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NASH EQUILIBRIA IN MODELS OF FISCAL COMPETITION

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This paper analyzes Nash equilibria in a simple model of an economy with jurisdictions engaging in fiscal competition. Small-number Nash equilibria in which tax rates are the strategic variables are shown not to coincide with Nash equilibria in which public expenditure levels are the strategic variables.

1. Introduction

Recently, the problem of tax competition among jurisdictions has been the subject of a number of important contributions. These include papers by Beck (1983), Wilson (1985, 1986), Zodrow and Mieszkowski (1986), and others.[1] Many of these studies analyze the 'purely competitive' case where the number of jurisdictions engaging in tax competition is large. However, some more recent research, notably Mintz and Tulkens (1986), de Crombrugghe and Tulkens (1987), and Bucovetsky (1986), has examined the problem in a small-number context in which strategic interactions cannot be ignored.

A typical assumption in this literature is that each jurisdiction sets a single tax rate on some interjurisdictionally-mobile tax base, such as a traded commodity or capital. The revenue from the tax is used to finance expenditure on a public good. A perennial question in the literature is whether tax competition results in underprovision of public goods, i.e. tax rates and expenditure levels that are lower than optimal. A common though not universal result in the 'purely competitive' case is that public goods are underprovided in an equilibrium with tax competition. In the small-number setting matters are somewhat less clear, although results of a similar flavor (e.g. de Crombrugghe and Tulkens) have appeared.

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[1]See Wildasin (1986, 1987) for further discussion and references.

The present paper attempts to shed additional light on this question, and on the more general question of what is an equilibrium in the small-number case. In almost all research to date, an equilibrium with tax competition means a situation where each jurisdiction is choosing its tax rate optimally, given the tax rates chosen by all other jurisdictions. That is to say, the standard concept of equilibrium is that of a Nash equilibrium, where tax rates are the strategic variable. The Nash equilibrium concept is of course extremely widely used in non-cooperative game theory, and, since the goal of analyses of tax competition is to describe the determination of tax rates, it is very natural and appropriate that Nash equilibria in tax rates have been carefully studied.

On the other hand, tax rates are only one aspect of fiscal policy, which in turn is only one category of economic policy in general. Jurisdictions that find themselves in a competitive posture with respect to one another must be expected to consider the competitive implications of all aspects of policy. This would include expenditure policy, in particular.[2] Competitive pressures might be felt in the determination of spending levels as much as they are felt in the setting of tax rates. Indeed, the two will ordinarily be one-to-one: an increase in expenditure will require an increase in the tax rate, and conversely. Since the choice of one is tantamount to the choice of the other, it might seem that we could just as well imagine jurisdictions choosing expenditure levels 'directly', and then determining the tax rates required to finance these expenditures, as the other way around. 'Fiscal' competition, then, would show up in a reluctance to raise expenditures because the higher tax rates this would entail would reduce the amount of local capital (or exports, etc.). Based on these considerations, one might be led to formulate a model in which jurisdictions choose levels of public expenditure as their strategic variables. A Nash equilibrium in these strategies could be defined in the usual way.

The purpose of this paper is to compare this type of equilibrium – i.e. a Nash equilibrium in expenditures – with the type that has been most prevalent in the literature – i.e. a Nash equilibrium in tax rates. One might at first conjecture that the two would coincide since, as noted, the relationship between expenditures and tax rates will often be one-to-one. Since there do not seem to be strong reasons to prefer one over the other on economic grounds, it would be somewhat reassuring to be able to verify this conjecture. Thus, section 2 presents a simple model of jurisdictions which tax mobile capital and use their revenues to finance provisions of a public good. Nash equilibria in tax rates and public goods are defined, and characterized

[2]Oates and Schwab (1985) have analyzed the determination of environmental policy in a competitive setting, which provides another important example of ways that jurisdictions might interact. Scotchmer (1986) considers Nash equilibria in expenditures for local governments that maximize land values.

in terms of first-order conditions for optimization. It becomes clear from this that the two types of equilibria do not, in general, coincide. Section 3 specializes the model to the case where all jurisdictions are identical, and examines symmetric Nash equilibria. In this setting, it is possible to make a more precise comparison of the two types of equilibria, and to obtain more economic insight into the differences between them. It is shown that Nash equilibria in expenditures are more 'rivalrous', in a certain sense, than Nash equilibria in tax rates. The analysis permits a characterization of the way that these Nash equilibria depend on the number of jurisdictions. Interestingly, the difference between the two goes to zero as the number of jurisdictions becomes large.

2. Two concepts of equilibrium in fiscal competition

This section of the paper sets out a basic framework for analysis. Two types of equilibria are then defined. After simplifying the model, the two equilibria are compared in detail in section 3.

In order to focus attention on the essentials, the model is kept extremely simple. Suppose there are $n \geqq 2$ jurisdictions, each inhabited by a single household. There is a single homogeneous private good that serves as numeraire and that is produced in each locality. The production process uses only homogeneous capital as a variable input, together with some unspecified locationally-fixed inputs, such as labor or land, which are owned by the local resident. Let $f_i(K_i)$ be private good production in locality i as a function of capital employed, K_i, and suppose $f_i' > 0 > f_i''$. Capital is paid its marginal product, hence the local return to the fixed factors is $f_i(K_i) - K_i f_i'(K_i)$. The homogeneous private good can either be consumed or used as an input into the provision of a local public good. Assuming units of local public good are chosen appropriately, we can measure public good provision in locality i, z_i, in terms of units of private good input – i.e. local public expenditure. Each locality finances its public expenditure using a per unit tax on capital at rate t_i. The government budget constraint for locality t is

$$t_i K_i = z_i. \tag{1}$$

The net return to capital in locality i is thus $f_i'(K_i) - t_i$. Capital is assumed to be fixed in supply to the economy as a whole, so that

$$\sum_i K_i = \bar{K} \tag{2}$$

for some given $\bar{K}$. It is also assumed to be freely mobile, so that

$$f_i'(K_i) - t_i = \rho, \quad i = 1, \ldots, n, \tag{3}$$

where ρ is the economy-wide net return to capital. Eqs. (2) and (3) provide a system of $n+1$ equations sufficient to determine the equilibrium allocation of capital $(K_1,\dots,K_n)$ and the equilibrium net return to capital ρ as functions of the vector of tax rates $\tau=(t_1,\dots,t_n)$. One easily verifies that[3]

$$\frac{\partial K_i}{\partial t_i}=\frac{\varepsilon_i K_i}{\rho+t_i}\frac{\sum\limits_{j\neq i}\frac{\varepsilon_j K_j}{\rho+t_j}}{\sum\limits_{j}\frac{\varepsilon_j K_j}{\rho+t_j}}<0, \tag{4.1}$$

$$\frac{\partial K_j}{\partial t_i}=-\frac{\frac{\varepsilon_j K_j}{\rho+t_j}\frac{\varepsilon_i K_i}{\rho+t_i}}{\sum\limits_{s}\frac{\varepsilon_s K_s}{\rho+t_s}}>0, \quad j\neq i, \tag{4.2}$$

where $\varepsilon_i=[\partial\log f_i'(K_i)/\partial\log K_i]^{-1}$ is the elasticity of demand for capital in locality i.[4] (By concavity of f_i, $\varepsilon_i<0$.)

The total income accruing to capital owners in the economy is $\rho(\tau)\bar{K}$. Assume that the household in locality i owns the share $\theta_i\geqq 0$ of the capital stock. Some capital, perhaps all capital, may be owned by households other than those in the n localities (e.g. absentee owners), and indeed the results in section 3 are presented for the simplest special case where $\theta_i=0$, for all i. However, for the purposes of this section, one could assume $\sum_i\theta_i=1$, so that all capital income is 'present and accounted for' without any need for absentee owners of capital. In any case, the private good consumption of the household in locality i, denoted x_i, will equal income from local rents plus any net capital income received, thus:

$$x_i=f_i(K_i)-K_i f_i'(K_i)+\theta_i\rho\bar{K}. \tag{5}$$

The household in locality i has a twice-continuously differentiable strictly quasi-concave utility function $u_i(x_i,z_i)$ describing its preferences for private

[3]See Wildasin (forthcoming) for derivations. These relations are crucial for grasping the intuition behind the results. As discussed in Mintz and Tulkens (1986) and Wildasin (forthcoming), the fact that an increase in i's tax rate raises js tax base [as shown in (4.2)] can be interpreted as a fiscal externality. Residents of j will be made better off because of i's tax increase since they can have either more public goods at the existing tax rate, or a lower tax rate at the existing level of public good provision, or some combination of the two. This is called the 'public consumption effect' in Mintz–Tulkens. It is this sort of externality that tends to result in underprovision of local public goods in economies of this type: as Gordon (1983) puts it, each jurisdiction ignores some of the benefit that its own taxing and spending produces for other jurisdictions. The analysis in Wildasin (forthcoming) describes Pigovian remedies to such externalities, in the form of matching grants to local governments.

[4]That is, $f'(K_i)$ can be thought of as an inverse demand function for capital. Taking the elasticity of this function and inverting yields ε_i.

and public goods. Note that x_i is a function of τ through $K_i(\tau)$ and $\rho(\tau)$. We will make the fundamental behavioral assumption that each locality chooses its strategies so as to maximize its utility.

The goal of the analysis will be to compare two different types of Nash equilibria, corresponding to two different specifications of the instruments or strategies to be used by the localities.[5] In the first specification, the strategies will be tax rates: locality i will choose t_i to maximize utility subject to fixed values of t_j, $j \neq i$. In the second specification, each locality i will choose its expenditure level, z_i, subject to fixed values of the expenditure levels z_j of other jurisdictions $j \neq i$. In either case, it is natural to suppose that the government budget constraint (1) must be satisfied in equilibrium. If one substitutes $K_i = K_i(\tau)$ into (1), this provides a system of equations in $2n$ variables $(t_1, \ldots, t_n, z_1, \ldots, z_n)$. As written, it allows one to solve for $z_i = z_i(\tau) = t_i K_i(\tau)$, i.e. to express the vector $\zeta = (z_1, \ldots, z_n)$ in terms of the vector τ. If the Jacobian matrix of derivatives of (1) with respect to t is non-singular, this relationship can be expressed in inverse form as $\tau = \tau(\zeta)$. For the moment, let invertibility simply be assumed. Then we can define the two equilibrium concepts more precisely.

T-equilibrium. A vector τ^* is a T-equilibrium if, for all i, t_i^* is the solution to

$$P_T: \quad \max_{\langle t_i \rangle} u_i(x_i, z_i)$$

subject to

$$x_i = f_i(K_i[\tau]) - K_i(\tau) f_i'(K_i[\tau]) + \theta_i \rho(\tau) \bar{K}, \tag{6.1}$$

$$z_i = t_i K_i(\tau), \tag{6.2}$$

$$t_j = t_j^*, \quad j \neq i.$$

Z-equilibrium. A vector ζ' is a Z-equilibrium if, for all i, z_i' is the solution to

$$P_Z: \quad \max_{\langle z_i \rangle} u_i(x_i, z_i)$$

subject to

$$x_i = f_i(K_i[\tau(\zeta)]) - K_i(\tau[\zeta]) f'(K_i[\tau(\zeta)]) + \theta_i \rho(\tau[\zeta]) \bar{K}, \tag{7.1}$$

[5]Since the objective of the analysis is to compare two different types of equilibria, existence of equilibria will be assumed. For more detailed discussion of the existence problem, see Mintz and Tulkens (1986) and, in a model close to the present one, Bucovetsky (1986).

$$\tau(\zeta) \quad \text{solves the system (1)}, \tag{7.2}$$

$$z_j = z'_j, \quad j \neq i.$$

In short, in a T-equilibrium, each locality optimizes with respect to its tax rate, taking other tax rates as given and letting expenditure levels vary passively via (1), whereas in a Z-equilibrium it is the expenditure levels that are optimized, with tax rates responding passively to keep the government budgets in balance.

Note that each equilibrium can be characterized in terms of the first-order conditions for the associated maximization problems P_T and P_Z. In particular, the two equilibria will coincide if $\zeta' = \zeta(\tau^*)$, i.e. $\tau^* = \tau(\zeta')$. In this case, the first-order conditions for P_T and P_Z must both be satisfied at the same (equilibrium) values of τ and ζ.

To derive the first-order condition for P_T, substitute from (6.1) and (6.2) into the utility function and differentiate with respect to t_i. Define $MRS_i = (\partial u_i/\partial z_i)(\partial u_i/\partial x_i)^{-1}$ and note that $\partial(f_i - K_i f'_i)/\partial K_i = -f''_i/\varepsilon_i$ and that $\partial z_i/\partial t_i = K_i(1 + \partial \log K_i/\partial \log t_i)$.[6] It follows that, in a T-equilibrium,

$$MRS_i = \left(\frac{f'_i}{\varepsilon_i}\frac{\partial K_i}{\partial t_i} - \theta_i \bar{K}\frac{\partial \rho}{\partial t_i}\right)\left(K_i\left[1 + \frac{\partial \log K_i}{\partial \log t_i}\right]\right)^{-1}, \quad i = 1, \ldots, n, \tag{8.1}$$

where of course, all functions are evaluated at τ^*. Similarly, in a Z-equilibrium,

$$MRS_i = \left(\frac{f'_i}{\varepsilon_i}\frac{\partial K_i}{\partial t_i} - \theta_i \bar{K}\frac{\partial \rho}{\partial t_i}\right)\frac{\partial t_i}{\partial z_i} + \sum_{j \neq i}\left(\frac{f'_i}{\varepsilon_i}\frac{\partial K_i}{\partial t_j} - \theta_i \bar{K}\frac{\partial \rho}{\partial t_j}\right)\frac{\partial t_j}{\partial z_i},$$

$$i = 1, \ldots, n, \tag{8.2}$$

must hold, where all functions are evaluated at ζ'. A Z-equilibrium and a T-equilibrium can only coincide if (8.1) and (8.2) hold simultaneously. There seems to be no a priori reason why this should be the case, in general.

3. The special case of identical jurisdictions

T- and Z-equilibria are most easily compared when all localities have identical preferences and technologies, so that attention can be directed to symmetric equilibria such that $K_i = K_j$, $\varepsilon_i = \varepsilon_j = \varepsilon$, $t_j = t_j$, $z_i = z_j$, etc.[7] In this

[6]To relate $\partial \log K_i/\partial \log t_i$ to ε_i, note that $\partial \log K_i/\partial \log t_i = /\partial \log K_i/\partial \log(\rho + t_i)][\partial \log(\rho + t_i)/\partial \log t_i] = [t_i/(\rho + t_i)]\varepsilon_i$.

[7]This is not to claim that symmetric Nash equilibria are the only possible Nash equilibria.

case, (4) simplifies to

$$\frac{\partial K_i}{\partial t_i}=\frac{\varepsilon K_i}{f'(K_i)}\frac{n-1}{n}, \tag{9.1}$$

$$\frac{\partial K_j}{\partial t_i}=-\frac{\varepsilon K_i}{f'(K_i)}\frac{1}{n}, \quad j\neq i. \tag{9.2}$$

Let us now impose the simplifying assumption that $\theta_i=0$, all i. (Thus, imagine that there are capital owners not elsewhere present in the model who absorb all capital income.) Then (8.1) and (8.2) reduce to

$$MRS_i=\frac{(n-1)/n}{1+\partial\log K_i/\partial\log t_i} \tag{10.1}$$

$$MRS_i=\left(K_i\frac{n-1}{n}\right)\frac{\partial t_i}{\partial z_i}-\sum_{j\neq i}\left(K_j\frac{1}{n}\right)\frac{\partial t_j}{\partial z_i}. \tag{10.2}$$

To compare these expressions, it is necessary to solve for $\partial\tau/\partial z_i$, from (1). First, we define (for any i,j):

$$D=\left(K_i+t_i\frac{\partial K_i}{\partial t_i}\right)\left(K_j+t_j\frac{\partial K_j}{\partial t_j}+(n-2)t_j\frac{\partial K_j}{\partial t_i}\right)-(n-1)t_it_j\frac{\partial K_i}{\partial t_j}\frac{\partial K_j}{\partial t_i}. \tag{11}$$

Substituting from (9) and using symmetry,

$$\begin{aligned}D&=K^2\left(1+\frac{n-1}{n}\frac{t\varepsilon}{f'}\right)\left(1+\frac{1}{n}\frac{t\varepsilon}{f'}\right)-K^2\frac{n-1}{n^2}\left(\frac{t\varepsilon}{f'}\right)^2\\&=K^2\left(1+\frac{t\varepsilon}{f'}\right).\end{aligned} \tag{12}$$

Depending on the values of the critical parameters, D could be positive, negative, or zero. However, if the proportional tax rate on capital is sufficiently small, it will be the case that $|t\varepsilon/f'|<1$. This would be true, for example, if $t/f'<1/2$ and $|\varepsilon|<2$, both of which conditions are empirically quite reasonable.[8] The condition that $D>0$ is essentially one that rules out perverse relationships between local tax rates and local tax revenue.

[8]As discussed in Wildasin (forthcoming), it is plausible, in the context of U.S. property taxation, to assume $|\varepsilon|\leqq 1.5$ and $t/f'\leqq 0.4$.

Therefore, for the remainder of the analysis, let us assume that, at any T- or Z-equilibria,

$$D>0. \tag{13}$$

One can now solve (1) implicitly for τ in terms of ζ. It is easily checked that $\tau(\zeta)$ must satisfy

$$\frac{\partial t_i}{\partial z_i}=D^{-1}\left(K_j+t_j\frac{\partial K_j}{\partial t_j}+(n-2)t_j\frac{\partial K_j}{\partial t_i}\right), \tag{14.1}$$

$$\frac{\partial t_j}{\partial z_i}=-D^{-1}t_j\frac{\partial K_j}{\partial t_i}. \tag{14.2}$$

Substituting from (9) into (14) one obtains:

$$\frac{\partial t_i}{\partial z_i}=D^{-1}K\left(1+\frac{1}{n}\frac{t\varepsilon}{f'}\right)>0, \tag{15.1}$$

$$\frac{\partial t_j}{\partial z_i}=D^{-1}K\left(\frac{1}{n}\frac{t\varepsilon}{f'}\right)<0, \tag{15.2}$$

where subscripts have been dropped because of symmetry and where the inequalities follow from (13).

It is now possible to compare (10.1) and (10.2). Multiplying (10.1) by D/D, where D is given in (12), one obtains:

$$MRS_i=K^2\frac{n-1}{n}\left(1+\frac{t\varepsilon}{f'}\right)A, \tag{16}$$

where $A=D^{-1}(1+\partial\log K_i/\partial\log t_i)^{-1}$. Similarly, multiplying the top and bottom of (10.2) by $(1+\partial\log K_i/\partial\log t_i)$, and substituting from (9) and (15), we have

$$MRS_i=K^2\frac{n-1}{n}\left(1+\frac{n-1}{n}\frac{t\varepsilon}{f'}\right)A. \tag{17}$$

Obviously (16) and (17) cannot hold simultaneously, that is, T- and Z-equilibria do not coincide. In particular, the first-order conditions (16) and (17) differ by the term $(1/n)(t\varepsilon/f')$. Evaluated at a T-equilibrium τ^*, the right-hand side of (17) exceeds the left. Intuitively speaking, this means that the equilibrium value of the marginal rate of substitution is lower and the equilibrium tax rate is higher in a T-equilibrium than in a Z-equilibrium, suggesting that T-equilibria result in higher levels of public good provision than Z-equilibria.

This is an intuitively appealing conclusion if one takes (15) into account. These expressions show that an increase in the level of expenditure in i not only requires an increase in the own-tax rate t_i, it also induces a *reduction* in the tax rate of other localities. The reason for this is that an increase in t_i drives capital out of i and into other localities, enabling them to achieve any specified levels of public expenditure at lower tax rates. Thus, under the Z-equilibrium assumptions, an increase in one jurisdiction's tax rate meets a more 'aggressive' response from other jurisdictions – i.e. they cut their taxes – than is true under the T-equilibrium assumptions.[9] When tax rates are held fixed, as in a T-equililibrium, an increase in the tax rate in jurisdiction i still drives away capital, and increases the tax bases of other jurisdictions. However, in this case, they respond by simply letting public expenditure levels rise. Since the expenditure on local public goods does not affect the location of capital, this response of other jurisdictions to an increase in t_i does not itself exacerbate the loss of capital from i. Thus, in a T-equilibrium each jurisdiction sees itself as having a somewhat less mobile or elastic tax base than it would in the Z-equilibrium setting. The result one would expect under these conditions is that taxes and public expenditure should be lower in a Z-equilibrium than in a T-equilibrium, and this is what the analysis suggests.[10]

It should be noted, however, that this comparison of the two types of equilibria in terms of the equilibrium levels of tax rates and spending is only rigorously justifiable under a further restriction on the model. So far, in comparing (16) and (17), we have been in effect comparing cost–benefit *rules* for public expenditure. As is well known [e.g. Atkinson and Stern (1974)], a comparison of cost–benefit rules in two situations may suggest that public expenditure *levels* differ in a direction opposite to what in fact occurs at the allocations where the rules actually are satisfied. Essentially this arises because of general equilibrium effects which comparisons of cost–benefit rules, which are partial equilibrium in nature, do not take into proper account. In the present analysis, the possibility of such effects arises because the elasticity of demand for capital, which appears in both (16) and (17), may

[9]It is thus possible to think of Z-equilibria as equilibria in which tax rates are chosen subject to non-zero – in fact, negative – conjectural variations. The notion of Z-equilibrium can thus provide a rationale for assuming non-zero conjectural variations in tax rates. I owe this observation to Henry Tulkens. The other side of the same coin is that there are *positive* conjectural variations in expenditure levels in T-equilibria.

[10]If local public goods yield benefits to capital owners, then the increase in the z_j's that results from an increase in t_i in a T-equilibrium might give the T-equilibrium a character closer to that of a Z-equilibrium. This is easily seen in the case where public expenditures operate in a manner equivalent to a per-unit subsidy to capital, since then the distinction between cutting t_j (as in a Z-equilibrium) and increasing z_j (as in a T-equilibrium) disappears.

take on quite different values in T- and Z-equilibria. In addition, income effects may disturb the quantity comparisons. If we abstract from these problems, a quantity comparison may be made rigorously:

Proposition 1. Let all localities have identical technologies with a constant elasticity of demand for capital ε. *Let all localities have identical preferences with utility functions linear in private good consumption. Let* (t^*, z^*) *and* (t', z') *denote, respectively, the values of* (t, z) *in symmetric T- and Z-equilibria. These equilibria are unique and* $t^* > t'$, $z^* > z'$.

Proof. See the appendix.

Finally, we note that the difference between T- and Z-equilibria disappear as one approaches the 'perfectly competitive' extreme of a large number of localities. The term $(n-1)/n$ in (17) approaches 1 as $n \to \infty$. Thus, in models which treat n as very large, no distinction need be made between the two types of equilibria.[11] The intuition behind this result is simply that the impact of jurisdiction i on any other jurisdiction j's tax rate, in the Z-equilibrium case, becomes negligible – because i has a negligible share of the capital stock in the economy. Thus, whether or not other jurisdictions allow their tax rates to adjust to changes in t_i becomes increasingly irrelevant from the viewpoint of i's own behavior: if other jurisdictions do adjust their taxes, the adjustment will be so small that this effect is practically insignificant.

4. Conclusion

The analysis has shown that two superficially equivalent concepts of Nash equilibrium in fiscal competition are in fact different. The differences between them become small as perfect competition is approached. This has been shown in the simplified setting of a model with absentee owners of capital and with identical jurisdictions. However, the non-equivalence of Nash equilibria in tax rates and public expenditure levels must surely obtain in more complicated models if it occurs in simple ones.

For small-number analyses, then, it is in general necessary to specify the strategic instruments carefully and to note that the results may be sensitive to the specification. In the present case, roughly speaking, taking the expenditure levels of other localities as given implies a 'more competitive' fiscal environment than is true if the tax rates of other localities are taken as given.

[11]It is easily seen that, as $n \to \infty$, the T- and Z-equilibria converge to the perfectly competitive limiting case where individual localities take the net return on capital ρ as exogenously fixed. This can be verified by noting, for example, that, as $n \to \infty$, (10.1) reduces to the first-order condition for optimal policy in the purely competitive case.

In more general models of fiscal competition, there will be more than just two possible policy instruments that jurisdictions may control. For example, instead of assuming that each locality has only one uniform tax rate, there might be two or more tax rates applied to different categories of property, income, etc. The analysis here indicates that the precise specification of which instruments are held fixed and which ones are allowed to vary will be an important determinant of the predicted Nash equilibrium in a small-number model of fiscal competition.

Appendix

Proof of Proposition 1. Under the given assumptions, note that $MRS_i = \phi(z_i)$ for some function ϕ such that $\phi' < 0$. By symmetry, the equilibrium value of K_i must be $\bar{K}/n$ for all i in any T- and Z-equilibria, hence $f_i'(K_i) = f_i'(\bar{K}/n) = f'$ can be treated as a constant. Using (9), (16) and (17) can be expressed as

$$\gamma(t,z) \equiv \phi(z) - \frac{n-1}{n}\left(1 + t\frac{n-1}{n}\frac{\varepsilon}{f'}\right)^{-1} = 0, \tag{A.1}$$

$$\psi(t,z) \equiv \phi(z) - \frac{n-1}{n}\left(1 + t\frac{\varepsilon}{f'}\right)^{-1} = 0. \tag{A.2}$$

Now substitute $z = t\bar{K}/n$ into (A.1) and (A.2), using the government budget constraint. We can thus think of $\gamma(t,z) = \gamma(t, t\bar{K}/n)$ and $\psi(t, t\bar{K}/n)$ as functions of t alone. Both are monotonically decreasing in t. Therefore, each of the equations (A.1) and (A.2) have unique roots such that $\gamma(t^*, t^*\bar{K}/n) = 0$ and $\psi(t', t'\bar{K}/n) = 0$. These roots are the unique symmetric T- and Z-equilibria. Moreover, for any given t,

$$\gamma\left(t, t\frac{\bar{K}}{n}\right) > \psi\left(t, t\frac{\bar{K}}{n}\right). \tag{A.3}$$

Hence, $\gamma(t^*, t^*\bar{K}/n) = \psi(t', t'\bar{K}/n) = 0$ implies $t^* > t'$, and hence $z^* = t^*\bar{K}/n > t'\bar{K}/n = z'$. This completes the proof.

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