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ON PUBLIC GOOD PROVISION WITH DISTORTIONARY TAXATION

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I. INTRODUCTION

Conventionally, it is argued that an extra unit of a public good is desirable or not according as the sum across households of the marginal benefits of the unit (ΣMRS) (Σ marginal rates of substitution) is greater or less than the cost of the resources used up in providing it, what may be called the *direct* marginal cost (MRT) (marginal rate of transformation) of the public good. At an optimum, of course, $\Sigma MRT = MRT$. This familiar "Samuelsonian" welfare criterion lies at the foundation of normative public expenditure theory, including the principles of benefit-cost analysis. It was argued by Pigou (1947), however, that when public expenditures are financed by distortionary taxes, they impose an *indirect* burden on society in addition to their direct marginal cost. This indirect marginal cost is the loss of real income that results from raising additional revenue in an already distorted market.

Pigou's argument may be illustrated using a diagram similar to that presented by Browning (1976).[1] In figure 1, D is the *compensated* demand curve for a taxed good which is provided at a constant supply price of p. The good initially is taxed at rate t, resulting in an initial equilibrium quantity X. Now government expenditure increases, requiring an increase[2] in the tax rate of, say, Δt to generate the required amount of revenue. This causes a loss of *abdt* or, to a first-order approximation, $X\Delta t = acdt$, of consumer's surplus, which is the cost, in terms of lost real income to the consumer, of providing the extra revenue. The increment in tax revenue (= extra public expenditure) itself is approximately $X\Delta t + t\Delta X = acdt - defg$ where ΔX is the change in equilibrium quantity following the tax increase. The ratio of these two magnitudes is the loss of real income per dollar's worth of incremental public expenditure, and would clearly equal $X\Delta t/X\Delta t = \$1$ if the taxed good had a perfectly inelastic demand curve, since then $\Delta X = 0$. Thus, with non-distortionary taxes, there is no extra burden of incremental public expenditure, just the direct cost of a transfer of resources from the private to the public sector. More generally, however, $\Delta X \simeq (\partial X/\partial(p+t))\Delta t = X/(p+t)\epsilon^D \Delta t < 0$, where ϵ^D is the compensated elasticity of demand for X with respect to its tax-inclusive or consumer price $p + t$. In this case, the full social cost per marginal dollar of public expenditure is

$$(1) \qquad MSC = \frac{X\Delta t}{X\Delta t + t\Delta X} = \frac{1}{1 + \dfrac{t}{p+t}\epsilon^D},$$

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1. See also Gramlich (1981, pp. 22-24) for a more recent discussion along these lines. Other related studies include Johnson and Pauly (1969), Stuart (1981), and Usher (1982).

2. I assume throughout this paper that tax rates are sufficiently low, and that demand (or supply) curves for taxed commodities are sufficiently inelastic, that increments in tax rates lead to increments, not decrements, in tax revenues. See note 8 below.

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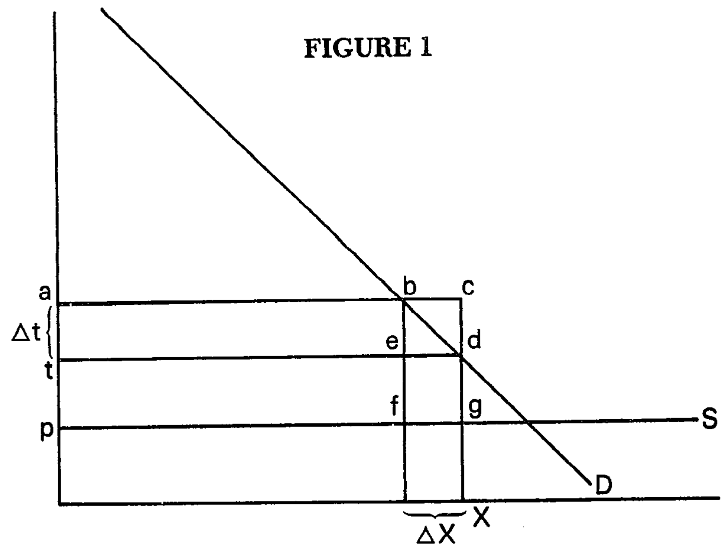
expenditure is given by the ratio of area *acdt* to the area *acdt* – *defg;* this ratio is greater than one since *defg* is not zero. The interpretation should be evident: at the initial distorted equilibrium, the demand price for X exceeds its supply price by $\$t$. Marginal reductions in the equilibrium quantity of X therefore generate excess burdens of $\$t$ per unit, and so the area *defg* is (approximately) the marginal excess burden associated with the given policy change. The full marginal social cost of a dollar's worth of additional public expenditure is greater than the direct cost of one dollar because this indirect cost is positive.

FIGURE 1



All of this discussion applies to the case where the taxed commodity is a good, but the extension to the case of a taxed factor is straightforward. If $\epsilon^s > 0$ is the compensated supply elasticity of the taxed factor, and if the demand curve for the factor is perfectly elastic, exactly analogous reasoning leads to the formula

$$MSC = \frac{1}{1 - \frac{t}{p-t}\,\epsilon^s} \qquad (2)$$

The Pigovian conclusion that $MSC > 1$ is reaffirmed in this case as well because ϵ^s is necessarily positive.

While this argument that MSC necessarily exceeds unity is most plausible, unfortunately it has been demonstrated by Atkinson and Stern (1974) that it is possible to have $MSC < 1$.[3] Moreover, this is not just a perverse theoretical curiosity. It occurs,

3. In focusing on this particular result of Atkinson and Stern, I should note that it is only a special case of their general analysis. It is certainly not *necessary* to have $MSC < 1$, as they clearly state. Nonetheless, this special case is of particular interest because of its apparent incompatibility with the Pigou-Browning view. It might also be noted that, like Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971), and Drèze and Marchand (1976), Atkinson and Stern proceed within a framework of simultaneous optimizing of both public expenditure and tax structure. The Pigou-Browning approach, by contrast, is more in the spirit of "piecemeal" reform, in that it seeks to evaluate incremental changes starting from a not-necessarily optimal initial position. Guesnerie (1979) also provides a more formal "incrementalist" anlaysis of public expenditure.

in their analysis, when the taxed commodity is labor, and when the ordinary (uncompensated) labor supply curve is backward-bending. Some would argue that this is, in fact, an empirically relevant case.

It is disturbing to find theoretical analyses which appear to be flatly contradictory, as these are, and one purpose of this paper is to resolve this contradiction. More generally, I reconsider the whole problem of public expenditure evaluation with distortionary taxation. Section II presents a general equilibrium model of an economy with identical households and distortionary-tax-financed public expenditures, and recalls a fundamental Slutsky-type equation describing the household's comparative-statics response to a change in public good provision. Section III then analyzes the conditions under which a marginal change in public expenditure (starting from an arbitrary initial level) increases or decreases welfare. The main contribution of this analysis is to emphasize the crucial importance of interactions between the level of public good provision and the demand for (supply of) taxed commodities. Obviously, figure 1 has assumed (implicitly) that the *compensated* demand curve for the taxed good does not shift as the level of public expenditure changes. Section III shows that formulae like (1) and (2), and the Pigou-Browning conclusion that $MSC > 1$ necessarily, are indeed valid given this assumption. At the same time, however, the analysis confirms that the Atkinson-Stern case of $MSC < 1$ can also occur, given their (explicit) assumption that the *ordinary* demand for the taxed good does not shift as the level of public expenditure changes. In fact, the difference between the Pigou-Browning and Atkinson-Stern results arises *solely* because they make different assumptions about public expenditure-private good demand interactions. While these assumptions identify useful theoretical polar cases, neither, in general, will be satisfied in practice. Thus, perhaps the most important contribution of section III is the derivation of general welfare criteria which not only include the Pigou-Browning and Atkinson-Stern results as special cases, but which show explicitly how quantitative estimates of the effect of public expenditure on private good demand should be included, in the general case, in the determination of the marginal social cost of public expenditure.

One important feature of the welfare criteria presented in section III is that they are stated in terms of magnitudes — demand elasticities and the like — which can, in principle, be determined empirically. For sake of illustration, and particularly to demonstrate the likely empirical importance of the public good-private good demand effects discussed above, section IV presents estimates of the marginal social cost of public expenditure for the United States. After extending the model to accommodate households with differing incomes, this computation is made for proportional, linear progressive (degressive, *i.e.*, constant marginal, but increasing average tax rates), and non-linear progressive taxes on labor income, paralleling and correcting the work of Browning (1976). This exercise identifies certain empirical magnitudes as ones that are important for policy and therefore worthy of further study. Section V discusses prospects for future work.

II. THE BASIC MODEL

Households. Assume that there are H identical consumers, each with a twice continuously differentiable strictly quasi-concave utility function $u(x_0, x_1, \ldots, x_n, z)$ defined over consumption of private goods ($x_i \geq 0$), supply of factors ($x_i \leq 0$), and public good consumption z. Suppose that only good 1 is taxed, so that consumers face the same prices as producers, $p = (p_0, p_1, \ldots, p_n)$ for all commodities except good 1, for which the consumer price is $q_1 = p_1 + t_1$, where t_1 is the rate of tax per unit. Let good 0 be the numéraire, with $p_0 = 1$. Each household chooses a private good consumption vector $x = (x_0, x_1 \ldots, x_n)$ to maximize utility subject to the budget constraint

$$t_1 x_1 + \Sigma_{i=0}^{n} p_i x_i = I. \tag{3}$$

I is income (denominated in units of good 0) from sources other than the sale of factors.[4] This maximization problem yields demand functions that depend on all prices, the tax rate, income, and public good provision. Then the indirect utility function may be defined as $v(q_1, p_2, \ldots, p_n, I, z)$.

This paper now briefly considers the demand function for good i, $x_i(q_1, p_2, \ldots, p_n, I, z)$. According to the well-known Slutsky equation, the derivative of x_i with respect to any price can be written as the sum of substitution and income effects. Although it is not nearly so well known, a similar decomposition may be performed for the derivative of x_i with respect to the level of public good, z. Clearly, other things constant, an increase in z increases a consumer's real income (utility). In fact, letting $MRS = (\partial u/\partial z)/(\partial u/\partial x_0) = (\partial v/\partial z)/(\partial v/\partial I)$ denote the marginal rate of substitution between the numéraire and public goods, it is known that a one-unit increase in z increases the household's real income by MRS, since this is the amount of numéraire that would have to be taken away to keep utility constant as z rises. Now let $(\partial x_i/\partial z)_{\bar{u}}$ denote the change in demand for good i as z changes, if income is simultaneously adjusted downward by MRS to keep utility fixed — *i.e.*, let $(\partial x_i/\partial z)_{\bar{u}}$ denote the derivative of the *compensated* demand function for good i with respect to z. This is a "substitution effect" that reveals any intrinsic substitutability/complementarity between good i and the public good. Thus, when z changes, the *ordinary* demand for good i changes partly on this account, and also because with money income constant, the household's real income rises by MRS. Hence, the derivative of the ordinary demand function may be written as:

$$\frac{\partial x_i}{\partial z} = \left.\frac{\partial x_i}{\partial z}\right|_{\bar{u}} + MRS\,\frac{\partial x_i}{\partial I}, \tag{4}$$

4. Since constant returns to scale in private production is assumed below, pure profits and hence lump-sum income I will be zero in equilibrium. Derivatives of demand and indirect utility functions with respect to I, evaluated at $I = 0$, are of course still well-defined.

where the right-hand terms are the substitution and income effects, respectively.[5] In general, (4) does not restrict the sign of either $\partial x_i/\partial z$ or $(\partial x_i/\partial z)_{\bar{u}}$; in this respect, it is unlike the Slutsky equation with its necessarily negative own-substitution term. It is clear, however, that $(\partial x_i/\partial z)_{\bar{u}} < 0$ for at least one i, since otherwise an increase in z would necessarily make the household better off.[6] A more formal derivation of (4) appears in Wildasin (1979a).

Finally, as a notational convention, the market demand for good i is denoted Hx_i, by X_i.

Private Production. Assume that private production takes place competitively subject to a linear technology. The vector p of equilibrium producer prices is thus constant.[7]

Government. Assume that to produce z units of public good requires $c(z)$ units of numéraire, so that $c'(z) > 0$ is the marginal rate of transformation (MRT) between the numéraire and public goods. The government is constrained to choose t_1 to balance its budget:

$$t_1 X_1 = c(z), \tag{5}$$

from which it is possible to solve for t_1 implicitly as a function of z, with derivative

$$\frac{dt_1}{dz} = \frac{MRT - t_1(\partial X_1/\partial z)}{X_1 + t_1(\partial X_1/\partial q_1)} \tag{6}$$

5. Equation (4), and the remaining discussion in this paper, is based on the assumption that z is made available to all consumers at no (direct) charge. It could, however, be supposed that z is price-excludable, and that a price p_z, less than the equilibrium price, is charged. Suppose, also, that all consumers are rationed so that each consumes the same amount of public good. Now the income effect of a change in z is dampened, since the consumer must spend an extra p_z dollars when z is increased by one unit. Formally, it can easily be shown that

$$\frac{\partial x_i}{\partial z} = \left.\frac{\partial x_i}{\partial z}\right|_{\bar{u}} + (MRS - p_z)\frac{\partial x_i}{\partial I}, \tag{4$'$}$$

of which (4) is now a special case, corresponding to $p_z = 0$. $MRS - p_z > 0$ as long as p_z is held below its equilibrium value, but as p_z rises, the income effect of a change in z diminishes, eventually vanishing altogether in the "incipient rationing" case of $MRS = p_z$. In this case the distinction between $\partial x_i/\partial z$ and $(\partial x_i/\partial z)_{\bar{u}}$, which plays a major role in the discussion below, becomes vacuous. (The extension sketched in this note was motivated by a reading of Lindbeck, 1982, who emphasizes the possible relevance of the case $p_z > 0$.)

6. This case can be seen formally by using (4) and the aggregation restriction

$$\Sigma_{i=0}^{n} p_i \frac{\partial x_i}{\partial z} + t_1\frac{\partial x_1}{\partial z} = \Sigma_{i=0}^{n}\left.\frac{\partial x_i}{\partial z}\right|_{\bar{u}} + t_1\left.\frac{\partial x_1}{\partial z}\right|_{\bar{u}} + MRS = 0.$$

7. Since relative prices are constant, one could treat goods 0, 2, . . . , n as a composite good. The goods are all carried separately in part because the notational burden is not heavy (the untaxed non-numeraire goods play no role in the actual analysis) and in part to remind the reader of the fixed-price assumption being made. More importantly, with only two goods, the tax rate on the taxed good that balances the budget is the optimal rate because it is the only feasible one. By explicitly including more goods, while restricting taxation to only one, we know that the tax structure is not (second-best) optimal. It is economically important, even if analytically trivial, to consider the problems in which not all tax rates are chosen optimally.

III. THE WELFARE EVALUATION OF MARGINAL PUBLIC EXPENDITURES

Now it is necessary to determine under what conditions a marginal increase in z, starting from some arbitrary level, is welfare-enhancing. Since all households are identical, it is convenient to take Hv, the sum of individual utilities, as the welfare indicator. Then an incremental unit of z is desirable if and only if the total derivative $H(dv/dz)$, or equivalently $(H/v_I)(dv/dz)$, is positive, where $v_I = \partial v/\partial I > 0$ is the marginal utility of income. Using (6) and $(\partial v/\partial q_1)/v_I = -x_1$ (Roy's identity), this paper has

$$\frac{H}{v_I}\frac{dv}{dz} = \Sigma MRS - \frac{MRT - t_1(\partial X_1/\partial z)}{1 + \frac{t_1}{q_1}\epsilon_1}, \tag{7}$$

where $\epsilon_1 = \partial log x_1 / \partial log q_1$ is the ordinary own-price elasticity of demand for good 1. At the (second-best) optimal level of z, the expression in (7) is, of course, zero.

The welfare criterion (7) is a fundamental result. A useful equivalent expression can be derived given a further weak assumption,

$$\frac{1 + \frac{t_1}{q_1}\epsilon_1}{1 + \frac{t_1}{q_1}\epsilon_1^c} > 0. \tag{8}$$

which is valid if the income derivative for good 1 and/or the tax rate t_1 are sufficiently small. (ϵ_1^c is the compensated own-price elasticity of demand for good 1, $\epsilon_1^c = (\partial log x_1/\partial log q_1)_{\bar{u}}$.) This assumption is valid for empirically reasonable values of the variables involved.[8] Given (8), substitution of (4) into (6) shows that

$$\frac{H}{v_I}\frac{dv}{dz} \gtreqless 0 \text{ as } \Sigma MRS - \frac{MRT - t_1(\partial X_1/\partial z)_{\bar{u}}}{1 + \frac{t_1}{q_1}\epsilon_1^c} \gtreqless 0. \tag{9}$$

Now it is necessary to interpret (7) and (9). By (7), there are two reasons why the change in welfare caused by a marginal change in public good provision is not given simply by $\Sigma MRS - MRT$. First, to the extent that the change in consumption of the taxed good induced by an increase in public good provision induces an "ordinary" increase (or decrease) in government revenues — *i.e.*, to the extent that

8. Assumption (8) is valid if both (a) an increase in the tax rate on the taxed commodity, with z and I held fixed, causes tax revenue to increase; and (b) an increase in the tax rate, with z fixed and I varying to keep utility constant, also causes tax revenue to increase. This is shown by computing the ratio of $d(t_1X_1)/dt$, and $d(t_1X_1^c)/dt_1$, using the fact that $X_1 = X_1^c$ at the equilibrium point where (8) is assumed to hold.

$t_1(\partial X_1/\partial z) > 0$ (or < 0) — *MRT* overstates (or understates) the true marginal cost of the public good. Of course, an analyst might believe that such effects are quantitatively insignificant and decide to ignore them, that is, to assume that the taxed good and z are *ordinary independents:*

$$\frac{\partial x_1}{\partial z} = 0. \tag{10}$$

Whether (10) is valid is, of course, an empirical question. For now, it must be noted that it is only theoretically possible, and that it is a standard (if implicit) maintained hypothesis in applied demand analysis, as discussed further in section V.

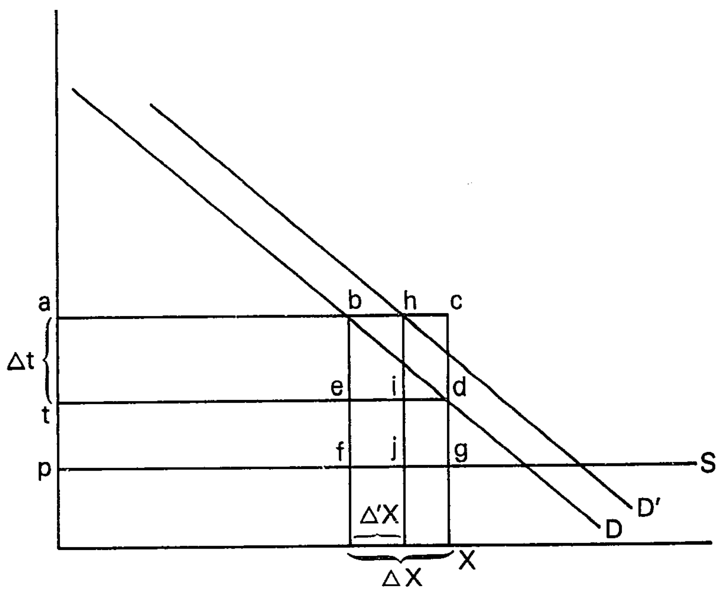
The second important element in (7) is the term $[1 + (t_1/q_1)\epsilon_1]^{-1}$, which is identical to the expression appearing in (1) and (2) except for the significant difference that ϵ_1 is an *ordinary* elasticity. Observe that $(t_1/q_1)\epsilon_1 < 0$ either when commodity 1 is a good ($x_1 > 0$, hence $t_1 > 0$) that is non-Giffen (hence $\epsilon_1 < 0$), or when commodity 1 is a factor ($x_1 < 0$, hence $t_1 < 0$) with an upward sloping supply curve ($\epsilon_1 > 0$). In either of these cases, if demand/supply curve shifts are ignored by assuming (10), (7) shows that *MRT* understates the true marginal social cost of the public good. If, however, commodity 1 is a taxed factor with a backward-bending supply curve (*i.e.*, $\epsilon_1 < 0$), then $(t_1/q_1)\epsilon_1 > 0$ and *MRT overstates* the social marginal cost of public good provision. Thus, this analysis, which closely parallels that of Atkinson and Stern, confirms their distinctive result, namely *MSC* < 1 when labor has a backward-bending supply curve and when ordinary independence (10) is assumed.

Next consider (9), which is identical to (7) except that compensated derivatives replace ordinary ones. Standard consumer theory implies, both for goods and factors, that $t_1\epsilon_1^c < 0$, so that the denominator in (9), unlike that in (7), is unambiguously less than 1. The $t_1(\partial X_1/\partial z)_{\bar{u}}$ term again can be interpreted as reducing the marginal cost of the public good to the extent that a — now compensated — increase in z increases tax revenues by changing the quantity demanded of the taxed good. An analyst might, however, choose to ignore such shifts of the compensated demand/supply curve, believing them to be quantitatively insignificant. This is the assumption of *compensated independence:*

$$\left.\frac{\partial x_1}{\partial z}\right|_{\bar{u}} = 0. \tag{11}$$

Whether or not (11) holds in practice is, of course, an empirical question. As already noted, however, this assumption implicitly underlies the construction of figure 1 and the derivation of (1) and (2). Given (11), (9) becomes identical to (1) and (2), and it yields the unambiguous result that $MSC > 1$, regardless of income effects and regardless of whether the taxed commodity is a good or factor. Thus this analysis confirms the Pigovian conclusion and the formulae derived from figure 1, provided that compensated independence (11) is assumed.[9]

FIGURE 2



9. It is instructive to show how (9) can be illustrated diagrammatically in the general case where $(\partial x_1/\partial z)_{\bar{u}} \neq 0$. Figure 2 illustrates the case where an increase in expenditure of ΔZ is financed by an increase in the tax rate of Δt above its initial level t, just as in figure 1. Unlike figure 1, however, we now assume that the compensated demand curve shifts out to D' as public spending rises (*i.e.*, $(\partial x_1/\partial z)_{\bar{u}} > 0$), so that the equilibrium quantity of the taxed good only falls by $\Delta X - \Delta' X$. The market's adjustment to the change in policy can be broken down into two parts. First, holding the demand curve fixed, the increase in the tax rate causes a loss of consumer surplus of $abdt \simeq acdt = X\Delta t$, as well as an increase in tax revenue of $abet - defg \simeq acdt - defg = X\Delta t + t(\partial X/\partial q)\Delta t$. This is identical to figure 1 so far. Now, taking the demand curve shift into account, we see that while there is no further change in consumer surplus (there is a quantity change, but no further price change that generates real income changes), tax revenue rises by $bhjf \simeq eijf = t(\partial X/\partial Z)\Delta Z$. Using the government's budget constraint, we have $\Delta Z = X\Delta t + t(\partial X/\partial q)\Delta t + t(\partial X/\partial Z)\Delta Z$, or, solving, $\Delta Z = [X\Delta t + t(\partial X/\partial q)\Delta t][1 - t\partial X/\partial Z]^{-1}$. It then follows that the loss of consumer surplus per dollar of marginal public expenditure is

$$\frac{X\Delta t(1 - t\partial X/\partial Z)}{X\Delta t + t\dfrac{\partial X}{\partial q}\Delta t};$$

or, recalling (1),

$$MSC = \frac{(1 - t\partial X/\partial Z)}{1 + \dfrac{t}{p+t}\epsilon^D},$$

which is just (9).

To summarize, two essentially equivalent expressions, (7) and (9), have been derived, both of which are quite general and either of which may be used to determine how MRT must be adjusted to reflect the true marginal social cost of the public good when it is financed by distortionary taxes. They highlight the importance of knowing how public good provision affects the demand function for the taxed good, since to estimate this adjustment, it is necessary to know *either* the compensated *or* ordinary derivative of that function with respect to the public good level, in addition to having information on the own-price elasticity of the function. It is possible, out of ignorance or otherwise, simply to assume that the compensated demand derivative for the taxed good is zero, in which case the formulae confirm the Pigou-Browning result that tax distortions cause the social marginal cost of a public good to exceed its direct marginal cost. It is possible, alternatively, to assume the ordinary demand function to be independent of the public good. In this case, the Pigou-Browning conclusion can be overturned, as, for instance, in the example produced by Atkinson and Stern. Both assumptions have been made in the literature. It is important to realize, however, that both are merely special cases, *neither* of which need be valid in any particular application. Moreover, *both* cannot be valid simultaneously, according to (4), provided that $MRS > 0$ (the household values the public good) and that $\partial x_1 / \partial I \neq 0$ (the income elasticity of the taxed good is non-zero). Thus, to assume compensated independence (11) is, implicitly, to assume that the ordinary demand curve shifts with variations in the level of public good, and conversely if ordinary independence (10) is assumed. In some form or other, therefore, theory demands recognition that public good provision necessarily affects the demand functions for private goods.

Of course, it remains possible that this theoretical necessity would be irrelevant in practical applications. Illustrative calculations in the next section show, however, that estimates of the marginal social cost of public expenditure are likely to be quite sensitive to this paper's assumption about the interaction between public and private goods.

IV. SOME ILLUSTRATIVE CALCULATIONS

The goal of this section is to derive some estimates of the social marginal cost *(SMC)* of public good provision, following Browning (1976) in assuming that public expenditures are financed essentially by a tax on labor income. However, a model in which households are identical in every respect, including their incomes, is too simple to allow a meaningful analysis of labor income taxation. It is necessary, therefore, to begin by generalizing the basic model.

First, the notation must be altered slightly by letting ℓ^h be the labor *supply* of household h, measured in positive units, with $(x_0^h, \ldots, x_n^h)$ the vector of other consumption. Assuming that all households face the same wages and prices $(w, p_1, p_2, \ldots, p_n)$, they can have differing wage incomes if their labor supplies differ.

In general, a household's marginal rate of tax depends on its income level, but in considering marginal variations around an initial equilibrium, we may assume that small changes in a household's income do not change its marginal tax rate — that is, no household starts with an income on the dividing point between two brackets. Let τ^h be the marginal rate for a household h and let the amount of tax-exempt income be $\bar{y}$. Then the budget constraint for household h, in the neighborhood of the initial equilibrium is

(12) $$\Sigma_{i=0}^{n} p_i x_i^h = w\ell^h - \tau^h(w\ell^h - \bar{y}),$$

while the government's budget constraint is

(13) $$\Sigma_h \tau^h(w\ell^h - \bar{y}) = c(z).$$

For later reference, we note (letting $v_I^h = \partial v^h/\partial I^h$)

(14) $$\frac{\partial v^h/\partial \tau^h}{v_I^h} = -w\ell^h + \bar{y};$$

(15) $$\frac{\partial \ell^h}{\partial \tau^h} = -w \frac{\partial \ell^h}{\partial \overline{w}^h} + \bar{y} \frac{\partial \ell^h}{\partial I^h},$$

where $\overline{w}^h = (1 - \tau^h)w$ is the net wage rate for household h, and where v^h is the household's indirect utility function.

While allowing households to differ in their preferences for public and private goods and in the amount of effective labor supplied, assume for simplicity that the ordinary and compensated (net) wage elasticities of labor supply, denoted $\epsilon_{\ell\overline{w}}$, $\epsilon_{\ell\overline{w}}^c$, are the same for all households. From the Slutsky equation, this implies that $\overline{w}(\partial \ell^h/\partial I^h)$, called the "total income elasticity of labor supply" by Cain and Watts (1973, p. 334), is the same for all households as well. These assumptions allow the analysis to proceed without unnecessary complications.

The model is now sufficiently rich to consider proportional ($\tau^h = \tau$, all h; $\bar{y} = 0$), linear progressive or "degressive" ($\tau^h = \tau$, all h; $\bar{y} \neq 0$), and non-linear progressive (τ^h's unequal, $\bar{y} \neq 0$) tax structures, hereafter referred to as the proportional, degressive, and progressive cases respectively. As in the simpler model of sections II and III, we wish to evaluate an incremental increase in z, accompanied by a tax rate change that keeps the government's budget balanced. In the proportional and degressive cases, we use (13) to solve implicitly for τ in terms of z. In the progressive case, follow Browning in assuming that all marginal tax rates are scaled up in proportion (as in the 1968 surcharge), again solving for the changes in the τ^h's from (13).

It is necessary, of course, to have a welfare indicator with which to evaluate increases in z. In order to derive simple formulae that resemble those of section III and that highlight the ordinary and compensated independence assumptions, use a Bergson-Samuelson social welfare function W which satisfies either of two special conditions. The first, "simple neutrality" (SN), says that the "social marginal utilities of consumption" (in the sense of Diamond, 1975) are equal, *i.e.*,

(SN) $$\frac{\partial W}{\partial v^h} v_I^h = \mu \qquad \text{all } h,$$

for some μ. The second condition, "extended neutrality" (EN), says that a \$1 lump-sum transfer from one household to another does not increase social welfare, taking into account the effect of the transfer on tax revenues and thus, *via* the budget-balance condition (13), on tax rates.[10] (*E.g.*, a transfer to household h induces h to work less, reducing tax revenue and requiring an increase in tax rates to maintain government revenue.) If $\partial\tau^{h'}/\partial I^h$ represents the required change in $\tau^{h'}$ as $1 is transferred to household h, then (EN) or "equal social marginal utilities of income" requires that, for some μ^*,[11]

$$\text{(EN)} \qquad \frac{\partial W}{\partial v^h} v_I^h + \Sigma_{h'} \frac{\partial W}{\partial v^{h'}} \frac{\partial v^{h'}}{\partial \tau^{h'}} \frac{\partial \tau^{h'}}{\partial I^h} = \mu^* \qquad \text{all } h.$$

To evaluate a change in z, first consider a proportional tax assuming W satisfies (SN). Totally differentiating W and dividing by μ yields

(16)

$$\frac{1}{\mu}\frac{dW}{dz} = \Sigma_h MRS^h - \Sigma_h w\ell^h \frac{d\tau}{dz} = \Sigma_h MRS^h - \frac{\Sigma_h w\ell^h}{\Sigma_h w(\ell^h + \tau \frac{\partial \ell^h}{\partial \tau})} [MRT - \tau w \Sigma_h \partial\ell^h/\partial z] = \Sigma_h MRS^h - \frac{[MRT - \tau w\, \Sigma_h(\partial\ell^h/\partial z)]}{1 - \frac{\tau}{1-\tau}\epsilon_{\ell\bar{w}}},$$

where the first equality uses (14), the second follows by solving for $d\tau/dz$ from (13), and the third follows from (15). With a degressive tax, there is instead

$$\text{(17)} \qquad \frac{1}{\mu}\frac{dW}{dz} = \Sigma_h MRS^h - \Sigma_h (w\ell^h - \bar{y}) \frac{d\tau}{dz}$$

$$= \Sigma_h MRS^h - \frac{\Sigma_h (w\ell^h - \bar{y})[MRT - \tau w\, \Sigma_h(\partial\ell^h/\partial z)]}{\Sigma_h w\ell^h(1 - \frac{\tau}{1-\tau}\epsilon_{\ell\bar{w}}) - \Sigma_h \bar{y}(1 - \frac{\tau}{1-\tau}\bar{w}\frac{\partial \ell^h}{\partial I^h})},$$

while with a progressive tax the formula becomes

(18)

$$\frac{1}{\mu}\frac{dW}{dz} = \Sigma_h MRS^h - \frac{[\Sigma_h \tau^h(w\ell^h - \bar{y})][MRT - \Sigma_h \tau^h w(\partial\ell^h/\partial z)]}{\Sigma_h \tau^h[w\ell^h(1 - \frac{\tau^h}{1-\tau^h}\epsilon_{\ell\bar{w}}) - \bar{y}(1 - \frac{\tau^h}{1-\tau^h}\bar{w}^h\frac{\partial \ell^h}{\partial I^h})]}$$

10. For earlier examples of the use of such an assumption, see Boiteux (1956), Marchand (1968), Mohring (1970), Drèze and Marchand (1976), and Wildasin (1977).

11. It is easy to produce examples of W functions satisfying (SN) or (EN). Note, however, that a given W cannot simultaneously satisfy *both* (SN) and (EN) in the progressive tax case. A $1 transfer to h causes tax revenue to change by $[\tau^h/(1-\tau^h)]\bar{w}^h\partial\ell^h/\partial I^h$, which is negative if leisure is normal. Given equal total income elasticities of labor supply, this revenue loss must be greater if h is in a higher tax bracket, in which case the proportionate scaling-up of tax rates required to satisfy (13), $\partial\tau^{h'}/\partial I^h$, must be greater as well. It is then clear from the formulae that (SN) and (EN) cannot simultaneously hold. On the other hand, the same argument shows that (SN) and (EN) are identical in the proportional and degressive cases, given the total income elasticity assumption.

This is the most general formula, of which (16) and (17) are special cases. Note that the *ordinary* derivative of labor supply with respect to z, $\partial\ell^h/\partial z$, and the *ordinary* wage elasticity of labor supply, $\epsilon_{\ell\bar{w}}$, appear in each expression.

Now assuming (EN), straightforward (but tedious) analysis shows that for the general case of a progressive tax

$$(19)\qquad \frac{1}{\mu^*}\frac{dW}{dz} = \Sigma_h MRS^h - \frac{[\Sigma_h \tau^h(w\ell^h - \bar{y})][MRT - \Sigma_h \tau^h w(\partial\ell^h/\partial z)_{\bar{u}}]}{\Sigma_h \tau^h(w\ell^h - \bar{y}) - \Sigma_h \tau^h w\ell^h \left(\frac{\tau^h}{1-\tau^h}\right)\epsilon^c_{\ell\bar{w}}}.$$

This takes on simpler forms if the tax structure is proportional or degressive. Notice that the compensated derivative of labor supply with respect to z, $(\partial\ell^h/\partial z)_{\bar{u}}$, and the *compensated* wage elasticity of labor supply, $\epsilon^c_{\ell\bar{w}}$, appear in (19), in contrast to (18). Formulae (18) and (19) thus parallel formulae (7) and (9) from the single consumer case.

Interestingly, (19), like (9), implies that the indirect marginal cost of the public good is unambiguously positive ($\epsilon^c_{\ell\bar{w}} > 0$) if labor and the public good are compensated independents. This is roughly compatible with Browning (1976), who derives formulae for the social marginal cost of a public good in terms of compensated labor supply elasticities but ignores the effect of public expenditure on labor supply, thus implicitly assuming compensated independence. His conclusion that the indirect marginal cost of the public good is positive is correct, given this assumption, but his formulae do not correspond with (19) because he errs by assuming at one point that "the tax base . . . is not affected by a small change in the tax rate" (p. 286). This amounts to assuming that the equilibrium quantity of the taxed good is independent of the tax rate, but in this case the demand for this good is perfectly inelastic, hence the indirect marginal cost of the public good must be *zero*. The essential differences between Browning's formulae and the author's "compensated" formula (19) can be traced to this error.

Given (16) through (19), it is fairly easy to estimate the social marginal cost of public expenditure, assuming, in the case of (16), (17) and (18), that labor and the public good are *ordinary* independents, and in the case of (19) that they are *compensated* independents. Suppose, following Browning, that the compensated wage elasticity of labor supply is .20, and consider a total income elasticity of –.20 or –.30. These figures result in an ordinary wage elasticity of 0 or –.10, which seem broadly consistent with representative empirical studies. For purposes of illustration, the assumptions of a wage-inelastic or slightly backward-bending labor supply seem to be of particular interest.

Consider now the case of a proportional income tax.[12] On the assumption that $\tau = .35$ because "total taxes as a percentage of net national product are about 35 percent" (Browning, p. 287), we find from (16) that MRT must be multiplied by 1 or by .95 to obtain a correct estimate of SMC in the ordinary independents case,

12. In applying the above formulae, one must recognize that labor income is not the only source of tax revenue in the United States. Thus, Browning's estimated effective income tax rates, applied to labor income *alone*, would yield less revenue than they do when applied to *all* income. In the absence of other taxes, this implies a smaller total government budget than is actually observed. To offset this, assume the existence of a lump-sum tax that, together with the labor income tax, brings total tax revenues up to the observed level, but that is not used to finance any incremental government spending. This assumption is implicit in Browning.

depending on whether the wage-elasticity of labor supply is 0 or −.10. In the compensated independents case, (19) when appropriately simplified yields an SMC of $1.12. These figures may be compared with the Browning estimate for this case (p. 287), based on the erroneous theoretical analysis noted above, of $1.07.

Now suppose there is a degressive tax with an exemption equal to 40 % of average total income. To raise 35 % of national income in taxes then requires a tax rate of 58 % on taxable income. Assume that additional public expenditures are financed at the margin by a proportional tax applied to 60 % of labor income, so that $\Sigma_h \bar{y} = .40\, \Sigma_h w\ell^h$ in (17). With $\tau = .58$, given ordinary independence,[13] SMC is $1.23 or $1.05 as $\epsilon_{\ell\bar{w}} = 0$ or −.10. For the compensated case, derived from (19) under the same assumptions on tax rates and exemptions, SMC is $1.85. Browning's (uncorrected) degressive tax estimate is (p. 289) $1.18.

In the progressive case, assume, with Browning, that each household enjoys a fixed dollar sum exemption that happens to equal 38.6 % of its labor income. This differs from the assumption underlying (18) and (19) of an equal exemption for all households, but it is not difficult to modify them[14] and, given Browning's data on tax rates and income by 27 income classes (p. 292, table 1, columns 3 and 4), to carry out the required calculations. On the ordinary independence assumption, SMC is $1.10 in the case where $\epsilon_{\ell\bar{w}} = -.10$. For the compensated independence case, (19) yields an SMC of $1.32. This may be compared to Browning's (uncorrected) estimate of $1.16.

The estimates so far parallel Browning's, but a comparison of these results (see table 1) is hampered by the fact that the average tax rate for the progressive case is only 24.5 %, and the exemption rate is 38.6 % instead of the 40 % assumed in the degressive case. Therefore, rows 4 and 5 of table 1 also present estimates for proportional and degressive tax structures with the same average tax and exemption rates as the progressive tax; the degressive tax rate on income above exemption is 38.8 % in this case.

13. Given equal labor-supply parameters for all households, an exemption of 40 % of income, and the ordinary independence assumption, (17) becomes

$$(17)' \quad \frac{1}{\mu}\frac{dW}{dz} = \Sigma_h MRS^h - \frac{.6}{(1 - \frac{\tau}{1-\tau}\epsilon_{\ell\bar{w}}) - .40(1 - \frac{\tau}{1-\tau}\bar{w}\frac{\partial \ell}{\partial I})} MRT.$$

An analogous version of (19) obtains for the compensated case.

14. For the uncompensated case, the critical expression is

$$(18)' \quad \frac{\Sigma_h \tau^h (.614) w\ell^h}{\Sigma_h \tau^h w\ell^h [1 - \frac{\tau^h}{1-\tau^h}\epsilon_{\ell\bar{w}} - (.386)(1 - \frac{\tau^h}{1-\tau^h}\bar{w}\frac{\partial \ell}{\partial I})]} .$$

The modification of (19) is similar.

Wildasin, David E., *On Public Good Provision with Distortionary Taxation* , Economic Inquiry, 22:2 (1984:Apr.) p.227

TABLE 1

Marginal Cost of Public Expenditure Under Alternative Assumptions[a]

	When Labor and Public Good are Ordinary Independents		When Labor and Public Good are Compensated Independents (Browning, Corrected)[b]	Original Browning Estimates
	$\bar{w}\frac{\partial \ell}{\partial I} = -.2$ (1)	$\bar{w}\frac{\partial \ell}{\partial I} = -.3$ (2)	(3)	(4)
Proportional Tax *ATR* = .35	1.00	.95	1.12	1.07
Degressive Tax *ATR* = .35 *ER* = .40	1.23	1.05	1.85	1.18
Progressive Tax *ATR* = .245 *ER* = .386	1.10	1.02	1.32	1.16
Proportional Tax *ATR* = .245	1.00	.97	1.07	NA
Degressive Tax *ATR* = .245 *ER* = .386	1.12	1.05	1.25	NA

[a]*ATR* = average tax rate; *ER* = exemption rate. See text for explanation and derivation of results.

[b]The first three numbers in this column are corrections of Browning's estimates, reported in column (4).

The estimates in table 1 reveal some expected results and some perhaps surprising ones. A comparison of columns (1) and (2) shows that the income elasticity of labor supply is important in determining the social marginal cost of public funds in the ordinary independents case; calculations with a different assumed compensated elasticity of labor supply would, no doubt, have shown that parameter to be important also. Comparing columns (3) and (4), it may be seen that the errors in Browning's analysis mentioned above are quantitatively serious. And, comparing rows, it is clear that the SMC depends significantly on the tax structure. Note that the SMC can be higher in the degressive than in the progressive case, even with identical average tax and exemption rates. This, logically, *cannot* occur (and, of course, *does* not occur in table 1) in the compensated independents case, but obviously can and does happen in the ordinary independents case.[15]

15. A proof of the former assertion is omitted but can easily be produced by considering the effects on the relevant expression in (19) of average-tax-rate-preserving increases in progressivity.

For the purposes of this paper, the key result revealed by table 1 is seen by comparing column 3 with either column 1 or 2. The full marginal cost of public funds varies substantially, and sometimes dramatically, depending on the assumption about the interaction between public expenditure and the supply of labor. It should be stressed again that the two independence assumptions are *logically incompatible*, so that to assume the absence of one is necessarily to accept the presence of the other. Of course, neither assumption is necessarily valid, since each is just a theoretical polar case, and empirical analysis (if feasible) is needed to determine the exact nature of the interaction between public and private goods. But the foregoing theoretical analysis and illustrative calculations show that *logically* it must be admitted that the effects of public expenditure on either the compensated or uncompensated demand, or both, for taxed goods, and that there is a strong presumption that this interaction is quantitatively important, at least for the problem of public expenditure evaluation.

V. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

The principal conclusion of this paper is that the welfare evaluation of public expenditure should take into account the effect of incremental public good provision on the demand for taxed goods. Logically, there is no doubt that either the ordinary or compensated demand (or both) for every private good, including in particular every taxed good, must change as the level of public ependiture changes.[16] Moreover, there is no reason to believe that the adjustment to the welfare criterion necessitated by this effect will be quantitatively insignificant. The illustrative calculations of section IV certainly suggest otherwise. To say this, however, is not to say that these effects can be easily accommodated in applied benefit-cost analysis. Much remains to be done.

First, the theoretical model developed here can be improved. The basic analysis of sections II and III assumes taxation of only one good, and at a uniform rate. In some respects this tax structure, and the assumption of an arbitrary initial level of public good provision, seems preferable to the assumption that the government simultaneously is optimizing tax rates on all goods and the level of public spending, as in the standard "optimal taxation" framework. Of course, existing tax structures, if not optimal, are more complex than the single uniform tax of sections II and III. Thus, it is important to extend the analysis to more complex problems — as illustrated in section IV's treatment of degressive and progressive taxes. Even there, the tax structure is far from realistic, since there is no source of income other than wages. Obviously, an extension to an intertemporal economy with both labor and capital income needs to be made.[17] Also, intersectoral factor-tax differentials (*e.g.*, a corporate income tax) should be considered. In making such extensions, however, an important but difficult issue arises: What tax instrument(s) should be assumed to vary as public expenditure varies? While a surcharge on an existing tax structure as

16. The author is grateful to Professor Browning for noting an exception to this assertion. Suppose the government engages in redistributive expenditure away from taxpayers and toward non-taxpayers. If taxpayers do not value such redistribution, *MRS* in (4) for any taxpayer is zero, and the ordinary and compensated effects are equal to each other, and equal zero. The ordinary and compensated demands for taxed goods are unaffected by the level of public expenditure.

17. An earlier version of this paper develops a simple overlapping-generations model without bequests and presents illustrative estimates of the marginal cost of public funds (Wildasin, 1979b). See also Usher (1982). In a dynamic setting, it would also be of interest to explore the implications of debt financing (*i.e.*, deferred taxation) for the social marginal cost of public expenditure.

in section IV may be considered, this is by no means the only or most relevant possibility. Different assumptions will lead to different estimates of the indirect cost of public expenditure.

Second, unlike the above model, there are many different public goods in real economies, and each affects the demand for taxed goods differently. Therefore, in the general case, no single number can measure the indirect cost of public spending on any and all public goods, and separate estimates should be prepared for each public expenditure category of interest.

Third, the estimation of the effects of public expenditure on uncompensated and compensated private good demand promises to be a formidable task. Applied consumption studies have generally excluded public expenditure as a determinant of private good demand. For cross-section analyses, this may be somewhat defensible since the level of public good provision may not vary a great deal across households (although this clearly depends on the public good in question). But for time series studies there is a more serious problem, since the levels of many public goods would vary substantially over any reasonably long sample period. It is conceivable, of course, that public goods simply do not enter the utility functions of consumers, so that neither compensated nor ordinary demands are in fact affected by public expenditure. Given the size of the public sector, this would be a disheartening conclusion, however convenient for applied demand analysis! Barring this expediency, consider briefly some popular functional specifications that have appeared in the literature. (a) The linear expenditure system, which estimates utility function parameters by expressing ordinary demand functions in terms of those parameters, is consistent with the ordinary independence assumption discussed earlier. (b) The direct and indirect translog utility functions studied by Chirstensen *et al.* (1975) are used to derive ordinary demand functions (actually budget shares) in which public expenditure does not enter as an independent variable. This amounts to a maintained hypothesis of ordinary independence. (c) The almost ideal demand system of Deaton and Muellbauer (1980), although motivated by duality relations between the consumer expenditure function and the ordinary demand function (actually budget share), involves estimation of ordinary demand functions from which public expenditure is excluded as an independent variable. Again this amounts to the maintained hypothesis of ordinary independence.

Thus, it is implicit in each of these approaches that compensated demands for private goods depend on public expenditure. This assumption, though possibly valid, ought to be tested.[18] Recent work by Lindbeck (1982) discusses several plausible examples of possible interaction between labor supply and public expenditure, and clearly suggests that these interactions will vary by type of public expenditure, so that no single simplifying assumption will be appropriate in all cases. The theoretical analysis and sample calculations above show this to be an empirical problem worthy of further investigation.

18. It is interesting to consider the case where the public good z is regarded as a perfect substitute, unit for unit, with some private good, say good n. Many forms of government expenditure, such as some types of health, education, and housing outlays, might plausibly be argued to have this characteristic. (Obviously, such goods are not Samuelsonian pure public goods, but the analysis of this paper applies equally to any publically-provided good.) Then $MRS = p_n$, and provision of one more unit of the good is clearly equivalent to increasing the consumer's income by the amount p_n. Hence $\partial x_1 / \partial z = p_n(\partial x_1 / \partial I)$ and $(\partial x_1 / \partial z)_{\bar{u}} = 0$. In other words, compensated independence holds in this case. Thus, although ordinary independence is assumed in many studies, this is not necessarily *a priori* plausible. (I am indebted to Professor Browning for suggesting this possibility.)

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