

## Local Public Goods, Property Values, and Local Public Choice

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This paper examines the relationship between the benefits and costs of local public good provision and local property values within the context of the Koopmans-Beckmann-Gale location-assignment model. Property values do not in general measure accurately the marginal net benefits of local public goods; special conditions sufficient for property values to measure or bound the marginal net benefits are stated, however. In addition, it is shown that under certain circumstances, households vote for property-value-enhancing levels of local expenditures. Under these conditions, a political equilibrium produces a Lindahl solution to the local public good problem.

### I. INTRODUCTION

This paper presents a formal analysis of the relationship between property (or land) values and local government spending and taxation, with two basic goals in mind. The first is to see whether and under what conditions one can make inferences about the optimality of (changes in) local public good provision by observing (changes in) property values. The second is to define a concept of Tiebout equilibrium, to see how it might be achieved, and to examine its implications for property value determination. Recent studies<sup>2</sup> have attempted to test "the Tiebout hypothesis" by estimating equations which relate property values to local fiscal variables, but unfortunately the theoretical underpinnings

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<sup>2</sup> See Gustely (1976) and references there to work by Oates, Heimberg, Orr, Pollakowski, and others; McDougall (1976), Meadows (1976), Smith and Deyak (1976), King (1977), Rosen and Fullerton (1977), Edel and Selar (1974), and Hamilton (1975a; 1976a, b). Other tests of the Tiebout hypothesis, not relying on estimates of the effects of local fiscal variables on property prices, can be devised; see Hamilton (1975b), Hamilton, Mills, and Puryear (1975), and Epple, Zelenitz and Visscher (1977).

of these studies have not been carefully elucidated, leading to debate over the interpretation of their results. Here I shall develop a model of Tiebout equilibrium which yields unambiguous predictions that are amenable to testing by cross-section regressions of the usual type.

Much of the analysis below parallels work done by economists studying the connection between air pollution and property values.<sup>3</sup> Some writers in this area, particularly Lind (1973, 1975), have employed the linear assignment model (Koopmans and Beckmann 1957, Gale 1960) in an effort to establish a relationship between property values and the benefits from pollution abatement. Others (Freeman 1974, Pines and Weiss 1976, Polinsky and Shavell 1976) have exploited the fact that the level of air pollution may vary continuously over space, permitting households, in their locational decisions, to make marginal adjustments in their consumption of clean air. It has been shown that cross-section data on property values can be used to estimate the marginal benefits of pollution abatement under much more general conditions than is true in the case of the assignment model (which, as usually formulated, does not explicitly allow for smooth locational adjustments by households).

In this paper I shall adapt and blend these two approaches in order to investigate the issues raised initially. Section II introduces a location-assignment model in which multiple local governments provide public services which are financed by taxes on property. This model is used in Section III to demonstrate several propositions akin to those established by Lind. In Section IV I drop the assumption that the number of communities is arbitrary but finite, hypothesizing instead that there are sufficiently many communities, with sufficiently diverse levels of public good provision, that households face a continuous spectrum of choice with regard to public good consumption. With some additional simplifying assumptions, it is possible to establish considerably sharper results on the link between property values and the efficiency of public good supply. Section IV also closes the model with a public choice theory of local public expenditure. It is demonstrated that individual maximizing behavior, in both market and non-market contexts, produces a Lindahl-Tiebout equilibrium such that no household is dissatisfied with the level of public services in the community in which it resides, this level is optimal, and the conditions for a Lindahl solution are met. Moreover, one can test for the existence of such an equilibrium with straightforward econometric techniques. Section V offers some concluding comments.

<sup>3</sup> For references and an overview of the discussion, see Polinsky and Shavell (1975) and Polinsky and Rubinfeld (1977).

II. THE  $K$ -COMMUNITY MODEL

Here and in the next section it will be assumed that there is a fixed (finite) number  $K$  of communities.<sup>4</sup> Each community  $k = 1, \dots, K$  contains a fixed number  $J_k$  of dwelling units,  $j = 1, \dots, J_k$ . The  $j$ th unit of community  $k$  will be designated by the double subscript  $jk$ . There are  $H$  households ( $h = 1, \dots, H$ ), and it is assumed that the total number of dwelling units in the system just equals the number of households:  $H = \sum_k J_k$ . (For simplicity I shall abstract from the existence of firms.) As usual, it is assumed that each household must reside in at most one unit, and that at most one household can reside in any given unit. An assignment of households is a matching up of households and dwelling units subject to these conditions. More formally, let  $\xi_{hjk} = 1$  if and only if household  $h$  is assigned to dwelling unit  $jk$ , and  $\xi_{hjk} = 0$  otherwise. Then an *assignment of households* is described by a set of  $H^2$  variables  $\xi_{hjk}$  satisfying the restrictions

$$\begin{aligned} \xi_{hjk} &= 0 \quad \text{or} \quad 1 && \text{for all } h \text{ and } jk; && (1) \\ \sum_h \xi_{hjk} &\leq 1 && \text{for all } jk; && \sum_{j,k} \xi_{hjk} \leq 1 \quad \text{for all } h. && (2) \end{aligned}$$

An  $H \times H$  matrix  $X = (\xi_{hjk})$  is called an *assignment matrix* if its entries satisfy (1) and (2).

The residents of each community  $k$  consume a single public good, the quantity of which is denoted  $\gamma_k$ ; we let  $g = (\gamma_1, \dots, \gamma_K)$ . I shall assume that *only* the residents of a community benefit from the good provided there; i.e., there are no interjurisdictional spillovers. It will also be assumed that the cost of public good provision,  $c_k(\gamma_k)$ , is financed by taxes on local property. The budget constraint for the  $k$ th local government is

$$\sum_{j=1}^{J_k} \tau_{jk} = c_k, \tag{3}$$

where  $\tau_{jk}$ , the tax on unit  $jk$ , is paid by the occupant of the unit. Thus, denoting the price of unit  $jk$  by  $\pi_{jk}$ , the full cost of residing there is  $\pi_{jk} + \tau_{jk}$ . We let  $t_k = (\tau_{1k}, \dots, \tau_{J_k k})$  and  $t = (t_1, \dots, t_K)$ . Let aggregate property values in community  $k$ , and in the whole economy, be defined by

$$P_k = \sum_{j=1}^{J_k} \pi_{jk} \quad \text{and} \quad P = \sum_k P_k,$$

respectively; also, define the  $H$ -vector  $p = (\pi_{jk})$ .

<sup>4</sup> I have borrowed extensively from Gale's extremely lucid discussion at several points below.

Let  $\alpha_{hjk}(\gamma_k)$  denote the maximum amount that household  $h$  is willing to pay to reside in unit  $jk$ ; as the notation indicates, this magnitude depends on  $\gamma_k$  as well as the other attributes of the unit. By residing in unit  $jk$ , the household enjoys a net benefit or surplus of  $\alpha_{hjk} - \pi_{jk} - \tau_{jk}$ . The maximum surplus that the household can obtain given  $p$ ,  $t$ , and  $g$  is given by  $\mu_h = \max_{jk} (\alpha_{hjk} - \pi_{jk} - \tau_{jk})$ . It is assumed that households choose to reside in units yielding maximum surplus.<sup>5</sup> Thus, let every community announce its  $\gamma_k$  and  $t_k$ ; with these given, we say that the entry  $\alpha_{hjk}$  in the  $H \times H$  value matrix  $A = (\alpha_{hjk})$  is *admissible with respect to  $p$*  if and only if  $\alpha_{hjk} = \mu_h + \pi_{jk} + \tau_{jk}$ . Then we make the

*Definition:* Given  $g$  and  $t$ , a non-negative  $H$ -vector,  $p^* = (\pi^*_{jk})$  is an *equilibrium price vector* if there exists an assignment matrix  $X^*$  such that

$$(E.i) \quad \mu^*_h \equiv \max_{jk} (\alpha_{hjk} - \pi^*_{jk} - \tau_{jk}) \geq 0 \quad \text{for all } h;$$

$$(E.ii) \quad \xi^*_{hjk} = 1 \Rightarrow \alpha_{hjk} \text{ is admissible with respect to } p^* \text{ for all } h \text{ and } jk;$$

$$(E.iii) \quad \sum_h \sum_{j=1}^{J_k} \tau_{jk} \xi^*_{hjk} = c_k \quad \text{for all } k.$$

According to (E.i) and (E.ii), it must be possible to assign households to units so that each household is in a surplus-maximizing location. Condition (E.iii) ensures that the budget constraint (3) is satisfied: no dwelling unit that has a positive tax levied on it can be left vacant, so that no taxes are left unpaid. It should be kept in mind that all of the endogenous variables,  $p^*$ ,  $\mu_h^*$ , and  $X^*$ , depend on the parameters  $g$  and  $t$ , and  $A$  of course depends on  $g$ .

Some brief discussion of the question of existence of equilibrium may be in order. We use the following assumption.

$$(A): \text{ For all } h, j, \text{ and } k, \alpha_{hjk} \geq \tau_{jk}.$$

In words, for every household, the value of each of the dwelling units in the system is at least as great as the taxes on it.

Existence of equilibrium is usually demonstrated using the optimal assignment model. For an arbitrary non-negative value matrix  $A$ , the optimal assignment problem is to choose an assignment matrix  $X$  to

<sup>5</sup> Surplus-maximization might appear to be rather an *ad hoc* assumption. It can be shown, however, that in a world with only two goods, property and a composite private good ("all other goods"), utility-maximization and surplus-maximization are equivalent decision criteria if the utility function is linear in the composite good. See Wildasin (1976, 114-118).

maximize

$$V(A, X) = \sum_{h,j,k} \alpha_{hjk} \xi_{hjk}.$$

$V(A, X)$  is called the value of the assignment  $X$ . For our purposes, it is useful to formulate an optimal assignment problem using the *net* value matrix  $B = (\beta_{hjk}) \stackrel{\equiv}{=} (\alpha_{hjk} - \tau_{jk})$ :

$$\max_{(X)} V(B, X) \quad \text{subject to (1) and (2)}. \tag{4}$$

Suppose that at the solution to (4), some of the constraints (2) are satisfied as strict inequalities, implying that some households are unassigned. By assumption (A) above, it is clear that  $B$  has all entries non-negative, so that the value of the assignment cannot be decreased by locating any unassigned households. So we may as well assume that constraints (2) do indeed hold as equalities. Then, since every unit is occupied, condition (E.iii) is satisfied. It then follows that

$$V(B, X) = V(A, X) - \sum_k c_k.$$

Now one can simply follow the proof of Koopmans and Beckmann (1957): The optimal  $X^*$  and  $2H$  dual variables  $(\mu^*_1, \dots, \mu^*_H)$  and  $p^*$  satisfy the complementary slackness conditions

$$\xi^*_{hjk} = 1 \Rightarrow \mu^*_h + \pi^*_{jk} = \beta_{hjk} \equiv \alpha_{hjk} - \tau_{jk}, \tag{5.1}$$

$$\mu^*_h > \beta_{hjk} - \pi^*_{jk} \equiv \alpha_{hjk} - \pi^*_{jk} - \tau_{jk} \Rightarrow \xi_{hjk} = 0. \tag{5.2}$$

It is easy to verify that the  $X^*$ ,  $\mu^*_h$ , and  $p^*$  thus determined meet the remaining equilibrium conditions (E.i) and (E.ii).

One might note that the role of assumption (A) is to ensure that the matrix  $B$  is non-negative, which in turn ensures that  $\mu^*_h$  and  $\pi^*_{jk}$  are non-negative. While (A) is sufficient, we see from (5.1) that it is only necessary that  $\alpha_{hjk} - \tau_{jk} \geq 0$  whenever  $\xi^*_{hjk} = 1$ ; that is, only  $H$  of the  $H^2$  conditions in (A) are necessary.

Finally, I stress that several important problems are ignored by this model (and the variant presented in Section IV). First, the cost of public good provision is assumed not to vary with local population; i.e., either local public goods are purely public (complete non-rivalness), or else costs depend on the number of locations in the community (which is fixed and hence irrelevant) but not population. Second, the tax base in each community—a certain number of locations with invariant (private) characteristics—is perfectly fixed. Once these two assumptions are relaxed, we must face the possibility of inefficient locational choices due to congestion externalities and distortionary taxes. I believe that it is useful to

suppress these difficulties in order to simplify the discussion of the special concerns of this paper, but the restrictiveness of the assumptions should be kept in mind.

### III. PROPERTY VALUES AND THE BENEFITS OF LOCAL PUBLIC EXPENDITURES

In this section, the model developed above is used to study the relationship between property values and the net benefits of local public good provision. If there is a change in the level of public good provision in some community, changes in the value matrix  $A$  and the equilibrium assignment matrix  $X$  will result. Letting superscripts 0 and 1 refer to the initial and terminal situations, respectively, the net benefit of the change can be defined as  $V(B^1, X^1) - V(B^0, X^0)$ . This differs from the gross benefit measure,  $V(A^1, X^1) - V(A^0, X^0)$ , in that allowance is made for changes in the costs of public good provision. (In previous studies, where these costs are neglected, there is of course no distinction between gross and net benefits.)

For an economy with no local governments and no property taxes, Lind has established a number of interesting propositions on the relationship between property values and the benefits of public projects, measured by  $V(A, X)$ , which may be summarized briefly. Let  $Q$  denote the set of parcels of property that are enhanced by a given project. (A parcel is enhanced if at least one household's valuation of it is increased by the project.) The following propositions have been demonstrated:

- (I) If no household anywhere in the system enjoys a surplus either before or after the project, then the change in aggregate property values is an exact measure of the benefits of the project. (Lind, 1973 (4.1).)
- (II) If the above zero-surplus condition is met for all locations in  $Q$  (i.e., not necessarily for all parcels throughout the system), then the change in property values in  $Q$  is an upper bound on the benefits of the project. (Lind, 1973 (4.7).)
- (III) If the above zero-surplus condition is met for all locations in  $Q$ , and if, in addition, no household originally residing in  $Q$  relocates as a result of the project, then the change in property values in  $Q$  is an exact measure of the project's benefits. (Lind, 1973 (4.4).)
- (IV) If  $Q$  is a set of *identical* parcels, and if  $Z$  is a set of parcels which are identical to those in  $Q$  after the project is undertaken (i.e., the parcels in  $Z$  are already improved), then the difference between the initial value of property in  $Z$  and the initial or pre-improvement value of the parcels in  $Q$  is an upper bound on the benefits of the project. (Lind, 1975.)

In this section I prove analogues to I-IV. I shall establish four propositions obtained from those above by substitution of the words "net benefits" for "benefits" and "increase in public good provision" for "project," where  $Q$  of course is to be interpreted as a particular community and  $Z$  is an otherwise identical community with the higher level of public good provision and taxes. In other words, I shall show that, to the extent that property values reflect the benefits of local public goods, they also reflect the costs of provision of these goods.

Let us consider an increase in the level of public good provision in community  $s$  from  $\gamma_s^0$  to  $\gamma_s^1$ , accompanied by a change in taxes from  $t_s^0$  to  $t_s^1$  which leaves the local budget balanced. This results in a change in a typical household's valuation of dwelling unit  $js$  from  $\alpha_{hjs}(\gamma_s^0)$  to  $\alpha_{hjs}(\gamma_s^1)$ . Equilibrium prices, the equilibrium assignment, and the value of the equilibrium assignment will, in general, be changed. For notational convenience, let us use superscripts to identify variables which depend on  $\gamma_s$ ; for example, denote  $\alpha_{hjs}(\gamma_s^1)$  as  $\alpha_{hjs}^1$ . Then the aggregate net benefit of this change is

$$NSB = \sum_{k \neq s} \sum_{j=1}^{J_k} \sum_h (\alpha_{hjk} \xi_{hjk}^1 - \alpha_{hjk} \xi_{hjk}^0) + \sum_{j=1}^{J_s} \sum_h (\alpha_{hjs}^1 \xi_{hjs}^1 - \alpha_{hjs}^0 \xi_{hjs}^0) - (c_s^1 - c_s^0), \quad (6)$$

where of course the assignment matrix  $X^i$ ,  $i = 0, 1$ , describes the equilibrium assignment of households when  $\gamma_s = \gamma_s^i$ .

First let it be assumed that the zero-surplus condition is maintained throughout the system, so that for  $i = 0, 1$ ,  $\xi_{hjk}^i = 1$  implies  $\alpha_{hjk} = \pi_{jk}^i + \tau_{jk}$  for  $k \neq s$  and  $\alpha_{hjk}^i = \pi_{jk}^i + \tau_{jk}^i$  for  $k = s$ . Then clearly (6) can be rewritten, using (E.iii), as

$$(I') \quad NSB = \sum_k (P_k^1 - P_k^0) = P^1 - P^0.$$

This establishes our analogue to (I).

Next suppose that the zero-surplus condition holds for locations in community  $s$ , but not necessarily throughout the entire system. A direct derivation of a result parallel to (II) would necessitate working through a number of distinct cases involving households which move about within community  $s$ , households which move from community  $s$  to another community, etc. Lind has already performed this laborious exercise for a system without property taxation, however, and it is possible to carry over his analysis to the current problem. In (4.4) and (4.7) of Lind (1973), it is established that for numbers  $p = (\pi_{jk})$  satisfy-

ing conditions (5), the following must hold :

$$\begin{aligned}
 P_s^1 - P_s^0 = \sum_{j=1}^{J_s} \pi_{js}^1 - \pi_{js}^0 &\geq \sum_{k \neq s} \sum_{j=1}^{J_k} \sum_h (\beta_{hjk}^1 \xi_{hjk}^1 - \beta_{hjk}^0 \xi_{hjk}^0) \\
 &+ \sum_{j=1}^{J_s} \sum_h (\beta_{hjs}^1 \xi_{hjs}^1 - \beta_{hjs}^0 \xi_{hjs}^0). \quad (7)
 \end{aligned}$$

But writing out the right-hand side of (7) using (E.iii), we have, by (6), that

$$(II') \quad NSB \leq P_s^1 - P_s^0.$$

This is the analogue to (II).

To develop a counterpart to (III) for the present model, let us suppose that no household in community  $s$  relocates as a result of the change in  $\gamma_s$ . (This is not imposed as a constraint, but is simply assumed to occur given the particular data at hand.) Although the location of households outside of community  $s$  may change, the total value of the assignment of these households cannot change—that is, the first expression in (6) must be zero.<sup>6</sup> But then, since households in  $s$  do not relocate and enjoy no surplus, (6) reduces to

$$\begin{aligned}
 (III') \quad NSB &= \sum_{j=1}^{J_s} \sum_h [(\pi_{js}^1 + \tau_{js}^1) - (\pi_{js}^0 + \tau_{js}^0)] \xi_{hjs} - (c_s^1 - c_s^0) \\
 &= P_s^1 - P_s^0
 \end{aligned}$$

which is (III), appropriately modified.

Finally, to complete the extension of (I)–(IV) to the local public good case, suppose that all units in community  $s$  are identical, and pay identical taxes, both before and after the change in public good provision (i.e.,  $\alpha_{hjs}^1 = \bar{\alpha}_{hs}^i$  for all  $h$ , and  $\tau_{js}^i = \bar{\tau}_s^i$  for all  $j = 1, \dots, J_s$ ). Suppose further that there is a community  $s'$  which is identical in every respect to community  $s$  in the terminal situation; that is,  $J_{s'} = J_s$ ,  $\gamma_{s'} = \gamma_s^1$ ,  $\alpha_{hjs'} = \alpha_{hjs}^1 = \bar{\alpha}_{hs}^1$ , and  $\tau_{js'} = \tau_{js}^1 = \bar{\tau}_s^1$ , for all  $h, j$ . This implies, of course, that all units in  $s'$  are identical to one another.

<sup>6</sup> To see this, suppose on the contrary that

$$\sum_{k \neq s} \sum_{j=1}^{J_k} \sum_h \alpha_{hjk} \xi_{hjk}^1 > \sum_{k \neq s} \sum_{j=1}^{J_k} \sum_h \alpha_{hjk} \xi_{hjk}^0$$

Then it would be possible to increase the value (either gross or net) of the initial assignment by replacing  $X^0$  with  $X^1$ : the value of the assignment of households outside of  $s$  would rise, while the value of the assignment of the residents of  $s$  would remain constant (since, by hypothesis,  $\xi_{hjs}^1 = \xi_{hjs}^0$ ). But this contradicts the fact that  $X^0$  is an equilibrium assignment.



Under these assumptions, the net value matrix  $B$  inherits all of the special features of  $A$ ; for example, it is easily seen that  $\beta_{hjs'} = \beta_{hjs}^1 = \tilde{\beta}_{hs}^1$ , all  $h$ . In short,  $B$  possesses all of the properties needed for one to apply the result of Lind (1975), according to which

$$\pi_{js'}^0 - \pi_{js}^0 \geq \sum_k (\beta_{hjs}^1 \xi_{hjs}^1 - \beta_{hjs}^0 \xi_{hjs}^0). \quad (8)$$

Summing (8) and using (6), we have

$$(IV') \quad P_{s'}^0 - P_s^0 \geq NSB,$$

the desired result.

The lesson to be learned from these exercises, of course, is that when estimating the benefits of local public expenditures which are financed by property taxes, one must allow for the fact that taxes depress property values just as the enhancement of the community from better public services raises them. Suppose, for example, that it is observed that property values remain constant when local public good provision in some community is increased and that the conditions of no relocation and zero surplus obtain. Then the correct conclusion is *not* that households are satiated with respect to the local public good (which would therefore be greatly over-supplied), but that the supply of the good is optimal. Thus, while not particularly difficult, it is obviously crucial to modify Lind's propositions to cover the case of property-tax financed local expenditures; failure to do so could lead to gross misinterpretations of empirical results.

#### IV. CONTINUOUS VERSION OF THE MODEL

In this section I wish to extend the discussion of the relationship between local spending and property values to the case where there is a "large" number of communities, so that households can vary their public good consumption at will by relocating. I shall also close the model by assuming that expenditure/tax levels in each locality are determined through a referendum open to local residents. It seems to me that the essence of the "Tiebout hypothesis" is that decentralized individual locational decisions sort households into jurisdictions that are homogeneous with respect to preferences for local public goods; homogeneity of preferences is then supposed to lead to efficient local expenditure decisions. Without going into detailed discussion of what Tiebout really meant, I believe that, consistent with the above remarks, a reasonable formulation of the Tiebout hypothesis is that household locational choice in a world with many communities leads to a Lindahl solution to

the local public good problem.<sup>7</sup> I shall show below that this kind of Tiebout equilibrium can occur, but I shall have to impose some additional assumptions.

First, it will be required that, within each community, all property is both physically and fiscally homogeneous: all dwelling units are identical to one another, and are equally taxed. (Hamilton (1975a) has emphasized the role of zoning as a device which will ensure that all dwelling units in a given community will be homogeneous.) One can then speak unambiguously of the quality of property in a community, denoted  $q$ . Second, it is assumed that public goods are homogeneous everywhere and that the public good cost function is the same for all communities. Third, it is assumed that in all communities in which the quality of property is  $q$ , the number of dwelling units is equal to a given number  $N(q)$ . Finally, it is assumed that the number and diversity of communities is sufficiently great that, for any particular quality type  $q$ , households are confronted with a continuous spectrum of choice with respect to the level of public good provision,  $g$ .

Under these assumptions, communities differ only in regard to the quality of their property and their level of public good provision. It is clear, then, that just as dwelling units within a single community must have the same price (because they are homogeneous and are equally taxed), so the prices of dwelling units in different communities with the same  $g$  and  $q$  must be identical. Thus the price of property is a function (as opposed to correspondence) defined on  $g$  and  $q$ , denoted  $p(g, q)$ . If  $c(g)$  is the cost function for public good provision, then the cost of purchasing a house of quality  $q$  in a community providing  $g$  of the public good—or, briefly, the cost of buying a house in a  $(g, q)$  community—must be  $p(g, q) + c(g)/N(q)$ , that is, the price of the unit plus tax. Since all of the units in each community are taxed equally, the tax per unit is just the total cost of public good provision, which equals total taxes when the local budget is balanced, divided by the number of units in the community.

<sup>7</sup> Space limitations preclude a full review of the literature; suffice it to say that Tiebout's seminal paper has generated several lines of inquiry into the local public good problem. A number of authors, including Edel and Sclar (1974), Hamilton (1975a, b; 1976a, b), Mills and Oates (1975a), and Epplé et al. (1977), have discussed the problem of marginal-cost pricing of local public goods. McGuire (1974) has formulated a model (without a property market, however) in which locational choice leads to the satisfaction of the Samuelsonian conditions for local spending. Sonstelie and Portney (1976) show how efficient local public good provision will be achieved by households who vote for property-value-maximizing expenditure levels, although they do not discuss the Lindahl characteristics of the equilibrium. Finally, Hamilton et al. (1975) and Wheaton (1975) have been concerned with stratification of households by income class.

Assume as before that households reside in surplus-maximizing dwelling units. If  $\alpha_h(g, q)$  describes household  $h$ 's valuation of a dwelling unit in a  $(g, q)$  community, then the household's location problem is to

$$\max_{\{g, q\}} \alpha_h(g, q) - p(g, q) - \frac{c(g)}{N(q)}.$$

The household's optimal location  $(g^*, q^*)$  can be characterized in part by the condition

$$\frac{\partial \alpha_h}{\partial g} - \frac{\partial p}{\partial g} - \frac{c'}{N(q^*)} = 0, \quad (9)$$

where  $c' = dc/dg$ .

Notice from (9) that, except when  $\partial p/\partial g = 0$ , it will not be true that  $\partial \alpha_h/\partial g = c'/N(q^*)$ . But  $\partial \alpha_h/\partial g$  is the household's marginal valuation of the public good, and  $c'/N(q^*)$  is the marginal tax-price (=the household's share in the tax base,  $1/N$ , times the marginal cost of the public good). Thus in the general case a Lindahl-Tiebout equilibrium will not occur: households do not in general locate in such a way as to equate  $\partial \alpha_h/\partial g$  and  $c'/N(q^*)$ .

Next, note that if (9) is summed over all households  $h$  residing in a particular  $(g, q)$  community,

$$N(q) \frac{\partial p}{\partial g} = \sum_h \frac{\partial \alpha_h}{\partial g} - c'. \quad (10)$$

Thus, if one regresses aggregate community property value on  $g$  and  $q$ , the  $g$  coefficient provides a correct measure of the marginal net benefit of public good provision to the existing residents of each community. This extends the conclusions of Freeman (1974), Polinsky and Shavell (1976), and Pines and Weiss (1976) for the continuous case in the same way that those of Lind for the discrete case were extended in Section III: that is, results concerning gross marginal benefits have been replaced by results concerning net marginal benefits. As before, this is because the costs of the improvement are reflected in property values, as well as the benefits. Notice that this strong result obtains without any zero-surplus assumption. Also note that, under the homogeneity assumptions maintained here, proposition (IV') already implies that the slope of the property value gradient bounds the marginal net benefits of public good provision from above; it is the continuum assumption that makes the relationship exact.

Now let us suppose that public expenditure levels in all communities are determined by popular vote. The residents of a community, after voting on the level of public good provision, are free to sell their property and relocate if they wish. In view of the large number of communities, let us suppose that they assume that their decision does not alter the  $p(g, q)$  function, that is, each community's residents act as price-takers. This being the case, it is clear that the consumption possibilities facing households are not (perceived to be) altered by the public expenditure decision made in their community, except insofar as it influences their income or wealth. Each household will therefore vote for the level of public good provision which most enhances the value of its initial property holdings.<sup>8</sup> Since property in any given community is homogeneous, all households there seek to maximize the *same* price, and hence they will be unanimous in their demands. Thus, all households in a  $(g, \bar{q})$  community will vote for  $g^*$  such that  $p(g^*, \bar{q}) = \max_{(g)} p(g, \bar{q})$ . Hence, in a full politico-economic equilibrium,  $p(g, q)$  must be a constant function for all  $q$ . The results of this political supply-adjustment are thus consistent with the supply-adjustment process sketched by Edel and Selar (1974), Hamilton (1976a), and others.

Note that the traditional marginal benefit and tax-price considerations do not determine household's demands for local public goods in this model. It is, therefore, something of a paradox to find that the politico-economic process produces an equilibrium in which marginal benefit and tax-price are equated for every individual. For when  $p(g, q)$  is constant, (9) becomes

$$\frac{\partial \alpha_h}{\partial g} = \frac{c'}{N(q^*)}. \quad (11)$$

And of course summation of (11) across all households in the community yields

$$\sum_h \frac{\partial \alpha_h}{\partial g} = c',$$

the Samuelsonian condition for optimal public good provision. Hence with free mobility of households, the conditions for a Lindahl solution are satisfied—but the public choice implications of this solution are lost

<sup>8</sup> This is utility-maximizing behavior under the conditions of n. 4 above. (See Wildasin, 1976, 128–130, and Sonstelie and Portney, 1976). Note the similarity to the "Separation Theorem" of Hirshleifer (1970, pp. 14, 63). Hamilton (forthcoming) has argued that the residents of a community will also seek to establish property-value-maximizing zoning policies.

by virtue of the very assumptions used to establish its existence. I would hesitate therefore to call this a strict Tiebout equilibrium (as defined above), although the result is certainly in the Tiebout spirit.

Observe, finally, that our theory of Tiebout equilibrium does have a testable implication. Controlling for quality (broadly defined), the  $g$  coefficient in the simple regression model mentioned above should be zero.

## V. CONCLUSION

I conclude on a cautionary note. In the model of Section III, all but one of the propositions relating property value changes and the marginal net benefits of local public goods depend on a zero-surplus assumption. As noted by Freeman (1975), it is by no means obvious that this assumption will typically be satisfied.

In the continuum model of Section IV, of course, it was possible to establish an exact relationship between property values and the net benefits of public goods without any zero-surplus assumption. Voting behavior in that model is determinate as well, and leads to a Lindahl-type solution in a way that seems consistent with the intuitive notion of a Tiebout equilibrium. (It is worth noting, however, that exclusive reliance on "voting with the feet" does not produce the Tiebout result: the model has to be augmented with a political theory of public expenditure determination.) But the continuum model relies heavily on several strong homogeneity-of-property assumptions, suggesting that the real-world applicability of the "Tiebout hypothesis" is rather limited. Certainly a demonstration of the Tiebout result under weak assumptions remains to be presented.

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