depends sensitively on complement–substitute relations between public and private goods. If the *ordinary* demand functions are insensitive to the level of public expenditure, income or revenue effects enter into the adjustments that must be made to marginal benefits; with backward-bending factor supply curves, one may well find that MRS (or ΣMRS in a many-person context) *understates* the marginal benefit of public expenditure. If instead the *compensated* demand functions (for the taxed goods) are insensitive to the level of public expenditure, one has that MRS (or ΣMRS) necessarily *overstates* marginal benefits. Thus, even the *qualitative* effects of distortionary taxation depend on private–public good complementarity–substitutability. There is, of course, no reason to expect in general that either ordinary or compensated demand functions are independent of public good provision, so that matters are even more complex than the polar cases discussed above would indicate.

Most bothersome of all is the fact that we have very little empirical information on the interaction between public good provision and private good demand. One would be hard-pressed, on the basis of available empirical information or even introspection, to dismiss or support either of the hypotheses of ordinary and compensated demand independence of public good provision. Note that one or the other of these hypotheses may be valid for some public goods but not for others. One can only conclude that we are far from having sufficient information to take practical account of the effects of distortionary taxation in cost–benefit calculations. More positively, the analysis here suggests an important area for applied consumer research.

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