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Income Taxes and Urban Spatial Structure

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This paper examines the impact of an income tax in a monocentric city where households equilibriate their allocation of time between work, commuting, and leisure. An increase in the income tax rate lowers the implicit value of time, and hence transportation costs. "Compensated equilibrium" comparative statics analysis shows that under certain conditions, this results in a larger, more dispersed urban area, with lower land rents at the city center and less population within any given distance from the center. The welfare effect of an income tax rate change is also studied, and an expression for the marginal excess burden is derived. The income tax produces welfare losses both because it induces substitution in favor of leisure *and* in favor of travel—the latter accompanied by excessive spatial dispersion and consumption of space. The marginal excess burden depends not only on the compensated demand elasticity for leisure, but also on that for space. Finally, the problem of benefit measurement for transportation projects in this tax-distorted spatial economy is examined. Benefit measures should be deflated to adjust for the fact that further transportation improvements lead to reduction of land use intensity, exacerbating the problem of spatial resource misallocation in an already excessively dispersed urban area

I. INTRODUCTION

This paper examines the positive and normative implications of an income tax in the setting of an urban economy. The rationale for studying this problem is easily stated. Urban economics models rely on the structure of transportation costs to explain fundamental characteristics of urban spatial structure, including land rent and housing price gradients, intensity of land use, the determination of the urban/rural boundary, etc.[2] Transportation costs are generally assumed to take the form of pecuniary expenses and of foregone leisure. Empirically, the latter is a crucial component of total transportation cost, and savings of travel time bulk large, e.g., in estimates of benefits of highway investment.[3] In such empirical work the problem of travel time evaluation arises, and, while many methods and

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[2] I am referring to the well-known family of models discussed, e.g., in Muth [12], Henderson [6], Wheaton [17], Mills [9], etc.

[3] See, e. g., Mohring [11, Chap. 5] for a discussion of the issues and literature. See also Meyer, *et al.* [8], Beesley [2] and Straszheim [13], inter alia.

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results appear in the literature, there is a general presumption that the value imputed to a marginal hour of travel time must reflect the market value of time as measured by wage rates, adjusted to reflect the disamenities of labor as compared with travel.[4] The value of travel time is related to wages because a household in equilibrium allocates hours among work, leisure, and travel so as to equate the value of an hour in each use. However, it is the *after-tax* wage rate that matters for travel time valuation, since work is valued for the real purchasing power that it creates. Hence, changes in income tax rates, which change the net wage rate, should affect urban spatial structure. The analysis of this relationship is the main concern of the present paper.[5]

Given the empirical importance of travel time as a component of transportation costs, and given the size of marginal tax rates, the effect of an income tax on urban spatial structure is of great potential relevance for both positive and normative urban economics. From the viewpoint of positive analysis, intuition based on the standard "new" urban models suggests that an income tax should have an effect analogous to a reduction in transport costs: lowering the value of travel time should result in less competition for central locations, thus flattening the land rent gradient, pushing the urban/rural boundary outward, lowering housing density, increasing the equilibrium amount of travel, etc.[6] From a normative viewpoint, since the market equilibrium urban structure is known to be efficient in the absence of congestion effects or other market imperfections (see e.g., Kanemoto [7]), an income tax, by changing the equilibrium urban structure as indicated above, would be expected to result in spatial misallocation of resources. Intuitively, we would expect the urban structure to be too dispersed in the presence of an income tax.

An examination of income taxes in an urban setting is thus important for the welfare analysis of labor income taxes which has traditionally focused on the tax distortion of the labor/leisure margin. In a world where

[4] It is generally found that travel time is valued at something less than the wage rate, presumably because travel is less odious than work. It does not matter for our purposes whether travel and work are equally undesirable, though the formal model we work with assumes this to be true, for simplicity. Rather, all that is essential is that *changes* in net wages cause corresponding *changes* in the value of leisure and travel, i.e., that households equilibriate their time allocations. That is, empirically, we should expect to find higher estimated values of travel time, cet. par., in high wage cities, countries, or time periods.

[5] Two recent studies, Haurin [5] and Buttler [3], examine income taxation in an urban area. Their concerns are quite different than those of the present paper, however.

[6] These effects could, however, be offset by the income effect of the tax: higher income taxes lower utilities, reducing the demand for land at each location, shifting the land rent gradient downward, etc. In addition, there can be various cross effects: a change in the net wage can change the demand for land and housing through substitute/complement relationships, as we discuss below.

households can allocate time among labor, leisure, and travel, there should in addition be a substitution effect in favor of more travel, since its price has fallen. This travel effect, however, must be related to the increased dispersion of population resulting from the tax, since the reason more travel would be undertaken is that people live farther from the city center. We therefore seek to understand the relationship between spatial resource allocation and the household allocation of time in order to assess the full welfare effects of an income tax.

The investigation of these issues is the main topic of the paper. In Section II we set up the simplest possible model for our purposes. We exploit the standard monocentric city framework with identical households who travel, subject to no congestion effects, to a city center at which they earn exogenously fixed wages. We allow households to allocate time between work, leisure, and travel, which is our main departure from the standard model. Section II presents some preliminary analysis of the model. (To facilitate reading, most of the mathematical argument and results are presented in the Appendix.)

Section III investigates the comparative statics and welfare effects of a tax. Part A deals with positive issues. An important difference between the analysis here and that appearing in Wheaton [14] is that we center our attention on the *compensated equilibrium* effects of the income tax. We employ a closed-city approach, keeping the total urban population constant, and yet we simultaneously keep equilibrium utility constant. In the usual analysis, these two assumptions are contradictory, since one either assumes a closed city with a fixed population and endogenously determined utility, or an open city, where utility is exogenous, with population varying to equilibrate the model. Here, we suppose instead a system of compensation payments to the fixed number of urban residents which keeps their utility constant in the face of tax policy changes and the ensuing effects on equilibrium prices and location patterns. From the positive viewpoint, this approach is useful because it focuses on substitution effects, permitting more clear-cut results and thus highlighting certain channels of influence of taxes on the urban economy.[7] It is also the correct approach from the viewpoint of welfare analysis.

In part B, we compute the marginal excess burden of the tax, and find a welfare loss not only because of the distortion of the labor/leisure margin, but also because the income tax distorts spatial resource allocation and the allocation of time and resources to transportation. We identify the crucial empirical parameters that determine the size of the marginal excess burden.

[7]The theoretical results become ambiguous when income effects are included. Part A also treats the case where incremental tax revenues are a pure loss to the urban economy, explaining the nature of the ambiguities that arise.

Given that income taxes do distort resource allocation, what are the implications for the measurement of the benefits of transportation improvements in tax-distorted urban areas? This is a particular example of the general problem of project evaluation in second best economies, and in Section IV we derive a quite intuitive result: since transportation investments cause still further dispersion in a city that is already of inefficiently low density, their benefits must be adjusted downward.

Section V concludes the discussion by considering some of the limitations, possible extensions, and further implications of the analysis.

II. THE MODEL

We imagine a standard monocentric city model with identical non-landowning residents and absentee landlords. Each resident household chooses a location at distance x from the city center (which could be a point or disk) at which it consumes an all-purpose good $c(x)$, land $h(x)$, and leisure $l(x)$. Residents must travel to the city center to work, incurring a pecuniary travel expense $k(x)$ and travel time $t(x)$, with $k', t' > 0$ all x. At the center, the household works $a(x)$ hours at an exogenously fixed gross wage of w, which is its sole source of income aside from a lump-sum transfer payment L from the government, which we discuss further below.[8] All households pay a proportional tax at rate τ on their gross earnings, so that the net wage $\bar{w} = (1 - \tau)w$. Suppose that each household has a total time endowment of T. If $p(x)$ is the price of land at x, a household locating there faces the budget constraint

$$c(x) + p(x)h(x) = (1 - \tau)wa(x) - k(x) + L, \tag{1.1}$$

or, using the time constraint $T = a(x) + t(x) + l(x)$,

$$\begin{aligned} c(x) + p(x)h(x) &+ (1 - \tau)wl(x) \\ &= (1 - \tau)w(T - t(x)) - k(x) + L \underset{\text{df}}{=} I(x). \end{aligned} \tag{1.2}$$

Maximizing utility subject to (1.2) yields the indirect utility function $v(p(x), \bar{w}, I(x))$. As a condition of equilibrium, utility is equal in all locations, so

$$v(p(x), \bar{w}, I(x)) = u \qquad 0 \leq x \leq \bar{x}, \tag{2}$$

where $\bar{x}$ is the boundary of the residential area and u is the equilibrium utility level. From (2) we can solve implicitly for the equilibrium land price

[8] The assumption of a fixed gross wage is effectively valid only if production technology at the city center is linear. Relaxing this assumption should not change the qualitative results, however.

at any location:

$$p(x) = p(\overline{w}, I(x), u) \qquad 0 \leq x \leq \overline{x}, \tag{3}$$

which depends on the policy parameter τ through both $\overline{w}$ and $I(x)$, and which depends on L through $I(x)$. The derivatives of p with respect to τ, L, and u are displayed in the Appendix.

The rural/urban boundary $\overline{x}$ occurs where the urban land rent falls to the exogenously fixed agricultural rent, s,

$$p(\overline{w}, I(\overline{x}), u) = s. \tag{4}$$

From (4), $\overline{x}$ is implicitly a function of τ, L, and u. Again, the derivatives are shown in the Appendix.

Let $n(x)$ denote the density of population at x, let N denote the total population of the city, and let $\theta(x)$ denote the density of land available for consumption by households at x. We assume $\theta(x)$ continuous, and N fixed, corresponding to a closed-city approach. The equilibrium condition in the land market is

$$n(x)h(x) = \theta(x), \tag{5}$$

which may be used to eliminate $n(x)$. Note here that $h(x)$ is a *compensated* demand function, evaluated at the equilibrium utility level u. (Similarly, $l(x)$ and $a(x)$ below are compensated demand and supply functions.) Hence, $n(x)$, as given by (5), depends on $(\overline{w}, p(x), u)$ because $h(x)$ does.

Finally, the equilibrium utility level is determined, conditional on τ and L, by the condition that $N - \int n = 0$. Written out in full,

$$N - \int_0^{\overline{x}(\tau, L, u)} \frac{\theta(x)}{h(\overline{w}, p[\overline{w}, I(x), u], u)}\, dx = 0. \tag{6}$$

If we regard (6) as a function $\phi(\tau, L, u)$, we can apply the implicit function theorem to solve for u in terms of τ and L, provided that $\phi_u \neq 0$.[9] In the Appendix, it is shown that $\phi_u > 0$ provided that land is not a Giffen good, an assumption we impose hereafter.

It is also shown in the Appendix that $\phi_L < 0$ while ϕ_τ is ambiguous. This is because while τ affects the compensated demand for housing through a necessarily negative own-price effect, there is also a cross-price effect which can be zero or of either sign. The cross effect is the change in demand for land resulting from a net wage rate change (= the derivative of the demand

[9] To simplify notation, subscripts will be widely used to denote differentiation.

for leisure with respect to the price of land.) Empirical evidence on the sign and size of this effect is nonexistent, though intuition might suggest that leisure and land (or residential housing services) are complements. In any event, in order to obtain cleaner results, we shall often assume henceforth that leisure and land are Slutsky–Hicks independents, i.e.,

$$\frac{\partial h(x)}{\partial \bar{w}} = \frac{\partial l(x)}{\partial p(x)} = 0, \tag{7}$$

or, more loosely, that the cross effects are not so large that they dominate the own effects.[10] Given (7), $\phi_\tau > 0$. Given (7) we also have

$$a'(x) = -t'(x) - \frac{\partial l(x)}{\partial p(x)} p'(x) = -t'(x) < 0; \tag{8}$$

that is, households further from the city center supply less labor. In fact, (7) implies that leisure is invariant across locations, so that the total time spent in work plus travel is the same for all households (hence $a'(x) + t'(x) = 0$). This conclusion is used below and is of interest in itself. The general result, that labor supply might vary with the distance of one's residence from the CBD, has been discussed little in the literature. Yet it is a natural consequence of a specification where time is allocated across competing uses, as we have assumed here. The specific result, that leisure is invariant to location is dependent upon, and in fact is another way of stating, the assumption (7) that cross effects can be ignored. This assumption could be tested by observing $l(x)$, rather than estimating $\partial h(x)/\partial \bar{w}$ directly.

The signing of the derivatives of ϕ permits us to conclude that

$$\frac{\partial u}{\partial \tau} = \frac{-\phi_\tau}{\phi_u} < 0 \qquad \text{(assuming (7))} \tag{9.1}$$

$$\frac{\partial u}{\partial L} = \frac{-\phi_L}{\phi_u} > 0. \tag{9.2}$$

Both of these results are expected. The latter merely says that an exogenous increase in lump-sum income increases utility. The former is more im-

[10] It is quite possible to carry through the analysis of this paper in the general case where (7) is not assumed, but the theoretical results are less clear cut and the relevant mathematical expressions are more cumbersome. In an empirical application, however, one would want to take these cross effects into account. The analysis of the general case appears in the Appendix, and the needed qualifications to our results can be supplied by the reader.

portant, but also unsurprising: higher taxes reduce utility.[11] This reflects both the misallocative effects of the tax alluded to earlier and the reduction in real income from the net transfer of tax revenue to the government. In the next section we turn to the more interesting question of *excess* burden of the tax.

To complete the specification of the model, it remains, first, to observe that the gross rents accruing to landlords, in the aggregate, are

$$R = \int_0^{\bar{x}} p(x)\theta(x)\, dx + s\left(A - \int_0^{\bar{x}} \theta(x)\, dx\right) \tag{10}$$

where A is the total amount of land available for both urban and rural use. Second, we must specify the total tax revenue collected by the government. For our purposes, it is irrelevant whether or not land rents are taxed. We assume for concreteness that they are, and hence (again suppressing parameters)

$$T = \tau w \int_0^{\bar{x}} n(x)a(x)\, dx + \tau R. \tag{11}$$

These revenues may be used to finance an exogenously fixed level of public expenditure (which we can therefore ignore) and for payment of lump-sum transfers. This latter use of funds, considered at least hypothetically, will be important below.

III. COMPARATIVE STATICS AND WELFARE ANALYSIS

A. Positive Analysis: Compensated and Uncompensated Comparative Statics

We now use the system (3)–(6) to examine the effects of changes in (τ, L) on the equilibrium of the urban area. To avoid ambiguities arising from income effects, we first present *compensated* comparative statics results, in which we assume that lump-sum transfers are paid to the residents of the city such that their equilibrium utility is invariant to changes in τ. After this, we discuss the uncompensated approach. The text merely summarizes the qualitative conclusions of the analysis, leaving all details to the Appendix.

The intuitive arguments given earlier suggest that an increase in τ, by lowering the implicit cost of travel, should create a less densely developed and more dispersed urban area. We therefore examine the effect of a compensated increase in the tax rate on the location of the urban boundary, the land rent gradient, and the distribution of population as a function of

[11] It is actually possible to prove that u necessarily falls with τ, at least starting with $\tau = 0$, using an analysis of the optimum city à la Mirrlees [10].

distance from the city center. For unambiguous results, we assume that (7) holds. Taking u as exogenously fixed (at its pre-tax change equilibrium value), we use (6) to solve for L as an implicit function of τ with derivative

$$L_\tau = -\frac{\phi_\tau}{\phi_L}, \tag{12}$$

with $L_\tau > 0$ given (7).

We can now show that

$$\bar{x}_\tau|_{\bar{u}} > 0, \tag{13}$$

where $|_{\bar{u}}$ denotes evaluation of a derivative with *equilibrium* utility constant, i.e., with L varying according to (12). Furthermore

$$p_\tau(\bar{x})|_{\bar{u}} > 0 \tag{14.1}$$

$$p_\tau(0)|_{\bar{u}} < 0. \tag{14.2}$$

(14.1) merely confirms (13): the boundary can expand with τ iff the equilibrium price of land at $\bar{x}$ rises with τ. By (14.2), however, equilibrium land rents fall at the city center, showing that the income tax flattens the equilibrium price gradient.

One would expect a flatter equilibrium price gradient to be associated with lower population density in the city, that is, with a more dispersed population. This obviously does happen in that the same population inhabits an expanded urban area as τ increases. More can be said, however. Let

$$N(x) = \int_0^x \frac{\theta(\xi)}{h(\xi)}\, d\xi \tag{15}$$

be the cumulated population out to distance x from the center. Then, given (7), we can show that

$$N_\tau(x)_{\bar{u}} < 0 \qquad \text{all } x \in (0, \bar{x}); \tag{16}$$

that is, population density falls throughout the city as the income tax rises, in the sense that fewer people reside within any given distance from the center. Taken together, results (13), (14), and (16) confirm the intuition that, abstracting from income effects (by considering compensated equilibria) and from substitute/complement relationships between land and leisure (by assuming (7)), a higher income tax does indeed "expand" the city.

We now consider an uncompensated increase in τ, holding L constant and allowing u to vary according to (6). The substitution effects just discussed all continue to operate, but the total response now includes the loss of real income resulting from the tax as well. Recall, e.g., from Wheaton [14], that a reduction in income causes the bid-rent gradient of a closed city to shift downward and to steepen. We have shown in (14) that the substitution effect of a tax increase is to flatten and shift up the equilibrium price gradient. Since the total effect of an uncompensated tax increase is the sum of these, we cannot hope to say anything a priori about the net impact. It is possible, by way of confirmation of the preceding results, to show that the uncompensated change is the same as the compensated one if the income elasticity of demand for land is sufficiently close to zero. In the Appendix, we present for completeness the uncompensated derivatives of $\bar{x}$ and $p(x)$ with respect to τ, which verifies this intuition. But, unfortunately, unambiguous qualitative results are not generally available.

B. The Welfare Effects of Tax Policy

From (9.1) we already know that welfare falls as the tax rate rises. Quite aside from excess burden considerations, the extra tax revenue generated by a tax increase should make city residents worse off. These revenue gains in turn benefit the public, however, and it is traditional and useful to net out the extra revenue in order to compute an excess burden. Also, we must take into account changes in net land rents accruing to landowners. Thus, we define the marginal excess burden as the amount that would have to be paid, in the aggregate, to urban residents, taxpayers, and landowners to keep utility constant as τ increases. Formally,

$$\mathrm{MEB} = \left.\frac{dNL}{d\tau}\right|_{\bar{u}} - \left.\frac{dT}{d\tau}\right|_{\bar{u}} - \left.\frac{d(1-\tau)R}{d\tau}\right|_{\bar{u}}. \tag{17}$$

In the Appendix, we show that this can be expressed as

$$\mathrm{MEB} = \left.\frac{-\tau w\, d\left(\int_0^{\bar{x}} n(x)a(x)\,dx\right)}{d\tau}\right|_{\bar{u}} = \left.\frac{-\tau w\, dL^s}{d\tau}\right|_{\bar{u}} \tag{18}$$

where $L^s = \int_0^{\bar{x}} n(x)a(x)\,dx$ is the aggregate (compensated) labor supply in the city. This exactly parallels the standard expression for excess burden in the usual nonspatial labor supply model. τw is the tax per unit of labor, or the excess of the demand over supply price for labor, and the parenthetical expression in (18) is the change in the compensated equilibrium quantity of labor associated with an increase in τ.

For MEB to be positive, then, we require $(dL^s/d\tau)_{\bar{u}} < 0$. Since $a(x) = T - t(x) - l(x)$,

$$\left.\frac{-\tau w \, dL^s}{d\tau}\right|_{\bar{u}} = \tau w \int_0^{\bar{x}} n(x) \left.\frac{dl(x)}{d\tau}\right|_{\bar{u}} dx$$

$$- \tau w \left[\int_0^{\bar{x}} a(x) \left.\frac{dn(x)}{d\tau}\right|_{\bar{u}} dx + a(\bar{x}) n(\bar{x}) \bar{x}_\tau |_{\bar{u}} \right]. \quad (19)$$

The first term in (19) shows the effect of the tax on the leisure margin. A change in τ affects the demand for leisure directly through its effect on the net wage, resulting in an unambiguously positive substitution effect. In addition, an increase in τ changes $p(x)$ resulting in the cross effect $(\partial l/\partial p) p_\tau|_{\bar{u}} = (\partial h/\partial \bar{w}) p_\tau|_{\bar{u}}$. As we have seen, the sign of $p_\tau|_{\bar{u}}$ is likely to vary with x, and the sign of $\partial h/\partial \bar{w}$ is theoretically indeterminate in any case. If we assume (7), however, the own-substitution effect certainly dominates and the first term in (19) is then positive. Except for allowing explicitly for cross effects, this first term merely reflects the standard distortion of the labor/leisure margin discussed in the public finance literature.

The second term in (19) is of greater interest. It shows the effect on labor supply of the spatial rearrangement of households in response to the income tax. In the Appendix, this is shown to be equal to

$$\tau w \int_0^{\bar{x}} N_\tau(x) a'(x) \, dx. \quad (20)$$

Intuitively, if an increase in τ causes greater spatial dispersion of the population, labor supply should fall because households living farther from the city center tend to travel more and work less than those closer in. A sufficient condition for this intuition to be valid is that (7) hold, since then (8) and (16) imply that the integral in (20) is positive.

In summary, if (7) holds, we have

$$\text{MEB} = \tau w \left[- \int_0^{\bar{x}} w n(x) \frac{\partial l(x)}{\partial \bar{w}} dx + \int_0^{\bar{x}} N_\tau(x) a'(x) \, dx \right] > 0. \quad (21)$$

Note that this implies that the excess burden of the tax is positive even if the demand for leisure is perfectly inelastic, since the second term will still be positive. That is, the increased dispersion of the city resulting from higher income taxation is a welfare-reducing distortion in itself. The crucial behavioral parameter upon which the size of this distortion depends is the compensated price elasticity of demand for residential space. The intuitive reason for this is as follows: A given population becomes more dispersed only if households, on average, consume more space. A higher income tax reduces the value of accessibility to the city center, which tends to flatten the bid-rent curve and thus to reduce land rents. This in turn increases the amount of space demanded, to an extent that depends on the elasticity of demand for space, which then shifts up the equilibrium bid-rent curve and results in a less dense urban area. If the demand for space were perfectly inelastic, this mechanism obviously could not operate, and the income tax would then not distort the spatial allocation of resources and reduce welfare on this account.

IV. SOME IMPLICATIONS FOR TRANSPORTATION INVESTMENT CRITERIA

Suppose that the transport cost functions depend on a parameter α such that $\partial k(x, \alpha)/\partial \alpha \equiv k_\alpha(x) < 0$ and $\partial t(x, \alpha)/\partial \alpha \equiv t_\alpha(x) < 0$. Increases in α could represent the effects of highway or other transportation improvements. In the absence of income taxes or other distortions, the gross benefit of an incremental improvement of this type would be the value of transportation cost reductions enjoyed by all households. In the presence of an income tax, however, urban structure is inefficiently dispersed, and transportation improvements—which result in still more urban expansion—are expected to exacerbate this inefficiency. Presumably, the benefits of transportation projects ought to be discounted in some way. We now sketch an analysis that confirms this intuitive argument and shows how benefit measures should be amended to take tax distortions into account.

First, we recognize that $I(x)$ in (1.2) depends on α, such that

$$I_\alpha(x) \equiv \frac{\partial I(x)}{\partial \alpha} = -(1-\tau)wt_\alpha - k_\alpha > 0. \tag{22}$$

As a consequence, the parameter α enters the equilibrium price function $p(x)$ in (3), and helps determine the city boundary $\bar{x}$ in (4). The derivatives of the functions $p(\bar{w}, I(x), u, \alpha)$, $\bar{x}(\bar{w}, I(x), u, \alpha)$, and $\phi(\tau, L, u, \alpha)$ (from 6)) are presented in the Appendix. It is thus possible to derive expressions showing the effect of a compensated change in α on equilibrium prices, the rural/urban boundary, and the density of population, just as in the compensated tax analysis of Section IIIA. To get specific qualitative results, consider the case of a general upgrading of the transportation system throughout the urban area, which would reduce travel costs more for residents farther from the city center. That is, we assume[12]

$$I'_\alpha(x) > 0. \tag{23}$$

Then, if (8) also holds, it is easily shown (see Appendix) that

$$p_\alpha(0)|_{\bar{u}} < 0 \tag{24.1}$$

$$p_\alpha(\bar{x})|_{\bar{u}} > 0, \tag{24.2}$$

the latter of which implies

$$\bar{x}_\alpha|_{\bar{u}} > 0. \tag{25}$$

Note that these results are analogous to those presented earlier in (13) and (14). Moreover, we can show that

$$N_\alpha(x)|_{\bar{u}} < 0 \qquad \text{all } x \in (0, \bar{x}). \tag{26}$$

Now in order to measure the true benefit of an increase in α, we consider as a cost any increase in compensation paid to city residents, holding u constant, while increases in land rents and taxes are counted as benefits. Thus,

$$\mathrm{MSB} = -NL_\alpha + \frac{d(1-\tau)R}{d\alpha} + \frac{dT}{d\alpha}. \tag{27}$$

It is possible to establish that

$$\mathrm{MSB} = -\int_0^{\bar{x}} n(x)\left[wt_\alpha(x) + k_\alpha(x)\right] dx - \tau w \int_0^{\bar{x}} N_\alpha(x)|_{\bar{u}} a'(x)\, dx. \tag{28}$$

From (28) we see that if $\tau = 0$, the benefit of the project is just the sum, across households, of the value of travel time and pecuniary travel costs saved. If $\tau > 0$, however, the benefit is reduced by an amount equal to the tax revenue lost by the increased dispersion of population, and consequent reduction in labor supply, caused by the project. Accordingly, MSB must be adjusted unambiguously downward, which is to say that less transportation investment is desired in the tax-distorted economy than would otherwise be

[12] If the transportation cost functions t and k are linear in x, so that $t(x,\alpha) = t(\alpha)x$ and $k(\alpha)x$, (23) will certainly be satisfied. This, of course, is a sufficient but much stronger than necessary condition for (23).

the case. Let us observe that the amount of downward adjustment depends on $a'(x)$ (and thus the structure of transportation costs) and on the (compensated) equilibrium redistribution of population induced by the project, $N_\alpha(x)|_{\bar{u}}$. This in turn depends on the change in the transportation cost functions brought about by the project and on the elasticity of demand for housing. These magnitudes can be empirically determined, in principle. Note that, in general, no simple adjustment to the first (value of travel time and pecuniary travel costs saved) term in (28) will accurately capture the marginal spatial distortions represented by the second term. In particular, replacing the gross by the net wage in the first term and ignoring the second term will not provide an accurate benefit measure (as can be easily seen by noting that the magnitude of the second term is zero if $h_p(x) \equiv 0$, but large if $|h_p(x)|$ is large, for any given τ).

V. CONCLUSIONS AND EXTENSIONS

In a world where time is an important component of transportation costs, and where marginal tax rates on wages (including implicit rates induced by income maintenance programs) are high, taxes can have a significant effect on long run equilibrium urban form. This in turn has implications for economic welfare. The foregoing analysis has drawn out these implications in a very simple model, and it is worthwhile enquiring how the results might be modified in a more elaborate analysis.

First, the only distortion allowed in this model is the income tax. Other distortions, especially unpriced congestion, ought to be considered as well. In such a model, the welfare loss from an income tax increase would presumably be greater: since both the tax and congestion effects work in the direction of excessive dispersion and travel, they will be mutually reinforcing rather than offsetting. Second, allowance might be made for multiple income classes and a progressive tax structure. The general qualitative character of the conclusions presented here would be unaffected by such an extension.[13] Third, it would be of interest to take into account the special income tax treatment of housing. A crude way to capture this in the present model is to suppose that the effective price of space at location x is $p(x)(1 - \tau\alpha)$ where α is the portion of rent that is tax deductible.[14] But in any case we would expect this effect to work further in the direction of more spatial dispersion in urban areas.

Finally, it is useful to emphasize some of the implications of the analysis presented here for the welfare analysis of income taxation. As we have seen,

[13] The locational choices of households in different income classes have been analyzed, e.g., by Hartwick *et al.* [4], Wheaton [15, 16], and Arnott *et al.* [1]. It is interesting to note that a progressive tax structure tends to make more peripheral locations relatively more attractive to high income households.

[14] This approach would, however, fail to distinguish between owner-occupied (predominantly low-density) and rental housing, which might be important for some purposes.

the income tax imposes an excess burden at the margin even if the demand for leisure is perfectly inelastic. In this case, the tax still causes a reduction in labor supply because population density falls and more time is spent traveling. However, this effect on labor supply will occur only with a lag, for the equilibrium adjustment of urban structure to tax changes takes place slowly. In principle, this lag should be taken into account in empirical work on labor supply response to taxes.

APPENDIX

First, we present the derivatives of the functions p defined in (3), and $\bar{x}$ defined in (4), with respect to τ, L, and u. Denoting these derivatives with subscripts, we have

$$p_\tau(x) \underset{\text{df}}{=} \frac{\partial p(x)}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau} + \frac{\partial p(x)}{\partial I(x)}\frac{\partial I(x)}{\partial \tau} = -\frac{wa(x)}{h(x)} < 0. \tag{A.1.1}$$

$$p_L(x) \underset{\text{df}}{=} \frac{\partial p(x)}{\partial L} = \frac{1}{h(x)} > 0. \tag{A.1.2}$$

$$p_u(x) \underset{\text{df}}{=} \frac{\partial p(x)}{\partial u} = \frac{1}{v_p(x)} < 0. \tag{A.1.3}$$

$$\bar{x}_\tau \underset{\text{df}}{=} \frac{\partial \bar{x}(\tau, L, u)}{\partial \tau} = \frac{wa(\bar{x})}{I'(\bar{x})} < 0. \tag{A.1.4}$$

$$\bar{x}_L \underset{\text{df}}{=} \frac{\partial \bar{x}(\cdot)}{\partial L} = -I'(\bar{x})^{-1} > 0. \tag{A.1.5}$$

$$\bar{x}_u \underset{\text{df}}{=} \frac{\partial \bar{x}(\cdot)}{\partial u} = \frac{-h(\bar{x})}{v_p(\bar{x})I'(\bar{x})} < 0. \tag{A.1.6}$$

These results follow from standard properties of the indirect utility function.

The derivatives of the function $\phi(\tau, L, u)$ defined by (6) are given by

$$\phi_\tau = -\frac{n(\bar{x})wa(\bar{x})}{I'(\bar{x})} - w\int_0^{\bar{x}} \frac{\theta(x)}{h(x)^2}\left(\frac{h_p(x)a(x)}{h(x)} + h_{\bar{w}}(x)\right) dx \tag{A.2.1}$$

$$\phi_L = \frac{n(\bar{x})}{I'(\bar{x})} + \int_0^{\bar{x}} \frac{\theta(x)}{h(x)^3} h_p(x)\, dx < 0 \tag{A.2.2}$$

$$\phi_u = -n(\bar{x})\bar{x}_u + \int_0^{\bar{x}} \frac{\theta(x)}{h(x)^2}\big(h_p(x)p_u(x) + h_u(x)\big)\, dx. \tag{A.2.3}$$

It is obvious from (A.2.1) that $\phi_\tau > 0$ if (7) is assumed. Using (A.1.3) and

Roy's identity, the bracketed expression in (A.2.3) can be written as

$$v_I(x)^{-1}\big(h_u(x)v_I(x) - h(x)^{-1}h_p(x)\big)$$
$$= v_I(x)^{-1}h(x)^{-1}\big(h(x)h_u(x)v_I(x) - h_p(x)\big), \qquad \text{(A.3)}$$

in which the last parenthetical expression, by the Slutsky equation, is the negative of the price derivative of the uncompensated demand for land. Provided that land is non-Giffen, this expression will be positive, so that, using (A.1.6), $\phi_u > 0$.

We now derive the results in Section IIIA. First, use (A.1.4), (A.1.5), (12), (A.2.1), and (A.2.2) to get

$$\bar{x}_\tau|_{\bar{u}} = \bar{x}_\tau + \bar{x}_L L_\tau = I'(\bar{x})^{-1}(wa(\bar{x})\phi_L + \phi_\tau)$$
$$= I'(\bar{x})^{-1}\phi_L^{-1}\left(w\int_0^{\bar{x}} \frac{\theta(x)}{h(x)^2}\left[\frac{h_p(x)}{h(x)}(a(\bar{x}) - a(x)) - h_{\bar{w}}(x)\right]dx\right)$$
$$> 0 \qquad (\text{by (A.2.2.), (7), and (8)}). \qquad \text{(A.4)}$$

Next, for any x, we have by (A.1.1), (A.1.2), (A.2.1), and (A.2.2)

$$p_\tau(x)|_{\bar{u}} = p_\tau(x) + p_L(x)L_\tau = -\phi_L^{-1}h(x)^{-1}(wa(x)\phi_L + \phi_\tau)$$
$$= -\phi_L^{-1}h(x)^{-1}\left(\frac{n(\bar{x})}{I'(\bar{x})}w[a(x) - a(\bar{x})] + w\int_0^{\bar{x}}\frac{\theta(\xi)}{h(\xi)^2}\right.$$
$$\left.\times\left[\frac{h_p(\xi)}{h(\xi)}[a(x) - a(\xi)] - h_{\bar{w}}(\xi)\right]d\xi\right) \qquad \text{(A.5)}$$

from which (14.1) and (14.2) follow, given (7). (By (8), $a(0) > a(\xi) > a(\bar{x})$ for all $\xi \in (0, \bar{x})$.) Finally, to show (16), note that

$$N_\tau(x)|_{\bar{u}} = -\int_0^x \frac{\theta(\xi)}{h(\xi)^2}\left(h_p(\xi)p_\tau(\xi)|_{\bar{u}} - wh_{\bar{w}}(\xi)\right)d\xi$$
$$= \phi_L^{-1}w\int_0^x \frac{\theta(\xi)}{h(\xi)^3}h_p(\xi)\frac{n(\bar{x})}{I'(\bar{x})}[a(\xi) - a(\bar{x})]\,d\xi$$
$$+\phi_L^{-1}w\int_0^x \frac{\theta(\xi)}{h(\xi)^3}h_p(\xi)\left\{\int_0^{\bar{x}}\frac{\theta(\zeta)}{h(\zeta)^3}h_p(\zeta)[a(\xi) - a(\zeta)]\,d\zeta\right\}d\xi$$
$$-\phi_L^{-1}w\int_0^x \frac{\theta(\xi)}{h(\xi)^3}h_p(\xi)\left\{\int_0^{\bar{x}}\frac{\theta(\zeta)}{h(\zeta)^2}h_{\bar{w}}(\zeta)\,d\zeta\right\}d\xi$$
$$+\int_0^x \frac{\theta(\xi)}{h(\xi)^2}wh_{\bar{w}}(\xi)\,d\xi, \qquad \text{(A.6)}$$

using (A.5). If we assume (7), the last two of these terms can be ignored. By (8), the first term, which we shall denote as $A(x)$, is negative for $x < \bar{x}$. It remains to sign the second term. If we define

$$\Gamma(x) = \int_0^x \frac{\Theta(\xi)}{h(\xi)^3} h_p(\xi)\, d\xi, \tag{A.7}$$

we can rewrite (A.6) more neatly as

$$\begin{aligned} N_\tau(x)|_{\bar{u}} &= A(x) + \phi_L^{-1} w \\ &\quad \times \left[\Gamma(\bar{x}) \int_0^x \Gamma'(\xi) a(\xi)\, d\xi - \Gamma(x) \int_0^{\bar{x}} \Gamma'(\xi) a(\xi)\, d\xi\right] \\ &= A(x) + \phi_L^{-1} w \Gamma(x) \Gamma(\bar{x}) \\ &\quad \times \left[\frac{\int_0^x \Gamma'(\xi) a(\xi)\, d\xi}{\Gamma(x)} - \frac{\int_0^{\bar{x}} \Gamma'(\xi) a(\xi)\, d\xi}{\Gamma(\bar{x})}\right]. \end{aligned} \tag{A.8}$$

Now, to sign the bracketed expression in (A.8), note that it can be written as $G(x) - G(\bar{x})$, where

$$G(x) = \frac{\int_0^x \Gamma'(\xi) a(\xi)\, d\xi}{\Gamma(x)}. \tag{A.9}$$

The derivative of $G(x)$ is

$$\begin{aligned} G'(x) &= \frac{\Gamma'(x) a(x)}{\Gamma(x)} - \frac{G(x) \Gamma'(x)}{\Gamma(x)} \\ &= \frac{\Gamma'(x)}{\Gamma(x)} \left(\frac{\int_0^x \Gamma'(\xi) a(x)\, d\xi}{\Gamma(x)} - \frac{\int_0^x \Gamma'(\xi) a(\xi)\, d\xi}{\Gamma(x)} \right) \\ &= \frac{\Gamma'(x)}{\Gamma(x)^2} \int_0^x \Gamma'(\xi)(a(x) - a(\xi))\, d\xi < 0 \end{aligned} \tag{A.10}$$

by (8) and because $\Gamma' < 0$. With G monotonically decreasing, the bracketed expression in (A.8) is positive and, by (A.2.2), (16) follows. We note for future reference that

$$N_\tau(0)|_{\bar{u}} = 0 = N_\tau(\bar{x})|_{\bar{u}} + n(\bar{x})\bar{x}_\tau|_{\bar{u}} \tag{A.11}$$

which follow, respectively, from $N(0) \equiv 0$ and $N - \int_0^{\bar{x}} n \equiv 0$.

Turning now to the analysis of the uncompensated equilibrium response to a tax change, we begin by observing that, given (7),

$$\bar{x}_\tau|_{\bar{L}=0} = I'(\bar{x})^{-1}\phi_u^{-1}w\int_0^{\bar{x}} \frac{\theta(x)}{h(x)^3}\Big[h_p(x)\big(a(x)v_I(\bar{x})^{-1} - a(\bar{x})v_I(x)^{-1}\big) + h(x)h_I(x)a(\bar{x})v_I(x)^{-1}\Big]\,dx, \tag{A.12}$$

by (A.1.4), (A.1.6), (9.1), (A.3), (A.2.1) and (A.2.3) where $|_{\bar{L}}$ denotes an uncompensated derivative, and where $h_I(x)$ denotes $h_u(x)v_I(x)$, the income derivative of demand for land. Now, $h_I(x) > 0$ clearly is an income effect which, through the last term in brackets in (A.12), works to make $\bar{x}_\tau|_{\bar{L}=0}$ negative. The parenthesized term in (A.12) is equal to zero at $x = \bar{x}$, and will have a sign given by sgn $[a(x)v_I(x) - a(\bar{x})v_I(\bar{x})]$ for $x < \bar{x}$. If the derivative of $a(x)v_I(x)$ w.r.t. x is everywhere negative (positive), this sign will be positive (negative). It is shown in Wildasin [18] that $dv_I(x)/dx = -\varepsilon(x)v_I(x)I'(x)I(x)^{-1}$, where $\varepsilon(x)$ is the elasticity of demand for land with respect to income net of transportation costs, I. Therefore,

$$\begin{aligned} a(x)^{-1}v_I(x)^{-1}&\frac{d[a(x)v_I(x)]}{dx} \\ &= \frac{a'(x)}{a(x)} - \varepsilon(x)\frac{I'(x)}{I(x)} \\ &= -\frac{t'(x)}{a(x)} + \varepsilon(x)\frac{[(1-\tau)wt'(x) + k'(x)]}{(1-\tau)w[T - t(x)] - k(x) + L} \end{aligned} \tag{A.13}$$

In the special case where $\varepsilon(x) = 0$, it follows from (8) and (A.13) that the parenthetical expression in (A.12) is positive. Similarly, if $k(x) = L = 0$ and $\varepsilon(x) \leq 1$, the second term in (A.13) becomes $\varepsilon(x)t'(x)/[a(x) + l(x)] < t'(x)/a(x)$, and again the parenthetical expression is positive. (This result is preserved if $L > 0$.) In general, however, (A.13) cannot be signed.

A simple and insightful way to show the necessary ambiguity of the effect of an uncompensated change in τ on prices is as follows. First, note that

$$p_\tau(x)|_{\bar{L}} = p_\tau(x) + p_u(x)\frac{du}{d\tau} = p_\tau(x) - p_u(x)\frac{\phi_\tau}{\phi_u}$$

$$= p_\tau(x)|_{\bar{u}} + p_L(x)\frac{\phi_\tau}{\phi_L} - p_u(x)\frac{\phi_\tau}{\phi_u} \qquad (\text{by(A.5)})$$

$$= p_\tau(x)|_{\bar{u}} + \phi_\tau\phi_u^{-1}\phi_L^{-1}h(x)^{-1}v_I(x)^{-1}(\phi_u v_I(x) + \phi_L)$$

$$(\text{by(A.1.2) and (A.1.3)})$$

$$= p_\tau(x)|_{\bar{u}} + \phi_\tau\phi_L^{-1}p_L(x)|_{\bar{\tau}} \qquad (\text{A.14})$$

where $p_L(x)|_{\bar{\tau}}$ is the change in equilibrium price resulting from an increase in income, with tax rates held fixed. Wheaton [14] shows that we may expect $p_L(0)|_{\bar{\tau}} < 0 < p_L(\bar{x})|_{\bar{\tau}}$. Since $\phi_L < 0$, (A.14) shows that changes in land rents, both at the city center and the boundary, are the result of income and substitution effects going in different directions, which cannot be assessed a priori.

The first result we need to show in Section IIIB is (18). From (17), using (10) and (11),

$$\left.\frac{dT}{d\tau}\right|_{\bar{u}} + \left.\frac{d(1-\tau)R}{d\tau}\right|_{\bar{u}} = \int_0^{\bar{x}} wn(x)a(x)\,dx + \tau\left.\frac{d\left(\int_0^{\bar{x}} wn(x)a(x)\,dx\right)}{d\tau}\right|_{\bar{u}}$$

$$+ \int_0^{\bar{x}} \frac{dp(x)}{d\tau}|_{\bar{u}}\theta(x)\,dx + [p(\bar{x}) - s]\bar{x}_\tau|_{\bar{u}}. \qquad (\text{A.15})$$

By (4), the last term vanishes. Substituting from (A.5), the penultimate term becomes

$$\int_0^{\bar{x}} \left.\frac{dp(x)}{d\tau}\right|_{\bar{u}} \theta(x)\,dx = -\int_0^{\bar{x}} w\frac{\theta(x)}{h(x)}a(x)\,dx + \int_0^{\bar{x}} \frac{\theta(x)}{h(x)}L_\tau\,dx. \qquad (\text{A.16})$$

Now the first expression in (A.16) cancels the first expression in (A.15) while the second term is just NL_α. Substitution in (17) yields (18).

To show (20), note that $(dn(x)/d\tau)_{\bar{u}} = N_\tau'(x)|_{\bar{u}}$, and integrate the second term of (19) by parts to get

$$-\tau w\left[\int_0^{\bar{x}} a(x)\frac{dn(x)}{d\tau}|_{\bar{u}}\,dx + a(\bar{x})n(\bar{x})\bar{x}_\tau|_{\bar{u}}\right]$$

$$= -\tau w\left[N_\tau(x)a(x)\Big|_0^{\bar{x}} - \int_0^{\bar{x}} N_\tau(x)a'(x)\,dx + a(\bar{x})n(\bar{x})\bar{x}_\tau\Big|_{\bar{u}}\right]$$

$$= \tau w\int_0^{\bar{x}} N_\tau(x)a'(x)\,dx \tag{A.17}$$

by (A.11.1) and (A.11.2).

Finally, we investigate the effect of transportation improvements. To begin with,

$$p_\alpha(x) \equiv \frac{\partial p(x)}{\partial \alpha} = \frac{I_\alpha(x)}{h(x)} > 0 \tag{A.18.1}$$

$$\bar{x}_\alpha \equiv \frac{\partial \bar{x}}{\partial \alpha} = -\frac{I_\alpha(\bar{x})}{I'(\bar{x})} > 0. \tag{A.18.2}$$

$$\phi_\alpha = \frac{n(\bar{x})I_\alpha(\bar{x})}{I'(\bar{x})} + \int_0^{\bar{x}} \frac{\theta(x)}{h(x)^3}h_p(x)I_\alpha(x)\,dx < 0. \tag{A.18.3}$$

It follows that $L_\alpha = -\phi_\alpha/\phi_L < 0$, as expected. Next, analogous to (A.5),

$$p_\alpha(x)_{\bar{u}} = \phi_L^{-1}h(x)^{-1}\Bigg(\frac{n(\bar{x})}{I'(\bar{x})}[I_\alpha(x) - I_\alpha(\bar{x})]$$
$$+\int_0^{\bar{x}}\frac{\theta(\xi)}{h(\xi)^3}h_p(\xi)[I_\alpha(x) - I_\alpha(\xi)]\,d\xi\Bigg). \tag{A.19}$$

Given (23), (24) follows. Also,

$$N_\alpha(x)|_{\bar{u}} = -\phi_L^{-1}\int_0^x \frac{\theta(\xi)}{h(\xi)^3}h_p(\xi)\frac{n(\bar{x})}{I'(\bar{x})}[I_\alpha(\xi) - I_\alpha(\bar{x})]\,d\xi$$
$$-\phi_L^{-1}\int_0^x \frac{\theta(\xi)}{h(\xi)^3}h_p(\xi)\int_0^{\bar{x}}\frac{\theta(\zeta)}{h(\zeta)^3}h_p(\zeta)[I_\alpha(\xi) - I_\alpha(\zeta)]\,d\zeta\,d\xi \tag{A.20}$$

which, by an argument similar to that given in regard to (A.6), yields (26).

Lastly, to derive (28) from (27), use (4), (5), the fact that $p_\alpha(x)|_{\bar{u}} = p_\alpha(x) + p_L(x)L_\alpha$, and (A.18.1) to derive

$$\begin{aligned}
\mathrm{MSB} &= -NL_\alpha + \int_0^{\bar{x}} \theta(x) p_\alpha(x)|_{\bar{u}}\, dx + (p(\bar{x}) - s)\theta(\bar{x})\bar{x}_\alpha|_{\bar{u}} \\
&\quad + \frac{\tau w\, d\left(\int_0^{\bar{x}} n(x)a(x)\, dx\right)}{d\alpha} \\
&= \int_0^{\bar{x}} \theta(x) p_\alpha(x)\, dx + \frac{\tau w\, d\left(\int_0^{\bar{x}} n(x)a(x)\, dx\right)}{d\alpha} \\
&= -\int_0^{\bar{x}} n(x)[(1-\tau)wt_\alpha(x) + k_\alpha(x)]\, dx - \tau w \int_0^{\bar{w}} n(x)t_\alpha(x)\, dx \\
&\quad + \tau w \int_0^{\bar{x}} a(x)\frac{dn(x)}{d\alpha}\Big|_{\bar{u}}\, dx + \tau w n(\bar{x})a(\bar{x})\bar{x}_\alpha\Big|_{\bar{u}} \\
&= -\int_0^{\bar{x}} n(x)[wt_\alpha(x) + k_\alpha(x)]\, dx - \tau w \int_0^{\bar{x}} N_\alpha(x)\Big|_{\bar{u}} a'(x)\, dx.
\end{aligned} \tag{A.21}$$

Equation (28) follows from an integration by parts identical to that in (A.17).

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