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NON-NEUTRALITY OF DEBT WITH ENDOGENOUS FERTILITY

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I. Introduction

ALTHOUGH its full implications for a number of areas in economics have yet to be felt, the fact is that there is a great deal of evidence to suggest that fertility depends on economic variables, i.e., fertility is subject, at least in part, to choice. The purpose of this paper is to explore the consequences of this fact for the analysis of government policies involving intergenerational transfers. In particular, we investigate the Barro (1974) neutrality proposition on government debt, and the closely related question of the impact of an unfunded social security programme, in an economy with endogenous fertility.

This exercise is useful for several reasons. First, underfunded social security programmes and substantial government borrowing are characteristic of many developed countries. The theoretical analysis below shows that such policies can be predicted (in a world of intergenerational altruism) to lower fertility. It may be necessary, therefore, to take these policies into account in analyzing the determinants of fertility in developed countries. Second, many authors have advocated the introduction or expansion of social security programmes as policies to curtail fertility in LDCs. The results of this paper provide a possible rationale for such arguments, although they also indicate that not all social security programs will have such effects. Third, our analysis demonstrates that the effects of government borrowing and social security on economic growth depend on the response of fertility to these policies. In analyses in which fertility is exogenously determined, all of the important dynamic consequences of a government policy are known once one can determine how the policy affects national savings: generally, an increase in national savings raises the capital/labour ratio, total output, output per head, and the wage rate, while reducing the return to capital. In an economy with endogenous fertility, by contrast, it is possible for variables measured in "total" terms (e.g., total output) to move in a direction opposite to the corresponding variables that represent per head magnitudes (e.g., output per head). As is shown below for a simple special case, it is quite possible for an increase in debt to cause

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the aggregate capital stock to fall, while the capital/labour ratio rises. In this case, added government borrowing causes the interest rate to *fall*. Fourth, the analysis here shows that one cannot test the intergenerational altruism hypothesis simply by testing for neutrality of government debt or social security. This is a consequence of the fact that the neutrality proposition is false when fertility is endogenous.

The paper is organized as follows. Section II presents the basic model of household consumption and fertility behaviour. Section III presents a comparative statics analysis of the response to government borrowing. Sections IV concludes the paper by mentioning some extensions of the results, some of their limitations, and possible directions for future research.

II. A model of fertility behaviour

To keep the model very simple, suppose there exist only two generations, 1 and 2, referred to henceforth as parents and children. Suppose, moreover, that each generation lives only for one period, so that there is no overlap between generations. (These restrictions are not essential to the argument.) The members of each generation are identical, and u_i and c_i denote, respectively, the utility and consumption per head of each. u_2 depends only on c_2. However, parental utility u_1 is assumed to depend on c_1, u_2, and n, where n (treated as a continuous variable) is the number of children born to the parents. The presence of u_2 in the parental utility function is familiar from models of intergenerational altruism beginning with Barro (1974). Parents receive a gross wage of w_1 (which might also be taken to include any other sources of lump-sum income, such as bequests inherited from prior generations) and pay lump-sum taxes of T_1. At death, they pass on a total bequest B to their children, each of whom receives B/n. The total bequest is equal to savings out of net income, $w_1 - T_1 - c_1$, times 1 plus the interest rate r, since the bequest occurs at the end of generation 1. Letting $R = (1 + r)^{-1}$, the budget constraints for generations 1 and 2 are thus:

$$c_1 = w_1 - T_1 - RB \tag{1}$$

and

$$c_2 = w_2 - T_2 + \frac{B}{n}, \tag{2}$$

respectively. These collapse to the intergenerational budget constraint

$$c_1 + Rnc_2 = w_1 - T_1 + Rn(w_2 - T_2). \tag{3}$$

Parents choose c_1, n and B to maximize

$$\phi(c_1, c_2, n) \equiv u_1(c_1, n, u_2[c_2]) \tag{4}$$

subject to (1) and (2), or, equivalently, they choose c_1, c_2, and n and to maximize (4) subject to (3).

The first-order conditions for this maximization (optimized variables are shown in parentheses) are

$$(c_1):\quad \phi_1 - \lambda = \frac{\partial u_1}{\partial c_1} - \lambda = 0 \tag{5.1}$$

$$(c_2):\quad \phi_2 - \lambda Rn = \frac{\partial u_1}{\partial u_2} u_2' - \lambda Rn = 0 \tag{5.2}$$

$$(n):\quad \phi_3 - \lambda R(c_2 + T_2 - w_2) = \frac{\partial u_1}{\partial n} - \frac{\lambda RB}{n} = 0 \tag{5.3}$$

where subscripts on ϕ denote the derivatives and λ is the marginal utility of first-period income for generation 1. (An interior solution is assumed for all variables.) Note that since parents are assumed to have a taste for children, so that $\phi_3 > 0$, condition (5.3) implies that bequests must be positive.[1]

This completes the specification of the basic model.[2] The reader can easily verify that the Barro debt neutrality result holds in this model when n is exogenously fixed. The next task is thus to explore what happens when the endogeneity of n is taken into account.

III. Comparative statistics response to feasible policy changes

Throughout most of this section, we assume that equilibrium wage and interest rates are unaffected by changes in government policy. If the economy is small and open, facing a fixed world interest rate, or if the underlying production technology in the economy is linear (i.e., constant marginal products, and thus infinite elasticities of substitution), this assumption will be strictly correct. Otherwise, the analysis must be interpreted as being partial equilibrium in nature. Later on, this assumption is relaxed so that the implications of borrowing for equilibrium factor prices can be discussed.

Feasible policy changes: pure public goods

The basic goal of this section is to see how the equilibrium of the economy depends on government policy. In a Barro economy, government debt policy is synonymous with the intertemporal structure of taxation. A decision to borrow now, for given government (non-transfer) expenditures,

[1] Early versions of this paper (Wildasin (1985)) allowed each generation to live for more than one period, and, in particular, allowed for the lives of parents and children to overlap. In such a model, it is not bequests *per se* that must be positive. Rather, it is "net parental expenditures on children," which would be the present value of expenses for raising children and net bequests.

[2] Razin and Ben-Zion (1975), Pazner and Razin (1980), Nerlove *et al.* (1982, 1984, 1985, 1987) and Cigno (1983) study models that are similar to the present one in their specification of intergenerational preferences. They analyze rather different issues, however, such as efficiency of laissez-faire equilibria, population size in laissez-faire equilibria vs. Benthamite or Millian social welfare optima, or the implications of marriage for bequest behaviour. None of these studies address the debt neutrality problem that is the focus of the present discussion.

is a decision to cut taxes now and to raise them, with interest, in the future. Feasible policy changes must therefore satisfy an intertemporal government budget constraint. The nature of this constraint will depend on the nature of the public goods provided by the government. Two extreme cases will be explicitly analyzed here, and the reader can consider combinations of the two. The first case is that of pure public goods. In this case, we let E_1 and E_2 represent government expenditure in periods 1 and 2 per family, that is, per member of generation 1. Assume that E_1 and E_2 are exogenously fixed, so that any effect they might have on the utility of either generation can be subsumed within the structure of the utility function u_1 and u_2. The government's intertemporal budget constraint requires that

$$T_1 + RnT_2 = E_1 + RE_2 = \text{constant}. \tag{6}$$

Solving (6) implicitly for T_2 as a function of T_1,

$$\frac{dT_2}{dT_1} = -\left[\frac{1}{R} + T_2\frac{\partial n}{\partial T_1}\right]\left[n + T_2\frac{\partial n}{\partial T_2}\right]^{-1}. \tag{7}$$

As is generally the case when the base of a tax is not exogenously fixed, it is possible here that there could be a perverse relationship between the rate of taxation and the amount of tax revenue collected. As a minimum restriction for interesting analysis, assume henceforth that increases in either T_1 and T_2 alone would actually lead to increases in the present value of tax revenue, at least in the neighbourhood of any initial equilibrium we might wish to consider. This will be true if either T_2 or the derivatives of n are not too large. Then the numerator and denominator of the ratio in (7) are both positive, and $\frac{dT_2}{dT_1} < 0$. $\left(\text{In particular, } \frac{dT_2}{dT_1} < 0 \text{ when } T_2 = 0 \text{ initially.}\right)$

Feasible policy changes: quasi-private public goods

There is considerable empirical evidence to indicate that many public goods are not purely public. In fact, for many public expenditure categories, it is approximately true that the cost of providing a given level of public service is proportional to the population being served—that is, the public good is "quasi-private." Education, fire and police protection, and health care all exemplify public goods for which larger populations require larger expenditures. To formulate the government budget constraint in the case of quasi-private public goods, let G_1 and G_2 represent the level of public service in periods 1 and 2, and let G_1 and nG_2 be the cost per family, in each period, of providing the public goods. Thus, in particular, public expenditures are proportional to population in period 2, the period when population itself is variable. We take G_1 and G_2 as exogenously fixed, so that their effect on welfare is subsumed within the structure of the utility functions. The government budget constraint is now

$$T_1 + RnT_2 = G_1 + RnG_2. \tag{6'}$$

Thus, any change in T_1 must be accompanied by a change in T_2 such that

$$\frac{\mathrm{d}T_2}{\mathrm{d}T_1} = -\left[\frac{1}{R} + (T_2 - G_2)\frac{\partial n}{\partial T_1}\right]\left[n + (T_2 - G_2)\frac{\partial n}{\partial T_2}\right]^{-1} \tag{7'}$$

It will be assumed that $\frac{\mathrm{d}T_2}{\mathrm{d}T_1} < 0$. This assumption is valid when increases in T_1 or T_2 result in increases in tax revenue. A sufficient condition for this to be the case is that $\frac{\partial n}{\partial T_i}$, $i = 1, 2$ is sufficiently small, and/or that $T_i - G_i$, $i = 1, 2$ is sufficiently small.

Comparative statics: general parental preferences

The goal of the analysis in this section is to see how feasible policy changes affect real decisions, (c_1, c_2, n). To begin with, we establish certain conclusions that do not require any special restrictions on the structure of preferences. (Where possible, the pure public good and quasi-private public good cases are analyzed together.)

First, let us analyze the effect of policy on fertility, n. Regarding (3) and (5) as a 4-equation system in (λ, c_1, c_2, n), differentiate totally to obtain

$$M\begin{bmatrix} \mathrm{d}\lambda \\ \mathrm{d}c_1 \\ \mathrm{d}c_2 \\ \mathrm{d}n \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{Rn} \\ 0_2 & 0_2 \\ 0 & \lambda R \end{bmatrix}\begin{bmatrix} \mathrm{d}T_1 \\ \mathrm{d}T_2 \end{bmatrix} \tag{8}$$

where M is the Hessian matrix of the system (3) and (5), and where 0_2 denotes a column of 2 zeros. Then

$$\frac{\partial n}{\partial T_1} = \frac{M_{14}}{|M|} \tag{9.1}$$

$$\frac{\partial n}{\partial T_2} = [RnM_{14} + \lambda RM_{44}]\,|M|^{-1} \tag{9.2}$$

where M_{ij} is the cofactor of the (i, j) element of M. Assuming that (3) and (5) characterize a strict maximum of utility, the leading principal minors of M alternate in sign. Then, if $\frac{dn}{dT_1}$ denotes the *total* effect of a feasible increase in T_1, using (7) or (7′) and (9), we obtain

$$\frac{\mathrm{d}n}{\mathrm{d}T_1} \equiv \frac{\partial n}{\partial T_1} + \frac{\partial n}{\partial T_2}\frac{\mathrm{d}T_2}{\mathrm{d}T_1} = -\lambda R\frac{M_{44}}{|M|}\Delta > 0, \tag{10}$$

where Δ denotes the denominator in (7) or (7′), as the case may be. The inequality in (10) holds because $\Delta > 0$ by assumption and because the second-order condition for utility maximization implies that M_{44} is of sign opposite to $|M|$. Thus, a debt-financed tax cut for parents (a decrease in T_1) unambiguously *reduces* fertility.[3] Note that this result holds independently of the purity or impurity of public goods. It establishes that government debt is definitely *not* neutral, even though there is intergenerational altruism of the Barro type.

The intuition behind the negative effect of borrowing on fertility becomes clear from examination of the intergenerational budget constraint (3), or from the first-order condition for the utility-maximizing choice of n, (5.3). The tax imposed in the second period, T_2, is one of the costs of having a child, appearing as a price term in the budget constraint. An increase in borrowing in period 1 means an increase in T_2, which is to say that it amounts to an increase in the effective marginal cost of having a child. A balanced-budget change will leave only a substitution effect from the combined changes in taxation in the two periods, as shown in (10).

Second, let us consider the impact of a change in tax policy on consumption expenditure. Depending on whether public goods are purely public or quasi-private, substitute from the government budget constraint (6) or (6′) into the household budget constraint (3). We then find that

$$\frac{\mathrm{d}(c_1 + Rnc_2)}{\mathrm{d}T_1} = R(w_2 - G_2)\frac{\mathrm{d}n}{\mathrm{d}T_1}, \tag{11}$$

where it is to be understood here that the term $G_2 = 0$ in the case of purely public goods, corresponding to (6). By (10), it follows that the present value of family consumption (the left-hand side of (11)) must rise when T_1 rises if public goods are purely public. Moreover, the same will be true for quasi-private public goods if $w_2 > G_2$, i.e., if earnings per head exceed public expenditure per head in period 2.[4] However, although the present value of aggregate consumption increases when T_1 increases, we cannot determine what happens to c_1 or c_2 individually without more specific information on preferences, such as is provided by the restrictions imposed in the next subsection.

Third, let us consider the effect of feasible policy changes on the welfare of parents. Differentiation of the parental utility function ϕ, use of the first-order conditions (5), and differentiation of the budget constraint (3) shows that the real income change from a feasible increase in T_1 is

$$\lambda^{-1}\frac{\mathrm{d}\phi}{\mathrm{d}T_1} = -1 - Rn\frac{\mathrm{d}T_2}{\mathrm{d}T_1}. \tag{12}$$

To evaluate this expression, take first the case of pure public goods.

[3] See also Becker and Tomes (1976), Willis (1985), Battina (1985), and Becker and Barro (1988) for related results.

[4] Of course this condition need not hold for arbitrary w_2 and G_2, but it is reasonable to expect it to hold in practice.

Substitution from (7) and (9) yields

$$\lambda^{-1}\frac{\mathrm{d}\phi}{\mathrm{d}T_1} = -RT_2\frac{\lambda M_{44}}{|M|}\Delta > 0 \tag{13}$$

if $T_2 > 0$, that is, welfare is unambiguously decreased by feasible borrowing.[5] The intuition behind this result is as follows: with pure public goods, lump-sum taxes paid by children are not distortionless. As seen from (3) or (5.3), they artificially raise the cost of having children and thus distort the fertility decision. Equation (13) shows that as long as second-period taxes are positive, welfare is enhanced by reducing them (and by increasing T_1 to preserve budget balance). Once $T_2 = 0$, a first-best optimum is achieved and the first-order welfare effect of a feasible policy change is zero. This is a striking result, because it implies not only that the government should not (from generation 1's viewpoint) run a deficit, it should run a surplus, prepaying all future taxes and financing future expenditures from interest on and the sales of assets. Alternatively, if the government could tax *families* rather than *individuals,* it would be possible to collect positive taxes from the second generation in a non-distorting way.[6]

Finally, to analyze the welfare effect of changes in tax policy for the case of quasi-private public goods, use equations (7′) and (9) in (12) to find

$$\lambda^{-1}\frac{\mathrm{d}\phi}{\mathrm{d}T_1} = -R(T_2 - G_2)\frac{\lambda M_{44}}{|M|}\Delta. \tag{13′}$$

This expression wil have the sign of $T_2 - G_2$. In particular, welfare is stationary (and in fact is maximized) when $T_2 = G_2$. The intuition for this result is as follows: the decision to have another child, when public goods are quasi-private, entails a real social cost of G_2, that is, the cost of providing the fixed level of public services to one more individual. If $T_2 = G_2$, this cost is correctly internalized to the family. If $T_2 > G_2$, then having additional children yields social benefits in excess of social costs and raising T_1, which increases the number of children, raises welfare. If $T_2 < G_2$, the opposite reasoning applies. Thus, if $T_2 > G_2$, T_1 should be increased and T_2 decreased, while if $T_2 < G_2$, T_1 should be reduced and T_2 should be raised. In either case, welfare increases as T_2 is brought closer to G_2, and is maximal when they are equated. This contrasts sharply with the case of pure public goods.[7]

[5] Cigno (1983) shows that a laissez-faire equilibrium will not be optimal from the viewpoint of a social welfare function that attaches more weight to subsequent generations than the parental utility function. One can see that the same result obtains here.

[6] Nerlove *et al.* ((1987), p. 84) also make the point that "a head tax is not a lump-sum tax [when] the number of children is endogenous." However, they do not specifically analyze the nature of the non-neutralities arising from second-period head taxes, or the optimal intertemporal tax structure.

[7] The results here strongly parallel those in the theory of local public economics (see Wildasin (1986, 1987), where it is shown that local lump-sum taxes should be equated to (marginal) congestion costs in order to induce efficient locational choices and, thus, an efficient distribution of population among jurisdictions. In the present context, population variation

To summarize the results so far:

Proposition 1: With no special restrictions on the form of parental preferences for children, the following results obtain:

(i) Regardless of the initial tax structure and of the publicness or quasi-privateness of public goods, an increase in borrowing necessarily diminishes fertility. Borrowing is therefore non-neutral.

(ii) With pure public goods, an increase in borrowing reduces the present value of total consumption by all generations. With quasi-private public goods, the same is true provided that wages per head exceed public expenditure per head.

(iii) With pure public goods, an increase in borrowing is always welfare-reducing for generation 1 (if future taxes would otherwise be positive). With quasi-private public goods, a move towards period-by-period budget balance is welfare-enhancing. If current taxes exceed expenditure per head, higher borrowing (i.e. a smaller current budget surplus) is welfare-increasing. If current taxes are less than expenditure per head, a reduction in borrowing will raise welfare.

Comparative static analysis: *a special case*

It is of great interest to analyze the effects of government debt on consumption per head and on total consumption for each generation. Such analysis is complicated, however, by the fact that the effect of policy on c_1 and c_2 depends on complementarity/substitutability between own-consumption, the number of children, and welfare of children, as embodied in the general parental utility function (4). Short of empirical investigation, one can only perform illustrative analyses for special cases. Nonetheless, this can be enlightening. In particular, consider the following specification:

$$\phi(c_1, c_2, n) = \phi_1(c_1) + \delta n \phi_2(c_2). \tag{14}$$

This utility function looks like that of a utilitarian planner that discounts future utility by the factor $\delta < 1$, and hence let us refer to this specification as *utilitarian preferences*.[8] Of course, ϕ_1 and ϕ_2 are assumed strictly concave (ϕ_2 need not, however, be identical to u_2.)

Given utilitarian preferences, the first-order conditions for c_2 and n take the special form

$$\delta\phi_2' - \lambda R = 0 \tag{5.2$'$}$$

$$\delta\phi_2 - \lambda R(c_2 + T_2 - w_2) = 0, \tag{5.3$'$}$$

Footnote 7 (*continued*)
arises from fertility behaviour, whereas in the local government context it arises from migration, but the principles of optimal taxation in each case are the same. Nerlove *et al.* (1987) discuss the fact that equilibrium resource allocation is efficient with pure pubic goods and all taxes imposed in the first period. They do not discuss the case of quasi-private (or congested) public goods, but it is clear from the above analysis that the equilibrium is *not* efficient in the presence of such goods without appropriate taxes being imposed in the second period.

where ϕ_i' denotes $\dfrac{d\phi_i}{dc_i}$, $\phi_i''=\dfrac{d^2\phi_i}{dc_i^2}$. From (5.3)′, not both c_2 and λ can remain constant as feasible policy changes occur. In fact, totally differentiating (5.3)′ and using (5.2)′,

$$-\lambda R\frac{dT_2}{dT_1}+\frac{RB}{n}\frac{d\lambda}{dT_1}=0. \tag{15}$$

Since $\dfrac{dT_2}{dT_1}<0<B$, it follows that $\dfrac{d\lambda}{dT_1}<0$. Concavity of the utility functions then implies

$$\frac{dc_1}{dT_1}<0, \qquad \frac{dc_2}{dT_1}<0. \tag{16.1}$$

By (1) and (2), this is equivalent to

$$\frac{d(RB+T_1)}{dT_1}>0, \qquad \frac{d(B/n-T_2)}{dT_1}<0. \tag{16.2}$$

These results mean that incremental government borrowing causes parental consumption to rise, and consumption per child to rise, when parental preferences are utilitarian. However, by Proposition 1(ii), the present value of total consumption across both generations must fall in the face of additional government borrowing. Therefore, it must be the case that total consumption by children must fall (in present value terms), that is, that fertility falls proportionately more than c_2 rises. Parental bequests rise (in present value terms) by *less* than their taxes are cut, and bequests per child rise by *more* than their taxes rise.

It is now possible to show what happens to the capital-labour ratio in period 2. To analyze this, we need to know how the real capital stock changes between periods 1 and 2. The amount of saving in period 1, per family, is RB. If there were no debt instruments in the economy, all of this

[8] The assumption of utilitarian parental preferences is in some respects similar to the Becker and Tomes (1979) and Becker (1981) assumption that parental utilty depends on the total wealth of their children, i.e., the number of children times wealth per child, and appears explicitly in Cigno (1983). Becker and Barro (1988) consider a function like (14), but assume strict concavity in n. Note that strict concavity of ϕ in n is not necessary for a unique interior choice of n. The easiest way to see this is as follows. For notational ease, let $\bar{w}_2=R(w_2-T_2)$. Then the second-oder condition for a strict maximum of (14) subject to (3) is that (see, e.g., Intriligator (1971) p. 36) the last 3 leading principal minors of the following matrix alternate in sign, the last being negative:

$$G\equiv\begin{bmatrix} 0 & -1 & -Rn & \bar{w}_2-Rc_2 \\ -1 & \phi_1'' & 0 & 0 \\ -Rn & 0 & \delta n\phi_2'' & \delta\phi_2'-R\lambda \\ \bar{w}_2-Rc_2 & 0 & \delta\phi_2'-R\lambda & 0 \end{bmatrix}$$

Concavity of ϕ_1 and ϕ_2 guarantee satisfaction of the required condition on the sign of the first two principal minors. To show that $|G|<0$, note first that $\delta\phi_2'-R\lambda=0$ from the first-order condition for c_2. By direct computation it now follows that

$$|G|=-(\bar{w}_2-Rc_2)^2\phi_1''\delta n\phi_2''<0.$$

saving would go into real capital, so that capital per family, denoted by K, would equal RB. The period-2 capital/labour ratio would then be $K/n = RB/n$. However, given the existence of public debt, not all savings will go into real capital formation. Thus, let D denote the amount of public debt per family, payable in period 2.[9] By the parental budget constraint, $K + D = RB$. Hence, the capital/labour ratio in period 2 is $\frac{K}{n} = \frac{(RB - D)}{n}$.

If public goods are pure, $D = R(nT_2 - E_2)$, so that $\frac{K}{n} = R\left(\frac{B}{n} - T_2 + \frac{E_2}{n}\right)$. If public goods are quasi-private, $D = Rn(T_2 - G_2)$, so that $\frac{K}{n} = R\left(\frac{B}{n} - T_2 + G_2\right)$. In either case, using (10) and (16.2), we find that

$$\frac{\mathrm{d}(K/n)}{\mathrm{d}T_1} < 0. \tag{17}$$

Hence feasible government borrowing *increases* the capital/labour ratio in period 2. Usually it is argued that government borrowing will depress savings and investment, at least in a full-employment economy, and that this will inhibit capital deepening. Here, however, we find that this latter inference can be invalidated, since, even if borrowing does reduce investment, it also reduces population, and may reduce population proportionately more.

The analysis so far has assumed fixed factor prices, either because the economy is small and open or because the production technology is linear. Let us now relax this assumption. Suppose that the economy is closed, and that the production technology, although still characterized by constant returns to scale, is such that labour and capital are no longer perfect substitutes.[10] With such a technology, an increase in the period-2 capital/labour ratio must increase labour productivity and wages in period 2, while reducing the interest rate. The analysis presented above can be thought of as describing the initial impact of a change in policy on the capital labour ratio, with fixed factor prices. If borrowing raises the capital/labour ratio at fixed factor prices, then allowing factor prices to adjust in response to policy should tend to dampen, but not reverse, the increase in the capital/labour ratio and the other real effects of government borrowing. This intuition is in fact correct, as shown formally in the

[9] D could include debt carried forward from prior periods. If the economy is closed, K is the amount of capital stock per family in the economy. If the economy is open, K still denotes real capital per family, but this will be greater than the domestic capital stock per family if the economy is a capital exporter and conversely if it is a capital importer. The capital/labour ratio, $\frac{K}{n}$, is therefore the stock of wealth per worker.

[10] A linear production technology means that isoquants are straight lines, and hence that the elasticity of substitution is infinite. We now allow for curved, strictly convex isoquants, implying less than perfect substitutability and variable factor prices.

Appendix. This means that government borrowing will have an effect on capital intensity and factor prices that is the *reverse* of what is usually found.

In summary, the above discussion establishes

Proposition 2. With utilitarian parental preferences, with either pure or quasi-private public goods, an increase in government borrowing results in:

(i) an increase in parental consumption and in consumption per child,
(ii) an increase in the present value of total bequests that is less than the amount of additional government borrowing,
(iii) an increase in bequests per child that exceeds the increase in taxes per child,
(iv) an *increase* in the capital/labour ratio, and
(v) in a closed economy with less than infinite substitutability in production between capital and labour an *increase* in the wage rate and a *reduction* in the interest rate.

In recent years, there has been a great deal of discussion of the impact of public debt and social security on saving and capital accumulation. In the usual framework in which these discussions are undertaken, fertility is treated as exogenously fixed. Hence, increases or decreases of the aggregate capital stock imply corresponding changes in the amount of capital per head, output per head, factor prices, and so on. The simultaneous movement of all of these variables in the conventional directions have come to be regarded as symptoms of economic "growth" (or lack thereof, as the case may be). With endogenous fertility, however, these variables can move in unconventional directions, so that the concept of economic "growth" itself becomes ambiguous. *A fortiori,* the effect of public policies like tax cuts or unfunded social security have ambiguous impacts on "growth," as measured by conventional indicators.

To appreciate this, note first that (10) shows that borrowing reduces fertility, and thus the total amount of labour in the economy, quite generally. Furthermore, under utilitarian preferences, borrowing increases consumption per head for both parents and children. This entails an increase in utility per head for generation 2. However, if public goods are pure, or if public goods are quasi-private and there is an initial deficit ($T_1 < G_1$), additional borrowing *decreases* utility for generation 1 (Prop. 1(iii)). In addition, as seen in (17), it would (in a closed economy) increase the capital/labour ratio, labour productivity, and the wage-rental ratio. These factor price effects, and the change in the intertemporal distribution of utility per head, are generally regarded as aspects of economic growth. In this sense, borrowing promotes economic growth. This finding not only contradicts the Barro neutrality result for economies with altruistic bequests. It also reverses the standard conclusions about the effect of borrowing in economies with life-cycle utility maximizers who leave no bequests.

On the other hand, Proposition 1(ii) and Proposition 2(i) show that (given utilitarian preferences) borrowing reduces the present value of total consumption, which occurs because the total consumption of generation 2 is reduced. The reduction in population induced by the programme is thus sufficient to reduce aggregate future consumption. Moreover, if public goods are pure, or if they are quasi-private and there is an initial deficit, incremental borrowing reduces the *aggregate* utility of the second generation. (Proof: By (16.1), c_1 increases, hence ϕ_1 in (14) must rise. The fact that ϕ falls—Prop. 1(iii)—means that $n\phi_2$ must fall enough to offset the increase in ϕ_1.) Since c_1 increases (Proposition 2(i)), borrowing also results in a smaller *total* capital stock. In these respects, government borrowing tends to reduce economic growth.

In summary, then, it might be best to characterize the effect of borrowing, given utilitarian preferences, as conducive to economic growth in its micro aspects (e.g., in terms of the capital/labour ratio, factor prices, utility per head) but detrimental to economic growth in its macro aspects (total consumption, total factor supplies, total utility). The differences between the two types of effects arise, of course, because the programme reduces fertility, permitting aggregative measures of growth to fall while micro measures increase.

IV. Further applications and conclusions

Certain extensions of the above results, and their applications to different problems, are more or less immediate:

(i) Within the context of the model, introduction of an underfunded (pay-as-you-go) social security programme is identical in its effects to government borrowing. Propositions 1 and 2, and subsequent remarks, therefore apply directly.

(ii) It has been hypothesized (see Leibenstein (1957) for the original statement and Wildasin (1983) for a review of subsequent literature) that social security programmes may reduce fertility. The usual argument for this effect is based on an explicit or implicit assumption of imperfect capital markets.[11] Proposition 1(i) shows that capital market imperfections are by no means necessary for this result, provided that the social security programme is underfunded.

(iii) The results are essentially unchanged when the model is extended to multi-period overlapping life cycles (Wildasin (1985)).

(iv) Some of the more specific and perhaps surprising results of the analysis, appearing in Proposition 2, are obtained under the assumption of utilitarian parental preferences. It is clear, however, that these results must still be qualitatively unchanged by small departures from this preference structure. (For example, a small deviation from

[11] A detailed analysis of the level of fertility in economies with and without capital markets can be found in Nerlove *et al.* (1987).

utilitarian preferences might change the magnitude of the increase in consumption that results from borrowing, but would not reverse the sign of this effect.) The assumption of utilitarian preferences thus leads to conclusions which, although somewhat special, are indicative of those that must obtain in many other cases as well.

(v) A simple form of human capital formation can be added to the model. Let h be partental investment per child in human capital, and let the wage of children be an increasing concave function of h, $w_2(h)$. It is straightforward to show that h will be chosen such that $Rw_2'(h)=1$. This determines h independently of any policy parameters. Hence the results are unaffected by this modification.

It remains to note some of the limitations of the analysis. To begin with, it rests on certain behavioural hypotheses that (perhaps to put it mildly) do not command universal acceptance. Chiefly, the assumption that parents care about the welfare of their children, as the children themselves define it, is open to some question. Intergenerational utility maximization of this type is, of course, crucial to the Barro neutrality result when fertility is exogenous. In examining the neutrality proposition with endogenous fertility it is natural to maintain the intergenerational utility maximization assumption, since it is obvious that borrowing cannot be neutral otherwise. That is, we have analyzed the impact of borrowing in a model in which it is least likely to have real effects. It therefore seems very likely that neutrality of debt will occur only exceptionally in models with endogenous fertility.

Finally, what of empirical importance of the effect of government policy on fertility? Obviously, this issue cannot be settled on *a priori* grounds. From the viewpoint of armchair empiricism, one might argue that the effect of a small change in the level of debt on fertility must be small as well. However, the level of unfunded social security liabilities and outstanding government debt is certainly very large for many countries. If one asks, therefore, whether such public policies have significant effects on fertility, the answer might well be no, for small policy changes, but yes, for changes of the order of magnitude observed during, say, the past half-century.

There is already a substantial number of empirical studies bearing on this issue, and a typical finding is that social security programmes have significant effects on social security.[12] However, the link between theory and empirical work in this area is rather weak. For example, existing empirical studies make no reference to the degree of funding of social security programmes, typically using a simple measure of benefit payouts as a regressor to represent program size. Yet as shown in the analysis above, the degree to which a social security programme is funded or underfunded may well be a crucial determinant of the programme's impact on fertility. This fact well illustrates the need for explicit theoretical analysis. In brief, the empirical issues cannot be adequately investigated in the absence of

[12] See, e.g. Hohm (1975) and Entwisle and Winegarden (1984). Wildasin (1983) provides a detailed review of the literature and many more references.

fully-articulated theoretical models, of which this paper has provided one example.

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APPENDIX

This Appendix presents a formal proof of Proposition 2(v). We now assume that factor prices in period 2 are determined in competitive markets, and that production occurs under conditions of constant returns to scale. As is well-known, this implies that factor prices depend only on the capital/labour ration K/n. If $f(K/n)$ is the production function showing output per worker in period 2, then letting $k = K/n$,

$$r = f'(k) \tag{A.1.1}$$

$$w_2 = f(k) - kf'(k) \tag{A.1.2}$$

where, recall, $R = (1+r)^{-1}$.

The capital stock per family in period 2 is $w_1 - T_1 - c_1$. We can treat w_1 as exogenously given. Therefore

$$k = \frac{K}{n} = \frac{w_1 - T_1 - c_1}{n}. \tag{A.2}$$

Now it is clear that maximization of parental utility (4) to the intergenerational budget constraint (3) implies that c_1 and n will depend on T_1, T_2, r, and w_2, hence (A.2) can be written as

$$k = \psi(T_1, T_2, r, w_2). \tag{A.3}$$

Moreover, taking into account the dependence of n on all of these variables, we can use the government budget constraint, whether for the case of pure or of impure public goods, to solve implicitly for T_2 in terms of T_1, r, and w_2. Substituting into (A.3), we have

$$\begin{aligned} k &= \psi(T_1, T_2[T_1, r, w_2], r, w_2) \\ &= \theta(T_1, r, w_2), \end{aligned} \tag{A.4}$$

say. In this notation, equation (17) states precisely that

$$\frac{\partial\theta}{\partial T_1} < 0. \tag{A.5}$$

Using (A.1.1) and (A.1.2) to eliminate r and w_2, we can substitute into (A.4) to obtain

$$k = \theta(T_1, r(k), w_2(k)). \tag{A.6}$$

Stability of equilibrium requires that

$$1 - \frac{\partial\theta}{\partial r}\frac{\partial r}{\partial k} - \frac{\partial\theta}{\partial w_2}\frac{\partial w_2}{\partial k} > 0, \tag{A.7}$$

a standard condition that we assume is satisfied. We can thus use (A.6) to determine k implicitly as a function of T_1, taking variability of factor prices fully into account. We find that

$$\frac{\mathrm{d}k}{\mathrm{d}T_1} = \frac{\dfrac{\partial\theta}{\partial T_1}}{1 - \dfrac{\partial\theta}{r}\dfrac{\partial r}{\partial k} - \dfrac{\partial\theta}{\partial w_2}\dfrac{\partial w_2}{\partial k}} < 0,$$

where the inequality follows from (A.5) and (A.7). This establishes the required result.

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