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LOCATIONAL EFFICIENCY IN A FEDERAL SYSTEM

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A number of recent studies have concluded that differing local government tax and expenditure packages necessarily create incentives for households to locate in a non-optimal fashion. This paper shows, on the contrary, that the locational equilibrium *may* be optimal. For example, if migration produces no congestion costs, then as long as localities tax the locationally-fixed commodity, land, the equilibrium will be optimal. In fact, there are only two reasons why non-optimality may result: local taxes may be distortionary (by taxing the mobile rather than immobile factor), or there may be non-internalized externalities.

1. Introduction

Consider an economy partitioned by a number of localities. These localities, let us suppose, provide public goods and services to their residents, and levy taxes to finance the concomitant costs. Suppose further that households are mobile. An optimal allocation of resources in this system must have the characteristic that there is no alternative feasible bundle of private and public goods, and no alternative distribution of households (and their labor) across localities, such that some household is better off and no household worse off in the alternative situation. This is a sufficient condition for optimality.

A *necessary* condition for optimality is the following: for arbitrarily *given* levels of public good provision in all communities, there can be no alternative feasible bundle of private goods, and no alternative distribution of households across localities, such that someone is made better off and no one worse off. The present paper is concerned with this *necessary* condition for optimality, particularly with reference to the location of households. Thus, if an allocation of resources satisfies this necessary condition for optimality, the economy will be said to have achieved *locational efficiency*. Note that the question of locational efficiency is distinct from the question of whether public goods are provided efficiently to the households residing in each community.

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A number of recent studies, including Buchanan and Goetz (1972) and Flatters, Henderson, and Mieszkowski (1974), seem essentially to have concluded that differing local government tax expenditure packages necessarily create incentives for mobile households to locate in a non-optimal fashion. The intuitive rationale for this conclusion, which seems to be widely accepted, is that the migrant household produces 'fiscal externalities' for the existing residents of a community by reducing their tax burdens and/or by increasing the congestion of public facilities; because these social benefits and costs do not enter the household's private decision calculus, inefficiency must be the result.

In this paper I wish to suggest an alternative and, I think, a more natural approach to the analysis of this question. By doing so I shall be able to prove the following very traditional-sounding propositions:

Proposition 1. In the absence of congestion costs, the locational equilibrium will be optimal if local governments tax immobile factors.

Proposition 2. If migration does produce congestion costs, optimality can be achieved if localities levy the appropriate Pigovian corrective taxes, making up additional revenues from a tax on an immobile factor.

One can rationalize these results as follows. When migration produces no *real* externalities in the form of congestion, the locational equilibrium will be optimal if local taxes are not distortionary. Because households cannot avoid a tax on a fixed factor by moving from one community to another, their locational decisions are undistorted and hence efficient. When migration does produce real externalities, in the form of congestion of public facilities, corrective taxes are called for to internalize the externality, restoring optimality. By implicitly assuming that local governments tax only the mobile factor, previous studies have erroneously concluded that efficiency, in general, is unattainable. Further, Proposition 2 has not been stated previously.[1]

On course, it might be argued that this paper, with its emphasis on locational efficiency, misses an essential problem in not allowing for variability in local public good provision and may not be directly comparable with the earlier studies. Certainly the well-known Tiebout (1956) paper, which was inspired by Samuelson's (1954) public goods paper and which in turn motivated Buchanan and Goetz, was concerned precisely with the

[1]The relevant literature goes back to early contributions by Buchanan (1950, 1952). An important recent work is that of Buchanan and Wagner (1970). [See Feldstein (1970) for comments.] Also Miller and Tabb (1973) present a model of inefficient household migration which is very similar to the Buchanan (1950), Buchanan–Wagner, and Buchanan–Goetz analyses. For further citations, see the first section of the Buchanan–Wagner paper. It may be noted that Negishi (1972) establishes Proposition 1 in his model, although he does not draw attention to it and does not relate it to the literature cited here.

implications of household mobility for efficiency in local public expenditure. Thus, in the model of Flatters, et al. (1974), the Samuelsonian condition for public good provision ($\sum MRS = MRT$) is always met, making the level of public good in a sense endogenous to the model. Yet in the appendix to this paper, I show why the assumption that public goods levels are arbitrarily fixed is actually more general than the Flatters et al., assumption. The point in a nutshell is simply that the levels of public good provision at which the Samuelsonian condition is met are *among* those that might be arbitrarily specified. And in the concluding section of the main text, I suggest that the above-cited literature has made an important contribution to the locational efficiency question, but has not really addressed the basic problem stirred up by Tiebout.

The plan of the paper is to present, in the main text, a brief, informal, and intuitive presentation of the key ideas. In the mathematical appendix, the results are developed rigorously

2. Characterization of an optimum with pure public goods

To begin with let us set out the basic framework. It is assumed for convenience that there are only two communities or regions, 1 and 2, between which identical, utility-maximizing households are free to migrate at zero cost. There are two homogeneous factors of production, labor and land. Each household is assumed to possess one unit of labor which is intrinsically tied to it (and is therefore mobile). On the other hand, land is locationally fixed, and it is assumed that the quantity $\bar{S}_i$ available in each community is fixed. There is a constant returns to scale production function for each region $F^i(n_i, \bar{S}_i)$ relating the inputs of labor (n_i = number of residents in region i) and land to the homogeneous output. This output is either consumed, with x_i the consumption per head in region i, or used as the sole input Z_i into public good supply for the region. The level $\bar{Q}_i$ of public good provision in each community is assumed fixed. Also, provision of the public good in one community is assumed not to benefit residents of the other, i.e., there are no interjurisdictional spillovers. Finally, it is assumed for simplicity that there is no congestion in public good consumption, i.e., the public goods are *purely* public. This latter assumption is relaxed below.

Now define a household's willingness to pay for public good consumption in region i, $B_i(Q_i)$, as the maximum amount of the private good the household would be willing to give up in order to consume $\bar{Q}_i$ units of the public good rather than zero (or any other reference amount).[2] The

[2] This willingness to pay measure can be rigorously justified if each household's utility function is of the form

$$u_i(x_i, Q_i) = x_i + g_i(Q_i), \qquad g_i(Q^0) = g^0,$$

where x_i is the consumption of a household residing in region i and where Q^0 is some reference aggregate willingness to pay for $\bar{Q}_i$ is then the sum over all households in region i of the B_i's. Next define the total value of private good consumption as the number of units of the private good consumed in the aggregate by all of the households in the economy. Then one can define an efficient allocation of resources (including the allocation of households to regions) as an allocation such that the total value of consumption — that is, the total value of private good consumption plus aggregate willingness to pay for public good consumption in the two regions – is maximized. More formally, if $C^i(\bar{Q}_i)$ is the cost function for the public good in region i, the welfare maximand is [3]

$$W=\sum_{i=1}^{2}\{F^i(n_i,\bar{S}_i)-C^i(\bar{Q}_i)+n_iB_i(\bar{Q}_i)\} \tag{1}$$

since total private good consumption equals total output less the amount used for public good provision.

Given this measure, the problem of characterizing an optimal allocation of households between regions is quite simple. Holding constant the quantity of public goods supplied in each region, the obvious necessary condition for efficiency is that, for each household, the total value of consumption across all households be unchanged or decreased if that household is shifted from its current residence to the other community. That is, if it is possible to increase the total value of consumption when one household is relocated, then the original situation cannot be optimal.

How then does the total value of consumption change as a household is relocated? There are two aspects of the relocation that are relevant. First, the total value of private good consumption will change if the marginal product of the household's labor differs between the two regions. Remembering that the Q_i's and hence the total costs of public good provision are fixed, it is clear that private good consumption will increase (decrease, remain unchanged) if the marginal product of labor is higher (lower, the same) in the

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level of public good consumption. To see this, consider a reference consumption bundle (x_i^0, Q^0). Then the maximum amount the household would be willing to pay to consume Q_i units of the public good, starting from the reference situation, is

$$B_i(Q_i)=g_i(Q_i)-g^0$$

since its utility, when consuming Q_i but giving up $B_i(Q_i)$, is

$$x_i^0-B_i(Q_i)+g_i(Q_i)=x_i^0+g^0=u_i(x_i^0,Q^0).$$

[3]Given the utility functions of $n \cdot 2$, (1) can be written as

$$W=\sum_{i=1}^{2}n_i(x_i+g_i[Q_i])-\sum_{i=1}^{2}n_ig^0=\sum_{i=1}^{2}n_iu_i(x_i,Q_i)-K$$

so that W is an unweighted sum of individual utilities (less a constant).

new community: as a household moves from community 1 to community 2, the total value of private good consumption will change by $F_n^2 - F_n^1$ (where $F_n^i = \partial F^i / \partial n_i$). In a world with no public sector, private good consumption would have to be maximized at the optimum and efficiency would require that $F_n^1 = F_n^2$. When there is a public sector, however, one must also look at the effects of migration on public good consumption. In the absence of congestion, it is obvious that the public good consumption of only one household, the migrant, need be considered, for the consumption of all other households will remain unchanged as the migrant relocates. If we assume that labor is paid its marginal product, the necessary condition for optimal allocation of households, assuming positive population levels for both communities, is

$$w_1 + B_1 = w_2 + B_2, \tag{2}$$

where w is the wage in region i. If the equality did not hold in (2), then the total value of consumption could be increased by the appropriate relocation of households. This relocation should continue until the wage rate rises (falls) in the community of departure (arrival) sufficiently for equality to be achieved.

3. Optimality properties of equilibrium

Given that condition (2) characterizes the optimal location pattern, the question remains, will decentralized utility-maximizing households achieve this optimum? I shall approach this question by deriving the condition for equilibrium location; this condition will then be compared with (2).

It is necessary to note first that the self-interested individual will move to the community in which the value of *his* private good consumption plus public good consumption is maximized. The former is equal to his total income less taxes. In general, of course, households have claims to both wage and non-wage income. If the household resides in community i, it will be paid a gross wage equal to its marginal product, w_i. Further, if $\bar{s}_i$ is the household's endowment of land in community i, then its gross income from land rents will be $\sum_{i=1}^{2} r_i \bar{s}_i$. Note that this income is *independent* of the household's location, as long as it is possible to own property in a community without residing there. I will assume that a community can tax both the labor and the land within its boundaries. Let τ_w^i and τ_r^i be the tax rates on wages and rents, respectively, in community i. Then (remembering that each household possesses one unit of labor) the total after-tax income of a household, should it reside in community i, is

$$y_i = (1 - \tau_w^i) w_i + \sum_{j=1}^{2} (1 - \tau_r^j) r_j \bar{s}_j. \tag{3}$$

A household will of course consume all of its after-tax income. In equilibrium, households will be indifferent between residing in community 1 and 2, which means that

$$y_1 + B_1 = y_2 + B_2. \tag{4}$$

Thus, by (3), the condition

$$(1-\tau_w^1)w_1 + B_1 = (1-\tau_w^2)w_2 + B_2 \tag{5}$$

must be satisfied in equilibrium.

Now let us compare the equilibrium condition (5) with the optimality condition (2). There are three cases to consider.

Case 1. Taxation of the mobile factor. Clearly when τ_w^i is positive, conditions (2) and (5) do not coincide unless $\tau_w^1 = \tau_w^2$. There is no reason to expect this to occur, and so in general the equilibrium will not be optimal when labor is taxed.

This is essentially the conclusion reached in previous studies. Indeed, Flatters et al. (1974) quite correctly show that equal taxes per worker across communities is the special condition which ensures optimality.

Case 2. Benefit taxation. An argument which dates to Musgrave's 1961 contribution to the NBER *Public Finances* volume is that when localities levy taxes according to the benefit principle, the locational equilibrium will be optimal.

Since this result contradicts the conclusion reached in the more recent studies, it is worthwhile to examine it briefly.

Let $\partial B_i/\partial Q_i$ be the marginal benefit to a typical household of the public good provided in community i. If we let T_i be the benefit tax paid by a household residing in community i, then (assuming the public good is optimally supplied and producible at constant marginal cost, so that the local budget is balanced)

$$T_i = \bar{Q}_i \frac{\partial B_i}{\partial Q_i}. \tag{6}$$

Since households pay no tax on wage or rental income in this model, the expression for after-tax income becomes

$$y_i = w_i + \sum_{j=1}^{2} r_j \bar{s}_j - T_i$$

instead of (3), and the equilibrium condition (4) is explicitly written as

$$w_1 - T_1 + B_1 = w_2 - T_2 + B_2. \tag{7}$$

Now, if the individual demand curves for local public goods are unit elastic[4] and identical across regions, the benefit tax per household given in (6) will be a constant that is the same for both regions. Condition (7) then reduces to (2) and locational efficiency is attained. Such circumstances would presumably be fortuitous, however, and as long as benefit taxation entails $T_1 \neq T_2$, inefficiency must result. Of course, since non-resident households do not benefit from the public goods provided in a community, and therefore pay no taxes there, a benefit tax is in effect a tax on a household's presence in a community. Since households are mobile, their locational decisions may be distorted by benefit taxes just as they are distorted by a tax on labor income. Thus the non-optimality of the locational equilibrium under benefit taxation is not surprising.

Case 3. Taxation of land. Now let us turn to the case where localities tax the fixed factor. Because $\tau^i_w = 0$ in this situation, it is obvious from (3) and (5) that the equilibrium condition coincides with the optimality condition (2).[5] Intuitively, this follows simply because the household's land tax burden does not depend on its location, so that the tax does not distort its locational decision. This is, of course, the result Proposition 1 mentioned earlier.

When we compare the conclusions reached in each of these three cases, the source of the divergence between the present analysis and that of previous writers is clear: whereas they have implicitly assumed that households or their labor must be the source of local tax revenues, I have explicitly allowed for the possibility that localities may tax the fixed commodity. And whether one prefers to think of dead-weight welfare losses as measured by the 'welfare triangle' or, more rigorously, in terms of tax-induced deviations from the marginal conditions for optimality, the conclusions are equally unsurprising. As the literature on optimal taxation would suggest, a tax on elastically-supplied commodities leads to inefficiency, while a tax on a commodity in inelastic supply does not.

4. The case of impure public goods

The treatment of this case can be given fairly briefly since it amounts to a technically minor variation on the above analysis. The essence of congestion

[4]A condition which Flatters, et al. (1974, pp. 105–106) identify as sufficient for an optimum in their model, but which they do not relate to the Musgrave argument.

[5]This result is entirely consistent with the analysis of Flatters, et al. Their optimality condition (4) requires that $\tau^1_w = \tau^2_w$, which is clearly satisfied if $\tau^i_w = 0$ for both i.

costs is that, while all households in the community consume the same quantity of the good, the presence of an additional member in the consuming group reduces the amount of the good available for all. Thus all residents of a city use the same road system, but a reduction in population would in some sense leave more road services to be consumed by the remaining residents. Congestion is thus reflected in the public good cost function, which may now be written

$$C^i(Q_i, n_i),$$

where $C^i_n = \partial C^i/\partial n_i > 0$. To maintain a given level of public good provision requires greater expenditures, the greater the population. I assume in addition that C^i is convex, so that $C^i_{nn} \geqq 0$.

Now let us consider the condition for optimal location with congestion costs. A household entering a community increases the value of output by its marginal product, it enjoys the consumption of the local public good, and, in addition, it congests the public facilities necessitating an increase in public expenditures. Thus the optimality condition generalizes from (2) to

$$w_1 + B_1 - C^1_n = w_2 + B_2 - C^2_n. \tag{8}$$

Of course the migrant will ignore the congestion that his entry imposes, since the level of total expenditures will vary only minutely with a unit change in population. Thus the condition for equilibrium location is the same as for the pure public good case, i.e., (5).

To see whether or not the locational equilibrium is optimal, we need only compares eqs. (5) and (8). Clearly the equilibrium will typically be inefficient if localities rely solely on land taxation, for in this event $\tau^i_w = 0$ and (5) and (8) do not coincide. Only in the seemingly fortuitous circumstance in which the marginal (congestion) cost of additional population is the same in each community will the equilibrium be optimal.

On the other hand, a comparison of (5) and (8) indicates how an optimum can be achieved by appropriate tax policy. Each community imposes a tax of $\tau^i_w = C^i_n/w_i$ on labor, using land taxation to make up any additional needed revenue. That is, to internalize the real externality imposed by migrants and achieve efficiency, one need only follow the familiar principle of levying a classical Pigovian corrective tax. This is the result stated above as Proposition 2.

5. Conclusions

The discussion above points to two important empirical magnitudes — the mobility of the taxed factor and the size of the congestion costs imposed by migrants – which determine whether or not a locational equilibrium will be optimal. In this regard, recall that a number of studies[6] have provided evidence that local property taxes in the U.S. are capitalized to a significant extent. Because a tax on a fully mobile factor could never be capitalized, this evidence suggests that local taxes may not be highly distortionary. Of course *some* mobility of the tax base is necessary for efficiency if migration produces congestion costs; thus the empirical question of the efficiency or inefficiency of the U.S. federal system is a difficult one to answer. But certainly no evidence has been offered to support the presumption that the system is inefficient.

Thus, certainly on theoretical grounds and potentially on empirical grounds we have reason to doubt the conclusion of Buchanan and Goetz (1972, p. 38) that

> 'local governmental units simply do not, and cannot, behave in the manner that efficiency criteria would dictate. The organization and operation of a fiscal sharing group on [an efficiency] basis violates the central notion of *free migration*, the notion upon which the models of the Tiebout adjustment process are initially founded .... The fiscal discrimination between old residents and in-migrants or new residents that would be required for efficiency violates the central meaning of resource mobility.'

The fallacy here is that household mobility is distinct from tax base mobility. In the absence of congestion costs, only immobility of the tax base is necessary to secure efficiency.

Of course one would be surprised to find that any actual taxing arrangement succeeds in perfectly internalizing migration-induced externalities while producing no distortionary effects. But the demonstration that local tax systems are not ideally efficient is not a demonstration of the need for central government intervention — for example, in the form of inter-jurisdictional equalizing grants, as suggested by numerous writers. For such intervention is liable to introduce its own distortions and costs, and these must be weighted against the defects of the existing system. Thus difficult empirical questions must be answered before the need for any corrective policy can be established.

I claimed at the outset that while the literature in this area has contributed to our understanding of the locational efficiency question, it has not shed much light on the issue raised by Tiebout (1956). I should qualify this

[6]See Oates (1969), and the studies reviewed by Gustely (1976), for example. This area of research is unsettled, of course, since many things determine property values besides taxes. The effects of public expenditures on property values must be allowed for, as emphasized by Oates. Also the relative scarcity of communities providing certain levels of public good provision [see Edel and Sclar (1974)], or imposing zoning requirements of a particular type [Hamilton (1976)], will affect property values. These points simply reinforce the conclusion that empirical testing for efficiency or inefficiency will be difficult.

claim by remarking that the Tiebout paper is as subject to variable interpretion as the U.S. Constitution is said to be. Like the Constitution this may be one of its strengths: being incompletely worked out and largely speculative, different scholars have found in it hints for interesting research in several related, but distinct, areas. Thus one could probably make the case that Tiebout was principally concerned with locational efficiency. It seems more plausible to me, however, to suggest that what Tiebout 'really' tried to establish is that household mobility facilitates *revelation of preferences* for local public goods. Tiebout, repeatedly citing Samuelson's fundamental paper on public goods, was concerned to show the possibility of a market analogue for the provision of such goods that would be immune from the strategic free-riding stressed by Samuelson. It is only too evident that Tiebout did not succeed in this task, which is not surprising given the apparent difficulty of doing so, and it is just recently — beginning, say, with a paper by McGuire (1974) — that models have been constructed in which household locational choice bears importantly on the achievement of the conditions for efficient public expenditure.

It is obvious that locational efficiency and preference revelation are distinct issues, and that determination of public goods levels are crucial to the latter but not to the former. Nonetheless, the two questions are not unrelated. In particular, when locational efficiency is not achieved because of non-optimal taxes, the Samuelson conditions characterizing the optimal level of public good provision have to be modified. [See Diamond–Mirrlees (1971), Atkinson–Stern (1974), for example.] Thus, the locational efficiency problem must be fully understood and seen in its proper light before we can hope to get a satisfactory theory of local public expenditure determination via preference revelation in the manner suggested by Tiebout.

Appendix

The treatment given above, though correct in its essentials, is not as rigorous as might be desired. In this appendix I shall present a more formal demonstration of Propositions 1 and 2. To begin with, in Section A.1, some necessary conditions for an optimal allocation of resources are set out, and the results in the text on the non-optimality of wage or head taxation, except as a corrective for congestion, are established. It is also shown that, with a land tax coupled with a wage tax serving as a congestion charge, the necessary conditions for an optimum will be met. In section A.2, it is shown that the land tax-congestion charge system does in fact lead to an optimum.

A.1. Necessary conditions for an optimum

The model in this appendix is basically the same as that presented in the text, except that allowance is made for several regions rather than just two.

There are $\bar{n}$ identical households, each of whom possesses one unit of homogeneous labor service which can be employed only in the region in which the household resides. A household has preferences defined over private and public good consumption that can be represented by a once-differentiable utility function $u(x_i, Q_i)$, where x_i and Q_i are, respectively, the private and public good consumption of a household residing in region i, $i = 1, \ldots, R$. Let n_i denote the number of households residing in region i. When the arguments of the function $u(x_i, Q_i)$ are understood, we can write simply u^i; let subscripts denote partial derivatives and assume $u^i_x > 0$ always.

Private good production in region i, Y_i, is determined by a differentiable linear homogeneous production function $F^i(l_i, S_i)$, where l_i and S_i are the amounts of labor and land employed, respectively. Both partial derivatives are assumed strictly positive.

To provide Q_i units of the public good to n_i households residing in region i requires $Z_i = C^i(Q_i, n_i)$ units of the private good as input. When I assume that the levels of public good provision are fixed, I will let $\bar{Q}_i$ denote the fixed level for community i. Assume that $C^i(\cdot)$ is twice continuously differentiable, with $C^i_n \geqq 0$, $C^i_Q > 0$, and convex, so that $C^i_{nn} \geqq 0$, $C^i_{QQ} \geqq 0$, and $C^i_{nn}C^i_{QQ} - (C^i_{nQ})^2 \geqq 0$.

Now consider the problem of characterizing an optimal allocation of resources. Initially, take the Q_i's as fixed at $\bar{Q}_i$. If, ultimately, we are concerned with evaluating alternative tax schemes in a world in which there are no restrictions on household migration, the feasible set should include only those allocations that generate no interregional utility differentials. Other allocations, in which households in different regions have different utility levels, will be unsustainable because utility differentials will cause migratory flows. Thus, for an allocation of resources $a = ((x_i), (l_i), (n_i), (S_i), (Y_i), (Z_i))$ to qualify as an optimum – i.e., as an allocation that maximizes the common utility level of all households – not only must the technological and commodity-balance constraints be satisfied,

$$\sum_i n_i x_i + \sum_i Z_i = \sum_i Y_i, \tag{A.1a}$$

$$l_i = n_i \qquad \text{for all } i, \tag{A.1b}$$

$$S_i = \bar{S}_i \qquad \text{for all } i, \tag{A.1c}$$

$$\sum_i n_i = \bar{n}, \tag{A.1d}$$

$$Z_i = C^i(\bar{Q}_i, n_i) \text{ for all } i, \tag{A.1e}$$

$$Y_i = F^i(l_i, S_i) \text{ for all } i, \tag{A.1f}$$

in addition, we must have

$$u(x_i, \bar{Q}_i) = u(x_1, \bar{Q}_1), \qquad i = 2, \ldots, R \tag{A.1g}$$

to ensure equal utility levels everywhere. If we wish to allow the Q_i's to be determined, the constraints (A.1e) and (A.1g) should be written with Q_i's rather than $\bar{Q}_i$'s; otherwise, there is no change.

Assuming an optimum a^* exists, there is the question of characterizing it. Consider first the case with Q_i's fixed arbitrarily. Necessary conditions can be found by maximizing the Lagrangian

$$\begin{aligned} L = {} & u(x_1, \bar{Q}_1) + \eta \sum_i (Y_i - n_i x_i - Z_i) + \sum_i \lambda_i (n_i - l_i) \\ & + \sum_i \sigma_i (\bar{S}_i - S_i) + v\left(\bar{n} - \sum_i n_i\right) \\ & + \sum_i \gamma_i [Z_i - C^i(\bar{Q}_i, n_i)] + \sum_i \phi_i (F^i(l_i, S_i) - Y_i) \\ & + \sum_{i \neq 1} \xi_i [u(x_i, \bar{Q}_i) - u(x_1, \bar{Q}_1)], \end{aligned}$$

where Greek letters denote multipliers. At an interior optimum,

$$\frac{\partial L}{\partial x_1} = u_x^1\left(1 - \sum_{i \neq 1} \xi_i\right) - \eta n_1 = 0, \tag{A.2a}$$

$$\frac{\partial L}{\partial x_i} = \xi_i u_x^i - \eta n_i = 0, \qquad i = 2, \ldots, n, \tag{A.2b}$$

$$\frac{\partial L}{\partial l_i} = \phi_i F_l^i - \lambda_i = 0 \qquad \text{for all } i, \tag{A.2c}$$

$$\frac{\partial L}{\partial n_i} = \lambda_i - \eta x_i - v - \gamma_i C_n^i = 0 \qquad \text{for all } i, \tag{A.2d}$$

$$\frac{\partial L}{\partial S_i} = \phi_i F_s^i - \sigma_i = 0 \qquad \text{for all } i, \tag{A.2e}$$

$$\frac{\partial L}{\partial Y_i} = \eta - \phi_i = 0 \qquad \text{for all } i. \tag{A.2f}$$

$$\frac{\partial L}{\partial Z_i} = \gamma_i - \eta = 0 \qquad \text{for all } i \tag{A.2g}$$

Using (A.2c), (A.2f), and (A.2g) to eliminate λ_i and γ_i from (A.2d), the latter becomes

$$F_l^i - x_i - C_n^i - \frac{v}{\eta} = 0 \qquad \text{for all } i,$$

implying

$$F_l^i - x_i - C_n^i = F_l^j - x_j - C_n^j \qquad \text{for all } i, j. \tag{A.3}$$

If one allowed for the Q_i's to be determined optimally, as in Flatters, et al. (1974), one would still obtain the first-order conditions (A.2a) – (A.2g). Now of course, the functions u^i and C^i would be evaluated with Q_i at its optimal value. In addition, one would have the first-order conditions

$$\frac{\partial L}{\partial Q_1} = u_Q^1\left(1 - \sum_{i \neq 1} \xi_i\right) - \gamma_1 C_Q^1 = 0, \tag{A.2h}$$

$$\frac{\partial L}{\partial Q_i} = \xi_i u_Q^i - \gamma_i C_Q^i = 0, \qquad i = 2, \ldots, n. \tag{A.2i}$$

Using (A.2a), (A.2b), and (A.2g) yields the well-known conditions

$$n_i \frac{u_Q^i}{u_x^i} = C_Q^i \qquad \text{for all } i. \tag{A.4}$$

The important point to note is that (A.3), the locational efficiency condition, continues to hold just as before. Indeed, by allowing for the possibility that (A.4) may not obtain, the case where each Q_i is arbitrarily fixed at $\bar{Q}_i$ is actually more general then the case where the Q_i's are chosen optimally.

Equilibrium. Conditions (A.3) are necessarily satisfied at an optimum. The question that now needs to be addressed is whether these conditions will in fact be met by price-guided decentralized decisionmakers. To answer this question the institutional structure of the economy must be specified.

Suppose first that all households have identical endowments of land, where $\bar{s}_i$ is the endowment of land in region i of each household. It is assumed that all land in each region is initially held by some household.

Assume that private good production is competitively organized, that firms maximize profits, and that the aggregate production technology in each region is described by the functions F^i introduced above. In equilibrium, factors are paid the value of their marginal products; taking the private good as numéraire, this implies that $w_i = F_l^i$ and $r_i = F_s^i$, where w_i is the price of l_i

and r_i is the price per unit of S_i. Let τ_w^i and τ_r^i be the rates of wage and rent taxation in region i, respectively. It is assumed that wage taxes can be levied only on residents, while taxes on rents can be levied on any owner of land, regardless of location.

If a household resides in region i, it will be subject to the budget constraint

$$x_i = (1-\tau_w^i)w_i + \sum_j (1-\tau_r^j)r_j\bar{s}_j.$$

Substituting for w_i and r_j, the following relationship must obtain:

$$F_l^i - x_i - \tau_w^i F_l^i = -\sum_j (1-\tau_r^j)r_j\bar{s}_j.$$

This implies

$$F_l^i - x_i - \tau_w^i F_l^i = F_l^j - x_j - \tau_w^j F_l^j \qquad \text{for all } i,j. \tag{A.5}$$

Note that all of the above conditions hold, regardless of whether the level of Q_i, in equilibrium, is fixed optimally, or just assumes some arbitrary $\bar{Q}_i$ level.

Comparison. It now remains to compare (A.3), which is necessarily met at an optimum, and (A.5), which is necessarily met at an equilibrium.

Case 1. Wage taxation without congestion. Assume $\tau_r^i = 0$, all i. In the absence of congestion, $C_n^i = 0$. Comparing (A.3) and (A.5), it is evident that the former will necessarily be violated in equilibrium unless $\tau_w^i F_l^i = \tau_w^j F_l^j$. *Unless wage taxes per worker are the same in all regions, the equilibrium location pattern will definitely be non-optimal.* This is the conclusion reached in earlier studies.

Case 2. Taxation of land rent without congestion. Assume $\tau_w^i = 0$, all i. Since $C_n^i = 0$, it is clear that (A.3) will be satisfied in any equilibrium. *In the absence of congestion, the necessary conditions for an optimum will be satisfied in equilibrium if land rent alone is taxed.*

Case 3. Corrective wage taxation with congestion. Assume $\tau_w^i = C_n^i/w_i$, all i, and set up $\tau_r^i = (Z_i - \tau_w^i w_i n_i)/r_i\bar{S}_i$, all i. (i.e., set wage taxes so that each worker pays a tax equal to marginal congestion cost, using land rent taxes to make up any needed additional revenue — or to disburse any surplus.) A comparison of (A.3) and (A.5) under these assumptions reveals that *corrective wage taxes coupled with land rent taxation lead, in equilibrium, to the satisfaction of the necessary conditions for an optimum.* Note that this result really encompasses the preceding one (Case 2) as a special case in which $C_n^i = 0$.

A.2. Sufficient conditions for an optimum

The above discussion showed that, in the absence of congestion, taxes on mobile households generally entail inefficiency: the necessary conditions for optimality *must* be violated (except in highly special circumstances) and an optimum *cannot* be achieved. It was also shown that in an equilibrium in which taxes are levied on immobile land, with appropriate corrective wage taxes, the necessary conditions for an optimum are satisfied, so that an equilibrium *may* be optimal. Here this latter proposition is strengthened: an equilibrium of the type specified *is* optimal.

Arbitrarily fixed Q_i's. To demonstrate this, it is necessary to describe an equilibrium more completely. I consider first the case where one has specified levels of public good provision $Q_i = \bar{Q}_i$, all i. Then an equilibrium is an allocation vector $a = ((x_i), (l_i), (n_i), (S_i), (Y_i), (Z_i))$, a price vector $p = ((w_i), (r_i))$, and a tax vector $t = ((\tau_w^i), (\tau_r^i))$ satisfying conditions (A.1) and

$$x_i = (1 - \tau_w^i) w_i + \sum_j (1 - \tau_r^j) r_j \bar{s}_j \quad \text{for all } i, \tag{A.6a}$$

$$Z_i = \tau_w^i w_i n_i + \tau_r^i r_i \bar{S}_i \quad \text{for all } i, \tag{A.6b}$$

$$(l_i, S_i, Y_i) \max Y_i - w_i l_i - r_i S_i \quad \text{for all } i. \tag{A.6c}$$

The first condition is the budget constraint for a household residing in i, the second is the balanced-budget constraint for each government, and the third requires that firms be at a maximum-profit position in equilibrium.

Now suppose there exists an equilibrium $\tilde{a}, \tilde{p}, \tilde{t}$ such that

$$\tilde{\tau}_w^i = \frac{C_n^i(\bar{Q}_i, \tilde{n}_i)}{\tilde{w}_i} \quad \text{for all } i, \tag{A.7}$$

and suppose that this equilibrium is not an optimum. Then there exists a vector $\hat{a}$ satisfying (A.1) such that the common utility level of all households is greater at the $\hat{a}$ allocation; i.e.,

$$u(\hat{x}_i, \bar{Q}_i) > u(\tilde{x}_i, \bar{Q}_i) \quad \text{for all } i. \tag{A.8}$$

I shall deduce a contradiction, implying that the hypothesis — that the equilibrium is not an optimum — is false.

Note first, from (A.8), that

$$\hat{x}_i - \tilde{x}_i > 0 \quad \text{for all } i. \tag{A.9}$$

Further, taking a Taylor expansion of $C^i(\cdot)$ about $(\bar{Q}_i, \tilde{n}_i)$, we have

$$\hat{Z}_i - \tilde{Z}_i \geqq (\hat{n}_i - \tilde{n}_i) C_n^i(\bar{Q}_i, \tilde{n}_i). \tag{A.10}$$

Now define

$$\tilde{\pi}_i = \tilde{Y}_i - \tilde{w}_i \tilde{l}_i - \tilde{r}_i \tilde{S}_i,$$

and

$$\hat{\pi}_i = \hat{Y}_i - \tilde{w}_i \hat{l}_i - \tilde{r}_i \hat{S}_i,$$

$\tilde{\pi}_i$ is the aggregate profit earned in region i in equilibrium, while $\hat{\pi}_i$ is the profit earned when prices are at their equilibrium levels and inputs and output are at the alternative levels $(\hat{l}_i, \hat{S}_i, \hat{Y}_i)$. Using the fact that $\tilde{a}$ and $\hat{a}$ satisfy (A.1b) and (A.1c), we have

$$\sum_i (\hat{\pi}_i - \tilde{\pi}_i) = \sum_i (\hat{Y}_i - \tilde{Y}_i) - \sum_i \tilde{w}_i (\hat{n}_i - \tilde{n}_i). \tag{A.11}$$

One can substitute from (A.1a) for the first term in the right-hand side of (A.11); one can also substitute from (A.6a) and (A.7) for $\tilde{w}_i$. Using the fact that

$$\left(\sum_j (1 - \tilde{\tau}_r^j) \tilde{r}_j \bar{s}_j\right) \sum_i (\hat{n}_i - \tilde{n}_i) = 0,$$

(A.10) becomes

$$\sum_i (\hat{\pi}_i - \tilde{\pi}_i) = \sum_i (\hat{n}_i \hat{x}_i - \tilde{n}_i \tilde{x}_i) + \sum_i (\hat{Z}_i - \tilde{X}_i)$$
$$- \sum_i \tilde{x}_i (\hat{n}_i - \tilde{n}_i) - \sum_i C_n^i (\hat{n}_i - \tilde{n}_i).$$

One now applies (A.9) and (A.10) to conclude that

$$\sum_i (\hat{\pi}_i - \tilde{\pi}_i) > 0,$$

which means that, for a least one i, the inout-output vector $(\hat{l}, \hat{S}_i, \hat{Y}_i)$ —

which is feasible, since $\hat{a}$ satisfies (A.1f) — is more profitable, at equilibrium prices $(\tilde{w}_i, \tilde{r}_i)$ than the equilibrium input output vector $(\tilde{l}_i, \tilde{S}_i, \tilde{Y}_i)$. This of course contradicts the equilibrium condition (A.6c). The equilibrium must therefore be an optimum.

Optimality of equilibrium with variable Q_i's. I now consider the case where an equilibrium has been attained such that the Samuelsonian conditions (A.4) are satisfied. I will exploit the assumption that individual utility functions are strictly quasi-concave and twice continuously differentiable. Indifference curves are therefore smooth and convex – i.e., non-pathological. An equilibrium is therefore described by a vector $Q=(Q_i)$ of levels of public goods, and vectors a, p, and t with the same meaning as before, satisfying (A.1), (A.4), and (A.6).

Now suppose there exists an equilibrium $\tilde{Q}$, $\tilde{a}$, $\tilde{p}$, $\tilde{t}$ such that

$$\tilde{\tau}^i_w = \frac{C^i_n(\tilde{Q}_i, \tilde{n}_i)}{\tilde{w}_i} \quad \text{for all } i, \tag{A.12}$$

and suppose that this equilibrium is not an optimum (in the extended range of economies admitting variable Q_i's). Then there exists a pair $\hat{Q}$, $\hat{a}$ satisfying (A.1) such that

$$u(\hat{x}_i, \hat{Q}_i) > u(\tilde{x}_i, \tilde{Q}_i) \quad \text{for all } i. \tag{A.13}$$

As before, I shall derive a contradiction, establishing the claim that the equilibrium is optimal.

Note first, by (A.13) and the strict quasi-concavity of u, that

$$\tilde{u}^i_x(\hat{x}_i - \tilde{x}_i) + \tilde{u}^i_Q(\hat{Q}_i - \tilde{Q}_i) > 0$$

(where a tilde above u^i or C^i will denote evaluation of the function at the equilibrium values of its arguments). See, e.g., Arrow and Enthoven (1961) for this well-known property. Given satisfaction of (A.4) at the initial equilibrium, it follows that

$$\tilde{n}_i(\hat{x}_i - \tilde{x}_i) + \tilde{C}^i_Q(\hat{Q}_i - \tilde{Q}_i) > 0. \tag{A.14}$$

From convexity of C^i, one has

$$\hat{Z}_i - \tilde{Z}_i \geqq \tilde{C}^i_n(\hat{n}_i - \tilde{n}_i) + \tilde{C}^i_Q(\hat{Q}_i - \tilde{Q}_i). \tag{A.15}$$

One can define $\tilde{\pi}_i$ and $\hat{\pi}_i$ just as before, and (A.11) continues to hold. Again use (A.1a) in (A.11), and use (A.6a) and (A.12) to eliminate $\tilde{w}_i$ and $\tilde{\tau}^i_w$.

It follows, using (A.15) to establish the weak inequality and (A.14) to establish the strong, that

$$\begin{aligned}\sum_i (\hat{\pi}_i - \tilde{\pi}_i) &= \sum_i (\hat{n}_i\hat{x}_i + \hat{Z}_i - \tilde{n}_i\tilde{x}_i - \tilde{Z}_i) \\ &\quad - \sum_i (\tilde{x}_i + \tilde{C}^i_n)(\hat{n}_i - \tilde{n}_i) \\ &\geqq \sum_i \hat{n}_i(\hat{x}_i - \tilde{x}_i) + \tilde{C}^i_Q(\hat{Q}_i - \tilde{Q}_i) \\ &> 0,\end{aligned}$$

which implies, as before, that (A.6c) is violated in the initial $\tilde{Q}$, $\tilde{a}$, $\tilde{p}$, $\tilde{t}$ situation. But this situation was assumed to be an equilibrium; hence a contradiction has been derived. The equilibrium must therefore be an optimum.

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