Distributional Neutrality and Optimal Commodity Taxation: Reply

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In a way Leslie Young's comment on my earlier paper is puzzling. My Proposition 3 is based on equation (19) which, reproduced here as (1′), shows that optimal taxation requires

$$(1') \quad (1-\theta)-\sum_h \frac{x_k^h}{X_k}\frac{\partial(tx^h)}{\partial I^h}=\frac{\partial(tX)/\partial t_k}{X_k}$$

given the assumption of extended neutrality. If (1′) is valid, it is obvious that the Marginal Revenue Proportionality Rule[1] (*MRPR*) *cannot* obtain unless $\Sigma_h(x_k^h/X_k)(\partial(tx^h)/\partial I^h)$ is independent of k, and it is just as obvious that this condition holds only under special assumptions on consumer preferences. The proof of Proposition 3 is ironclad given (1′), so the obvious (and only) way to disprove it is to expose an error in the derivation of (1′). Young neither proves nor asserts that (1′) is false; in fact it is not.

On the other hand, Paul Samuelson did show that optimal redistribution does lead to aggregate demand behavior that is consistent with the restrictions that demand theory places on a single utility-maximizing consumer, and Young is indeed correct in his belief that this allows one to establish the single person results (both *MRPR* and the Ramsey Rule (*RR*) characterize the optimal tax structure) in the many-person case. This does not contradict my earlier results, however, and Young's contention to the contrary rests on a misunderstanding of the nature of the optimal tax problem he discusses. To be sure that neither my results nor those that Young purports to establish are misinterpreted, I propose to show in some detail how the two optimal tax rules can be derived for the many-person economy on the assumption that optimal lump sum redistribution of a certain kind is *actually performed.* I will also show that the assumption that such redistribution takes place—which is crucial to the conclusion that Young seeks to establish—alters in a fundamental way the problem that I originally investigated. Thus Young's conclusion differs from mine not because my results are incorrect, but because he considers a different problem.

To get on with the analysis, which presumes familiarity with the earlier discussion, let there first of all be given an *arbitrary* social welfare function (*SWF*) W, defined over individual utility levels. Recall that households are assumed to maximize utility by choice of x^h subject to

$$(1) \qquad qx^h=s_0^h$$

where q and s_0^h are taken as parametrically given. This yields the demand functions $x^h(q,s_0^h)$ and also the indirect utility functions $v^h(q,s_0^h)$.

Now suppose that there is an agent who, in Young's words, makes sure that "at every price, income is redistributed so as to maximize social welfare" (p. 234). This agent then takes consumer prices q as given and chooses a vector $s_0=(s_0^1,\ldots,s_0^H)$ to

$$(\mathrm{R}) \qquad \max_{s_0} W(v^1[q,s_0^1],\ldots,v^H[q,s_0^H])$$

subject to

$$(2) \qquad \sum_h s_0^h=S$$

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[1]Interpreted in the natural way—that is, in terms of the sum of the derivatives of the ordinary individual demand functions (which is what we would actually observe in the marketplace in the absence of Youngian redistribution), as discussed in detail below.

where S is some fixed parameter. Ordinarily $S \equiv 0$ so (R) is a pure redistribution problem. The first-order conditions associated with this problem are (2) and

$$(3) \qquad W_h v_I^h - \mu = 0 \qquad h = 1, \ldots, H$$

where μ is the multiplier associated with (2). Assuming that $W(\{v^h(q, s_0^h\})$ is strictly quasi concave in s_0 for any given q,[2] one can solve for (μ, s_0) from (2) and (3) as continuously differentiable functions of (q, S). Then define

$$(4) \qquad V(q,S) = W(\{v^h(q, s_0^h[q,S])\})$$

and

$$(5) \qquad X(q,S) = \sum_h x^h(q, s_0^h[q,S])$$

Then the problem

$$(\text{P}) \qquad \max_{q, y^f} V(q,0)$$

subject to

$$(6) \qquad y^f + y^g - X(q,0) = 0$$

and

$$(7) \qquad \phi^f y^f \leq 0$$

is equivalent to Young's (7), and is the natural way to exploit the Samuelsonian representative consumer idea in trying to recast the many-person tax problem in a single person mold. The functions $V(q,S)$ and $X(q,S)$ are of course to play the role of indirect utility and demand functions for this representative consumer.

Using definition (4) and differentiating (2), we have

$$(8) \qquad V_S = \sum_h W_h v_I^h \frac{\partial s_0^h}{\partial S} = \mu$$

[2]This is not a terribly strong requirement. Young believes that one can dispose with it, but it seems clear that it is necessary if one wishes to derive optimal tax rules, such as (16) below, involving the derivatives of aggregate demand functions.

$$(9) \qquad V_i \underset{\text{def}}{=} \frac{\partial V}{\partial q_i} = \sum_h W_h v_i^h + \sum_h W_h v_I^h \frac{\partial s_0^h}{\partial q_i} = \sum_h W_h v_i^h$$

It follows that

$$(10) \qquad \frac{V_i}{V_S} = -X_i$$

From (5) we have

$$(11) \qquad \frac{\partial X}{\partial S} = \sum_h \frac{\partial x^h}{\partial I^h} \frac{\partial s_0^h}{\partial S}$$

$$(12) \qquad \frac{\partial X}{\partial q_i} = \sum_h \frac{\partial x^h}{\partial q_i} + \sum_h \frac{\partial x^h}{\partial I^h} \frac{\partial s_0^h}{\partial q_i}$$

Also, using (1) and (5), one has

$$(13) \qquad X_i + q_i \frac{\partial X}{\partial q_i} = 0$$

$$(14) \qquad q \frac{\partial X}{\partial S} = 1$$

It is now straightforward that a solution to (P) is characterized by

$$(15) \qquad \frac{\mu}{\rho_0} X_k = t \frac{\partial X}{\partial q_k} + X_k \qquad \text{all } k$$

where ρ_0 is the Lagrange multiplier associated with the 0th of equations (6). The marginal revenue proportionality result

$$(16) \qquad \frac{\mu}{\rho_0} = \frac{\partial(tX)/\partial t_k}{X_k} \qquad \text{all } k$$

follows immediately. Note that the derivatives of X appearing in (16) are *not* simply the pure price derivatives $\Sigma_h \partial x^h / \partial q_k$, as can be seen by recalling (12).

Now rewrite (16) as

$$(17) \qquad \frac{\mu}{\rho_0} = 1 + \frac{t}{X_k}\left(\frac{\partial X}{\partial t_k} + X_k \frac{\partial X}{\partial S}\right) - t\frac{\partial X}{\partial S} = 1 + \sum_i \frac{t_i S_{ik}}{X_k} - t\frac{\partial X}{\partial S}$$

where we define $S_{ik} = \partial X_i/\partial t_k + X_k(\partial X_i/\partial S)$. On the assumption that $S_{ik} = S_{ki}$, simple rearrangement yields

$$\frac{\mu}{\rho_0} - 1 + t\frac{\partial X}{\partial S} = \sum_i \frac{t_i S_{ki}}{X_k} \tag{18}$$

which is precisely the Ramsey Rule, with the S_{ik}'s playing the role of substitution terms.

I shall now work out an implicit solution for s_0 as a function of q and S for a simple special case and then show directly that $S_{ik} = S_{ki}$. Suppose $H=2$ and suppose the social welfare function is

$$W = \alpha_1 v^1 + \alpha_2 v^2 \tag{19}$$

If v^1 and v^2 are chosen so that the marginal utility of income diminishes (i.e., $v^h_{II} < 0$), then we can use (2) and (3) to solve for $s_0(q,S)$. This condition is met, for example, if the direct utility functions are of the familiar form

$$u^h = \beta^h_0 \log(x^h_0 + \omega^h) + \sum_{i=1}^{n} \beta^h_i \log x^h_i$$

with

$$\sum_{i=0}^{n} \beta^h_i = 1, \omega^h > 0 < \beta^h_i \qquad \text{all } i$$

It is straightforward to find

$$\frac{\partial s^1_0}{\partial S} = \frac{\alpha_2 v^2_{II}}{\alpha_1 v^1_{II} + \alpha_2 v^2_{II}}, \qquad \frac{\partial s^2_0}{\partial S} = 1 - \frac{\partial s^1_0}{\partial S} \tag{20}$$

$$\frac{\partial s^1_0}{\partial q_i} = \frac{\alpha_2 v^2_{Ii} - \alpha_1 v^1_{Ii}}{\alpha_1 v^1_{II} + \alpha_2 v^2_{II}}, \qquad \frac{\partial s^2_0}{\partial q_i} = -\frac{\partial s^1_0}{\partial q_i} \tag{21}$$

$$i = 1, \ldots, n$$

where $v^h_{Ii} = \partial v^h_I/\partial q_i$. Using $v^h_{Ii} = -v^h_I(\partial x^h_i/\partial I^h) - x^h_i v^h_{II}$, (3), and (20), we can write

(22)

$$\frac{\partial s^1_0}{\partial q_i} = \frac{\mu(\partial x^1_i/\partial I^1 - \partial x^2_i/\partial I^2)}{\alpha_1 v^1_{II} + \alpha_2 v^2_{II}} + x^1_i - X_i\frac{\partial s^1_0}{\partial S}$$

$$= \gamma\left(\frac{\partial x^1_i}{\partial I^1} - \frac{\partial x^2_i}{\partial I^2}\right) + x^1_i - X_i\frac{\partial s^1_0}{\partial S}$$

Letting $D_i = \partial x^1_i/\partial I^1 - \partial x^2_i/\partial I^2$ for brevity, using (20) and (21) to eliminate derivatives of s^2_0, and using (11) and (12), we can write out S_{ik} in detail:

$$S_{ik} = \left.\frac{\partial x^1_i}{\partial q_k}\right|_{\bar{u}} + \left.\frac{\partial x^2_i}{\partial q_k}\right|_{\bar{u}} - x^1_k\frac{\partial x^1_i}{\partial I^1} - x^2_k\frac{\partial x^2_i}{\partial I^2} + D_i\frac{\partial s^1_0}{\partial q_k} + X_k\left[D_i\frac{\partial s^1_0}{\partial S} + \frac{\partial x^2_i}{\partial I^2}\right] \tag{23}$$

Now using (22) it follows that

$$S_{ik} = \left.\frac{\partial x^1_i}{\partial q_k}\right|_{\bar{u}} + \left.\frac{\partial x^2_i}{\partial q_k}\right|_{\bar{u}} + \gamma D_i D_k = S_{ki} \tag{24}$$

using the symmetry of the individual substitution terms. We have established the symmetry of the substitution terms of the price derivatives of the representative consumer's demand function; the Ramsey Rule therefore characterizes an optimal tax structure, as shown in (18). Note from (24), however, that the S_{ik}'s are *not* the sum of the substitution terms of the individual consumers. We can summarize all this in the form of a

THEOREM: *Given lump sum redistribution of the type described in problem* (R), *an optimal tax structure—that is, a tax structure emerging from the solution of problem* (P)*—can be characterized by both MRPR and RR, these tax rules being interpreted in terms of the demand functions of the representative consumer.*

Does this valid theorem contradict any of my earlier results? No, because I did not allow for—that is, I implicitly ruled out —the Samuelsonian redistribution, embodied in the functions $s^h_0(q,S)$ obtained as solutions to problem (R), that is so crucial to the proof of the above theorem. To see how crucial this redistribution really is, recall that the "ordinary" and "compensated" price derivatives of the representative demand function that appear in (16) and (18) are *not* the sum of the corresponding derivatives of the individual demand functions

$x^h(q,0)$ of consumers who have lump sum incomes fixed identically at zero, independent of q—yet the latter are what would be observed in the absence of redistribution.

Young believes that the problem (P) is equivalent to the standard problem

$$\text{(P}'\text{)} \qquad \max_{q,y^f} W(\{v^h(q,0)\})$$

subject to (7) and

$$\text{(25)} \qquad y^f + y^g - \sum_h x^h(q,0) = 0$$

when the *SWF* exhibits extended neutrality. He argues that extended neutrality implies that $s_0^h = 0$, all h, solves (R), and that the solution set to (P′), in which each s_0^h is in effect constrained to be identically zero, will then coincide with the solution set to (P). To clear up a minor difficulty at first, note that simple—not extended—neutrality is needed to have $s_0^h = 0$ at a solution to (R). This is obvious from the first-order conditions (3), and should immediately alert the reader to the fact that Young has not shown what he thinks he has. But suppose that W does satisfy the simple neutrality condition at a solution $(\tilde{q},\tilde{y}^f)$ to problem (P), so that the functions $s_0^h(q,0)$ giving the solution to (R) have the property that $s_0^h(\tilde{q},0)=0$ all h. Is $(\tilde{q},\tilde{y}^f)$ then a solution to (P′)? No, because the proper choice of consumer prices in the problem (P) must take into account, *at the margin*, the effects of prices on lump sum incomes, whereas this is not true in the case of the problem (P′). The fact that, under simple neutrality, the functions $s_0^h(q,0)$ have a value of zero at a solution to (P) does *not* imply that the *derivatives* of these functions also vanish, and (20) and (22) show that the derivatives will not in fact vanish, in general.[3] But as shown by (16) and (18), interpreted in view of (12) and (23), these derivatives play a critical role in the determination of a solution to (P), and in the derivation of *RR* and *MRPR*. In the case of problem (P′), when redistribution is ruled out ($s_0^h \equiv 0$ all h), the aggregate demand functions are just $\sum_h x^h(q,0)$, which, unlike the $X(q,0)$ functions, do not have the properties of individual demand functions; specifically

$$\partial\Big(\sum_h x^h[q,0]\Big)/\partial q_k = \sum_h (\partial x^h/\partial q_k)_{\bar{u}} - \sum_h x_k^h(\partial x^h/\partial I^h)$$

$$\neq \sum_h (\partial x^h/\partial q_k)_{\bar{u}} - \Big(\sum_h x_k^h\Big)\sum_h (\partial x^h/\partial I^h)$$

that would be required for both *RR* and *MRPR* to hold. This, of course, is just what I originally showed in my paper.[4]

[3]As Samuelson observed, "a stipulated percentage breakdown of income [for example, Papa 10 percent, Mama 51 percent, Junior 39 percent] cannot be an optimal rule for a nonshibboleth social welfare function." Rather, "[i]ncome must always be reallocated among the members of our family society so as to keep the 'marginal social significance of every dollar' equal" (p. 11), in the face of changing prices. In other words, $\partial s_0^h/\partial q_i \neq 0$.

[4]It might be helpful to clarify the relationship between the kind of redistribution that I discussed on pp. 893–94 of my original paper in motivating the extended neutrality definition, and the kind of redistribution implied by Young's approach and described above in problem (R). Young obviously believes the two are equivalent. That they are actually different is most easily seen by comparing the conditions for optimal redistribution in each case: equation (18) in my original paper for the redistribution considered there, (3) above for the redistribution considered by Young. In contrast to the latter, the redistribution that I discussed takes into account the effect of redistribution on government revenue via the term $\partial(tx^h)/\partial I^h$ appearing in my original (18). (As a footnote to the footnote, Young errs in interpreting $\partial(tx^h)/\partial I^h$ as "the marginal welfare cost of the additional consumption that the household would carry out as the result of an addition to income" (p. 234) which must be *subtracted* from $(W_h v_I^h)\rho_0$ to arrive at marginal social *net* benefit of income. To the extent that additional income results in additional demand for tax-distorted goods, a marginal welfare *gain* is generated that must be *added* to $(W_h v_I^h)/\rho_0$.) When accompanied by optimal selection of (q,y^f), redistribution of the type that I originally discussed thus leads to a welfare level higher than that attained by optimal choice of (q,y^f) with *endogenous* consumer price-contingent redistribution as set out in problem (P). Thus the redistribution of problem (R) is optimal in a somewhat restricted sense: rather than requiring the redistribution program to satisfy (3) at every price, it would be better to choose prices, output, and transfers *simultaneously*.

In sum, Young is correct in his view that Samuelsonian redistribution does lead consumers, in the aggregate, to behave like a single consumer, and that both *MRPR* and *RR* characterize an optimal tax structure in this case. He fails to appreciate the fact that this redistribution alters the optimal tax problem in a fundamental way, so that rather than contradicting my original results, the new problem leads to new results.

Since redistribution must actually be carried out for the theorem stated above to be valid, its scope of application is presumably nil. This does not mean that it is uninteresting, however, for the thought of using a Samuelsonian representative consumer to analyze the optimal tax problem in the many-person economy is a natural one, the implications of which are worth exploring. Young deserves credit for suggesting this approach. Nothing in Young's comment or this reply would, however, lead me to change the statement that "[t]here is simply no unambiguous way to segregate the efficiency and distributional aspects of the optimal tax problem" (p. 896).

REFERENCES

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