THE DEMAND FOR PUBLIC GOODS IN THE PRESENCE OF TAX EXPORTING**

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ABSTRACT

Conventionally, tax exporting is thought to lower the effective cost of public services, thereby creating an incentive to increase public expenditure. This paper shows, however, that the effect of tax exporting on public spending depends critically on the nature of non-exported taxes. In general, tax exporting influences spending, if at all, by creating income effects and by affecting the marginal excess burden of non-exported taxes. If, for example, taxes on non-traded goods are distortionless, the marginal cost of public spending will not be reduced at all even though an additional dollar of revenue raised by taxation of a traded good may impose a very high burden on non-residents of the taxing jurisdiction. These results have a number of implications for empirical and policy analysis.

I. Introduction

THE purpose of this paper is to develop some improved understanding of the role of tax exporting as a determinant of the demand for public services by localities, provinces or states, or even entire countries. For concreteness, and because this case has been important in the empirical literature, let us henceforth think of the governments in question as localities—e.g., cities.

Tax exporting, of course, refers to the shifting of tax burdens by a locality to non-residents. Chiefly, there are two vehicles for tax exporting: changes in a locality's terms of trade with the rest of the economy and, at least in the U.S. context, federal income tax deductibility of state and local taxes. Most of the present discussion will focus on the first case.

Tax exporting is of interest to economists for several reasons. First, it affects the distribution of real income and is therefore important for assessing the equity impact of local tax policies. Second, the possibility of tax exporting can provide incentives for localities to alter their tax and expenditure policies, and thus may affect the efficiency of resource allocation. This essay is concerned with the incentive effects of tax exporting.

With almost no exceptions, the literature on tax exporting speaks with one voice about the incentive to undertake local public spending; by pushing some of the burden of public expenditures on to non-residents, tax exporting stimulates higher levels of spending than would otherwise be the case. This intuitively appealing view can be found, for example, in Bird and Slack (1983), Hogan and Shelton (1973), Ladd (1975), McLure (1967), Oates (1972), and Zimmerman (1983).

Apparently, the only exception to this view is provided by Mieszkowski and Toder (1983). They consider a jurisdiction that has monopoly power in the market for some good which is sold to non-residents. If either the sales to residents and non-residents can be differentially taxed, or if residents do not consume the good, Mieszkowski and Toder note that the optimal strategy for the locality is to set its tax policy so as to extract the maximum foreign contribution to local public spending. Suppose that \( R_{\text{max}} \) is the maximum revenue that can be extracted from non-residents. Then for levels of public spending less than \( R_{\text{max}} \), no local revenue sources should be used; all expenditures should be financed through tax exporting. The effective cost of public spending to the locality—what I will henceforth call the local marginal cost of public funds—is thus zero for levels of public spending less than \( R_{\text{max}} \). This of course implies that the level of public spending will at least equal \( R_{\text{max}} \), if the locality is non-satiable with respect to public spending. As the level of spending rises above \( R_{\text{max}} \), local or own-sources of revenue must be used. No additional

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taxes will be exported at the margin, since the maximum burden is already being imposed on non-residents. Thus, Mieszkowski and Toder conclude that tax exporting does not lower the effective price, and thus does not stimulate the demand for local public goods—at least in what is likely to be the relevant range—i.e., expenditure levels in excess of $p_{\text{max}}$.

While this conclusion certainly runs counter to the standard view of tax exporting, it seems to apply to quite special cases—namely, cases where no residents consume the exported good, or where residents and non-residents can be taxed differentially. One can imagine many important situations where such an assumption might not be met. For example, take the standard case of taxes on restaurants and hotel rooms in resort areas. While non-residents may bear a significant portion of such taxes, residents also bear some of the burden as estimated e.g., by Fujii et al. (1985). Or suppose that a locality taxes residential, commercial and industrial capital via a property tax. Perhaps some of the tax on business property is shifted to non-residents in the form of higher output prices, as estimated recently by Phares (1980). Nonetheless, residents may consume much of the output of local producers. Either of these examples illustrates a case where the Mieszkowski-Toder analysis would seem not to apply—and, by default, to be a case where the incentive effects of tax exporting would work to raise the level of local public spending.

Such a conclusion may be unwarranted, however. The next section of this paper presents a framework for the analysis of the effects of tax exporting on the local marginal cost of public funds. This framework encompasses not only the Mieszkowski-Toder case, but also the more general case exemplified by the room and restaurant tax and property tax illustrations mentioned above. The interaction between non-exported taxes (e.g., taxes on non-traded goods) and exported taxes plays a crucial role in the analysis. Although tax exporting may indeed stimulate public spending, as suggested by the conventional view, it nonetheless turns out that the essence of the Mieszkowski-Toder conclusion applies much more generally than appears at first sight. To make this clear, Section II discusses in some detail the precise mechanisms through which tax exporting may affect public spending. Later sections then discuss the importance of this analysis for empirical work, and its policy implications.

II. The Local Marginal Cost of Public Funds

An essential feature of the analysis to follow is its comparison of the costs of raising revenue from local sources as against the costs of raising revenue from taxes on traded goods or factors. Of these two cases, the easiest to analyze, and one that has been treated already in the literature, is the case of local taxation of non-traded goods. We shall therefore consider this case first, which also introduces the basic methodology of the paper, and then turn to the more interesting case of taxation of traded goods.

Suppose, then, that a locality taxes a commodity $X$ that is produced and consumed entirely within the locality. What is the effective marginal cost of public funds raised from this revenue source? To keep the analysis as simple as possible, let $X$ be produced at a constant marginal cost, $p_X$, and suppose general equilibrium effects in the local economy—such as the indirect effects of taxation of good $X$ on the terms of trade for traded goods and factors, or on the equilibrium quantities of other taxed commodities—can be ignored. (The analysis can accommodate such effects at the expense of some algebra.) Let $\epsilon_X$ be the elasticity of (local) demand for good $X$, and let $t_X$ denote the per unit tax on $X$. Hence $q_X = t_X + p_X$ will be the tax-inclusive price of good $X$ to consumers, on the assumption that individual firms producing $X$ are perfectly competitive profit-maximizers. Then, if $X$ also denotes the quantity of the good, the total revenue collected from the tax on good $X$ will be
If \( dRx \) of extra revenue were to be raised from this source, \( t_x \) would have to rise by \( dt_x \) such that

\[
dRx = Xdt_x + t_x dX
= X\left(1 + \frac{t_x}{q_x}\right)dt_x.
\]

(2)

(This formula uses the fact that \( p_x \) is constant, so that \( dq_x = dt_x \).) According to standard principles of welfare analysis, the loss of real income to local consumers from such a tax increase would be \( Xdtx \). Thus, if \( LMC_x \) denotes the local marginal cost of public funds raised from taxation of good \( X \), that is, the real income loss to residents of the locality per dollar of revenue produced,

\[
LMC_x = \frac{1}{1 + \frac{t_x}{\epsilon_x}}.
\]

(3)

This expression, or somewhat more general ones, have appeared widely in the literature (see, e.g., Wildasin (1984) and references therein).

Note that \( t_x = 0 \) implies \( LMC_x = 1 \); i.e., the first dollar of revenue "costs" just a dollar. But \( t_x > 0 \) and \( \epsilon_x < 0 \) imply \( LMC_x > 1 \); i.e., if the taxed good is elastically demanded, incremental dollars of revenue cost more than a dollar in terms of real income foregone by the locality. The reason for this, of course, is that the tax involves an excess burden. Typically, although not necessarily, the marginal excess burden, that is, \( LMC_x - 1 \), will rise as \( R_x \) rises, as can easily be seen for the case of a constant \( \epsilon_x \). This is portrayed in Figure 1, where \( LMC_x \) is graphed as a function of the amount of revenue collected.

Now consider the cost of raising revenue through taxation of a traded commodity—say commodity \( Y \). For concreteness let \( Y \) be a traded good (although the analysis is essentially unchanged for factors). Let \( Y_D \) denote the amount of the good consumed by residents, let \( Y_F \) denote the amount consumed by non-residents, let \( \sigma = Y_D/(Y_D + Y_F) \) denote the share of output taken by local consumers, and let \( \epsilon_D \) and \( \epsilon_F \) be the local and foreign demand elasticities. Let \( p_Y \) be the local marginal cost of production of good \( Y \), and assume for simplicity that this marginal cost is constant. As in the case of good \( X \), we assume that the local producers of good \( Y \) are perfectly competitive and profit maximizing firms. The analysis is only interesting if the locality has some market power with respect to good \( Y \), i.e., if \( \epsilon_F \) is finite, and this is assumed throughout the rest of the paper. Thus each individual local firm faces a perfectly elastic demand curve, while the aggregate of all local firms faces local and non-local demand curves with non-infinite elasticities \( \epsilon_D \) and \( \epsilon_F \)—yielding a market demand curve for local firms with an elasticity of \( \sigma \epsilon_D + (1 - \sigma) \epsilon_F \). Let \( t_Y \) be the local tax on good \( Y \)—not differentiated between resident and non-resident consumers. Let \( R_Y \) be the revenue from this tax. Again, let us ignore general equilibrium effects for simplicity. Then

\[
R_Y = t_Y(Y_D + Y_F).
\]

(4)

An increase in revenue from this source of \( dR_Y \) would require a change in \( t_Y \) such that

\[
dR_Y = (Y_D + Y_F)dt_Y + t_Y(dY_D + dY_F)
= (Y_D + Y_F)\left(1 + \frac{t_Y}{q_Y}\right)
\cdot [\sigma \epsilon_D + (1 - \sigma) \epsilon_F]dt_Y
\]

(5)

where \( q_Y = t_Y + p_Y \).

Now, when the tax on good \( Y \) increases, two distinct groups of individuals are hurt. An incremental increase \( dt_Y \) in \( t_Y \) causes a loss of real income to local residents of \( Y_D dt_Y \), and a loss to non-residents of \( Y_F dt_Y \). If \( LMC_Y \) denotes the local marginal cost of public funds raised from this revenue source, we have, by (5),
Similarly, the incremental burden borne by non-residents, which we may call the marginal export rate or MER_y, is

\[
\text{MER}_Y = \frac{Y_F \, dt_Y}{dR_Y} = \frac{1 - \sigma}{1 + \frac{t_Y}{q_Y} \left[ \sigma \epsilon_D + (1 - \sigma) \epsilon_F \right]}.
\]  

Note that LMC_y = \sigma < 1 when R_y = t_y = 0—i.e., the first dollar of revenue from the tax on the traded good costs only \sigma. For higher levels of revenue (R_y > 0), LMC_y will be greater than \sigma if the demand elasticities \epsilon_D, \epsilon_F are negative. In general, of course, these demand elasticities and the domestic and foreign shares of good Y consumption need not be constant as t_y, or R_y, vary. However, in the special case where these parameters are constant, and \epsilon_D < 0 > \epsilon_F, both LMC_y and MER_y will be increasing functions of R_y; as R_y rises t_y rises and the denominators get smaller. Figure 1 portrays a rising LMC_y curve.

Consider now the locality’s optimal mix of taxes. Whatever level of revenue is required, it should be raised at minimum cost. Hence, quite analogously to a multi-plant firm, the curve LMC, which is the horizontal summation of LMC_x and LMC_y, shows the marginal cost of public funds when the local tax structure is optimized. If sufficiently low levels of revenue are required (R \leq R_0), only the tax on good Y should be used, with the purely domestic good untaxed. If a higher level of revenue
is needed, then a mix of taxes from both revenue sources is optimal. For example, suppose that \( D \) represents a demand curve for local public expenditure.\(^2\) Then \( R^* \) is the equilibrium level of public spending, which is optimally financed by \( R_X^* \) dollars from the taxation of good X and \( R_Y^* \) dollars from taxation of good Y. Formally, the optimal tax mix is set so that \( LMC_X = LMC_Y \), or

\[
\frac{1}{1 + \frac{t_X}{q_X} \epsilon_X} = \frac{\sigma}{1 + \frac{t_Y}{q_Y} [\sigma \epsilon_D + (1 - \sigma) \epsilon_F]}
\]

(8)

Let us now examine the way that tax exporting influences the demand for local public spending. It is most instructive to consider a series of cases of gradually increasing complexity and generality.

**Case 1: \( \sigma = \epsilon_X = 0 \).** Note that this is essentially the case considered by Mieszkowski and Toder. Like most writers who have analyzed the demand for public expenditure by lower-level governments, they do not take explicitly into account the excess burdens arising from distortionary taxes. Thus, they implicitly assume that \( \epsilon_X = 0 \), so that \( LMC_X = 1 \), as shown in Figure 2. Moreover, they assume that all of the traded good is purchased by non-residents, so that \( \sigma = 0 \). This implies that \( LMC_Y = 0 \) up to \( R_{\text{max}} \), the maximum revenue obtainable from taxation of the traded good. (Note that \( t_Y/q_Y = \epsilon_F^{-1} \) at a revenue maximum.) At that point, \( LMC_Y \) becomes infinite. This is shown by the vertical line at \( R_{\text{max}} \) in Figure 2. Thus, the local marginal cost of public funds is zero up to \( R_{\text{max}} \) and $1 thereafter, as shown by the heavy line. If the demand curve for local public spending is \( D \), then the local marginal cost of public funds is unaffected, in the relevant range, by the existence of tax exporting; more than \( R_{\text{max}} \) will be spent, and each incremental dollar of spending above \( R_{\text{max}} \) costs the locality a full dollar. Hence tax exporting does not stimulate public spending at the margin in the MT case. Of course, there is an income effect that operates here. Since the inframarginal units of funds are obtained at less than full cost, the locality has higher real income, and this can result in higher demand for local public goods—depending on their income elasticities. However, if we assume a zero income

![Figure 2](image-url)
elasticity of demand for local public goods, the equilibrium level of spending in the M-T case, \( R^* \), is identical to what would occur if taxation of the exported good were prohibited.

Case 2: \( \sigma = 0 > \epsilon_X \). Let us now generalize the M-T framework by allowing for a non-zero elasticity of demand for the non-traded good, \( X \). Now the \( LMC_X \) curve rises from \$1, and the \( LMC \) curve, shown in Figure 3, will also be rising beyond \( R_{max} \). With a demand curve \( D \), the equilibrium level of spending is \( R_e \). If this same level of spending were to be supported without any use of the exported tax (i.e., if taxation of good \( Y \) were prohibited), the \( LMC \) curve would coincide with \( LMC_X \), and \( R_e \) units of expenditure would cost \( LMC_e \) at the margin rather than \( LMC_e \). Therefore, the equilibrium level of spending would be lower, even abstracting from the income effect of tax exporting. For example, if the income elasticity of demand for the public good were zero, the equilibrium with no tax exporting would occur at the spending level \( R_e'' < R_e \). Note that the locality exports the maximum possible amount of tax burden here, and no exporting is occurring at the margin.

It is important to understand clearly why tax exporting stimulates local public spending in this case. Simply put, the fact that localities need not exploit their distortionary own-source revenues as highly in the presence of tax exporting, and the fact that the marginal cost of obtaining such revenues is an increasing function, means that the marginal cost of funds will be lower at each level of public spending. It is this downward shift of the local marginal cost curve that induces a higher level of spending. Since this effect is critically dependent on the fact that the locality is using a distortionary tax to finance its expenditures, let us henceforth refer to it as the "distortion reduction" effect of tax exporting.

It is interesting to remark here in passing that if the locality receives a lump-sum grant from a higher-level government, this too will shift downward the local marginal cost of public funds. This could provide still another explanation for the so-called "flypaper effect." (See, e.g., Gramlich (1977), Oates (1979), Hamilton (1983) on this subject, and Wildasin (1986) for additional references.)

Case 3: \( \sigma > 0 = \epsilon_X \). So far, we have analyzed the effect of tax exporting with no local consumption \( (\sigma = 0) \) without \( (\epsilon_X = 0) \) and with \( (\epsilon_X < 0) \) distortionary taxes on non-traded goods. Now let us turn to the cases of greatest present interest, those in which residents also consume the traded
good (σ > 0). First, let us focus on the simple special case where \( \varepsilon_X = 0 \), so that \( LMC_X = 1 \). Figure 4 shows that the LMC curve coincides with \( LMC_Y \) up to \( R = R_Y^0 \), whereupon LMC is just fixed at $1. Here the locality raises its tax on the traded good from zero up to, say, \( t_Y^0 \), the tax rate at which \( LMC_Y \) rises to $1. At that point, all additional revenue requirements are met by taxation of non-traded goods, at a local marginal cost of $1. At \( t_Y^0 \), we have \( LMC = 1 \) or, by (6),

\[
\sigma = 1 + \frac{t_Y^0}{q_Y^0} [\sigma \varepsilon_D + (1 - \sigma) \varepsilon_F]. \tag{9}
\]

If the demand curve for local public spending is \( D \), the equilibrium level of spending is \( R_e \). If there are no income effects on the demand for local public goods, \( R_e \) is also the equilibrium when taxation of the traded good, and hence tax exporting, is disallowed. This conclusion parallels the result of Case 1 illustrated in Figure 2. There is a very important difference between the two cases, however. In Figure 2, the maximum amount of tax burden is shifted to outsiders, in equilibrium, and there is no possibility of exporting additional taxes at the margin. In Figure 4, that is no longer the case; there, the equilibrium level of public spending is unaffected by the ability to export taxes, even though each additional dollar of revenue raised by taxation of the traded good imposes some burden on outsiders. Indeed, the marginal export rate, \( MER_Y \), can be determined from (7) and (9) as

\[
MER_Y = \frac{1 - \sigma}{\sigma}. \tag{10}
\]

For instance, let \( \sigma = .5 \). Then the marginal dollar of revenue raised via the tax on the traded good will impose a burden of one dollar on non-residents. Even so, the demand for local public spending is unaffected by tax exporting. Indeed, to take a more extreme case—one close to the Mieszkowski-Toder world—suppose \( \sigma = .01 \). Then \( MER_Y = 99 \), i.e., the last dollar of revenue collected via the traded good tax imposes a $99 burden on non-residents. Still the demand for local public goods is unaffected.

This analysis clearly shows that the presence of significant tax exporting, even at the margin, does not by itself imply that the local marginal cost of public funds is reduced, nor does it imply that the demand for local public goods will be increased.
Case 4: $\sigma > 0 < \varepsilon_x$. Finally, let us return to the general case portrayed in Figure 1. It is true that the equilibrium level of public spending will be higher on account of tax exporting, even in the absence of income effects on the demand for local public goods. The reason for this is identical to that given in Case 2: tax exporting makes it unnecessary to exploit the non-traded goods tax base as heavily as would otherwise be the case, and hence the marginal cost of public funds from taxation of non-traded goods, $\text{LMC}_x$, will be lower. That is, the distortion reduction effect will be operative.

III. Implications of the Analysis

The analysis of Section II (and of the Appendix) makes it quite clear that tax exporting may, but need not, lower the relative price of public goods to the spending jurisdiction. The traditional view that tax exporting stimulates spending has led to the inclusion of proxy variables for tax exporting in analyses of the demand for local public spending—see, e.g., Ladd (1975) or Zimmerman (1983). However, the foregoing discussion indicates that the interpretation of the coefficients on such variables is somewhat problematic. It seems useful, therefore, to recall the ways that tax exporting might influence public spending, and to consider how one might attempt to detect their presence in the data:

(i) Income Effects. If the income elasticity of demand for local public spending is positive, then the shifting of some of the burden of local taxes to non-residents may stimulate local public spending. Note, however, that estimation of this effect requires one to measure the total cost savings that accrue to residents from the use of tax exporting. It is important to realize that the portion of local taxes falling on non-residents at the margin is not relevant as a determinant of local public spending. If the income elasticity is small (or negative) the income effect on local public spending will be small (or negative) as well.

Note also that one locality's gain of real income from tax exporting is another locality's loss. Indeed, when the taxes on traded goods are distortionary, there will be an overall loss of real income to the economy as a whole from tax exporting. Hence, a system of tax exporting by many jurisdictions could actually result in income effects which reduce the level of public spending in each and every locality. A reduction in public spending because of the adverse effects of a system of tax exporting would be difficult to observe empirically if one is dealing, as is usually the case, with a cross-sectional sample of jurisdictions. This is unfortunate, because a system-wide framework is the appropriate one to use for some purposes of policy analysis. For example, suppose that a state or group of states is considering the imposition of policies which would greatly reduce the ability of localities to export their taxes. (Prohibition of the use of certain taxes by localities would be one way to do this.) The net effect of such a policy change might be to increase, not decrease, local public spending.

(ii) Reduction of Distortion Effects. Examination of Cases 2 and 4 of Section II, and comparison of these with Cases 1 and 3, indicates the potential importance of the "distortion reduction" effect of tax exporting. Here, tax exporting does lower the local marginal cost of public funds, but not because a portion of the incremental dollar of revenue is exported. Rather, it lowers the local marginal cost of funds because it allows the jurisdiction to rely less heavily than would otherwise on an increasing-cost source of funds. Indeed, in Case 2, no taxes at all are exported at the margin, the entire burden of a marginal dollar of public spending falls only on residents, and there may not even be any income effect on local public spending because of a zero income elasticity of demand. Despite all this, tax exporting may still stimulate the demand for local public goods.

What empirical parameters would determine the magnitude of this effect? Obviously, the price-elasticity of demand for public goods is of prime importance here. In addition, the slope of the $\text{LMC}_x$ curve, which depends critically on the elasticity
Demand for the non-traded good, is of major importance. If $e_x = 0$, as is generally implicitly assumed in empirical work, $LMC_x = 1$ and the distortion reduction effect does not exist. By contrast, if $|e_x|$ is large, tax exporting can have a large effect on the local marginal cost of public funds. Of course, the size of the effect depends also on $R_Y$, since this is the amount of revenue that it is not necessary to collect via the distortionary tax on the non-traded good. The equilibrium value of $R_Y$ depends, in turn, on the elasticity of demand for the traded good. It would seem important for empirical analysis, therefore, to be able to quantify the elasticities of demand for all taxed goods. Ceteris paribus, lower demand elasticities for domestic taxed goods would imply a smaller stimulus from tax exporting, while the opposite would be true for traded goods.

IV. Conclusion

The analysis in this paper has shown that tax exporting may, but does not necessarily, stimulate local government spending. Indeed, as remarked above, it is possible that tax exporting by a system of jurisdictions might actually reduce local public spending through the negative income effects associated with the deadweight loss from the distortion of trade. The analysis has also shown that empirical estimation of the effect of tax exporting is a rather subtle business. For example, among the important determinants of this effect one must include the elasticity of demand for non-traded taxed goods. Although no empirical studies have been made so far using such a specification (to the author’s knowledge), such work would appear to be feasible.

Let us conclude by mentioning some of the public policy implications of this analysis. First, one cannot infer, simply from observation of the level of tax exporting (at the margin and a fortiori in total), that local governments are over-spending. Indeed, in the most commonly-used analytical frameworks (those which ignore the distortions caused by local taxes on non-traded goods), tax exporting would not result in efficiently high levels of local public expenditure. Hence, one would not wish to use the a priori claim of over-spending to justify policies that restrict tax exporting, nor would one wish to use this claim to justify policies aimed directly at reducing local spending.

Second, although this has not been a main theme of the paper, the analysis has shown that localities may exploit taxes on traded goods in a way that is socially very wasteful. A jurisdiction might well set a tax rate on a traded good at a level that results in a very high social marginal cost per dollar of revenue raised. This is the consequence of rational local behavior. Obviously, the existence of such distortions might call for policies to restrict the amount of tax exporting.

Finally, it should be mentioned explicitly that this paper has not been concerned at all with the distributional impact of tax exporting. This is of course an important topic and raises many interesting policy issues in its own right.

FOOTNOTES

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¹In order to provide a very concrete illustration of some of the more unusual results that can emerge from the analysis, an Appendix presents an explicit example in which a tax-exporting locality chooses its optimal level of public spending. This example has the property that in equilibrium, a very high percentage—e.g., more than 100 percent—of each additional dollar of revenue raised from taxation of a traded good is borne by non-residents. Despite this, the level of public spending, and the local marginal cost of public funds (in the relevant range) is identical to what would occur if taxation of the traded good, and hence tax exporting, were altogether ruled out.

This paper follows previous analyses by assuming the existence of such a curve. Certainly the assumption is rigorously justifiable for the special case where all of the households in the locality are identical and immobile. For the purposes of the present analysis, it is simplest to abstract from the problems created by more complicated local public choice environments.

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Appendix

The analysis in the text of this paper has been cast in somewhat informal terms. It relies, for example, on an intuitive appreciation of the welfare effects of distortionary taxation, and on various simplifying assumptions about preferences such as absence of cross-price effects, etc. The logic of the analysis is rigorous. However, skeptical readers may find a more formal approach more convincing and perhaps easier to understand. Therefore, this Appendix presents a detailed treatment of a simple example which illustrates some of the key ideas of the general analysis of Section II of the text. This example has the following properties. First, the utility function is quasi-linear in a numeraire private good. As is well known, this means that the income elasticities of demand for other goods, including in particular the income elasticity of demand for the public good, are zero.

Second, there is a non-traded taxable private good with a perfectly inelastic demand. Third, there is an exported taxable good which is also locally consumed. The first of these properties implies that the income effects of tax exporting do not affect the demand for the public good. The second property implies a constant marginal cost of $1 from domestic (unexported) taxation. Hence, no distortion reduction effect from tax exporting can arise. As the general analysis would lead one to expect, and as emerges explicitly in the example, the upshot is that tax exporting does not increase public spending at all. This is true even though an arbitrarily high proportion of the burden of the tax on the traded good is exported at the margin, in equilibrium.

To proceed with the example, then, imagine a jurisdiction containing a single household (or many identical households). Suppose this household has a perfectly inelastic demand for non-traded good \( X \), with a fixed demand. Suppose also that this household has a utility function defined over a traded good \( Y \), a non-traded numeraire good \( r \), and a local public good \( z \), of the form

\[
 u(r, y_D, z) = r - \frac{y_D}{\Theta} + \phi(z), \tag{A.1}
\]

where \( \Theta > 0 \) and \( \phi'(z) > 0 > \phi''(z) \). This is a well-behaved utility function, as is easily verified. Suppose the household is endowed with \( \bar{r} \) units of good \( r \) (its income), so that it faces a budget constraint

\[
 \bar{r} = r + q_XX + q_YY_D. \tag{A.2}
\]

Assume that \( \bar{r} \) is sufficiently large that the household is able to purchase positive quantities of all goods in equilibrium. Maximization of (A.1) with respect to \( r \) and \( y_D \), subject to (A.2), conditional on \( z \), yields a demand function for \( y_D \):

\[
 y_D = q_Y^{-1/\Theta}. \tag{A.3}
\]

from which it follows that the domestic demand elasticity for \( y_D \) is \( e_Y = \frac{1}{1 + \Theta}^{-1}. \) One can compute the indirect utility function directly from (A.1), (A.2), and (A.3) as

\[
 v = \bar{r} = q_XX - \frac{1 + \Theta}{\Theta} q_Y^{\Theta/1+\Theta} + \phi(z). \tag{A.4}
\]

Note that \( \partial v/\partial x = -x \) and \( \partial v/\partial e_Y = -y_D. \) If taxation of the traded good \( Y \) is disal-
Allowed, the locality faces the revenue constraint
\[ t_X x = z \]  
(A.5)

if we assume that the marginal cost of production of \( z \) is constant and equal to unity. Then it is easily verified that the locality's optimal level of \( z \), which maximizes \( u \) subject to (A.5), will be \( z^* \) such that

\[ \phi'(z^*) = 1. \]  
(A.6)

Given the assumption on \( \phi \), \( z^* \) is uniquely determined.

If, on the other hand, a tax can be imposed on the traded good \( Y \), the government revenue constraint is

\[ t_X x + t_Y (y_D + y_F) = z, \]  
(A.7)

where \( y_F(q_Y) \) is the foreign (net) demand for good \( Y \). Suppose that \( y_F(q_Y) = \alpha q_Y^\sigma \) so that \( \epsilon_F = -\rho \) is the foreign demand elasticity. To simplify matters, let \( \rho = (1 + \Theta)^{-1} \) so that \( \epsilon_D = \epsilon_F = \epsilon < -1 \) and \( \sigma = (1 + \alpha)^{-1} \).

Maximization of (A.4) subject to (A.7), ignoring any non-negativity restriction on \( t_X \), yields (after some manipulation)

\[ t'_Y = \frac{p_Y(1 - \sigma)}{\rho - (1 - \sigma)} > 0 \]  
(A.8)

as the optimal tax rate on the traded good, independently of the level of \( z \). One can then use (A.7) to solve for \( t_X \). Define \( z' \) by

\[ z' = t'_Y (1 + \alpha)(t'_Y + P_Y)^{-\sigma}. \]

Then for all \( z > z' \), the tax rate \( t_X \) on good \( X \) must be positive if \( t_Y \) is optimally set. Suppose, in particular, that \( z^* > z' \). The function \( \phi \) can always be chosen so that this is the case—e.g., by adding a term \( \beta z \) to the function, with \( \beta \) sufficiently large.

From (A.8) it is now obvious that the first \( z' \) units of public good will be financed by taxation of the traded good only. Any additional units of the public good will be financed by taxation of the non-traded good. It is therefore clear, and can be verified, that choosing \( z \) to maximize the indirect utility function \( v \) in (A.4) subject to the government budget constraint (A.7) will yield (A.6) as the first-order condition, given the assumption that \( z^* > z' \). Hence, \( z^* \) is still the unique optimal level of public good provision, even in the presence of tax exporting.

Finally, observe that when \( t_Y = t'_Y \), the marginal export rate on the traded good (applying (10) in the present particular case) is

\[ \text{MER}_Y = \frac{1 - \sigma}{\sigma} = \alpha. \]

In our example \( \alpha \) is still a "free" parameter in the sense that it can be set at any positive value without invalidating other assumptions. Thus, for example, we could have \( \alpha = 1 \), meaning that \$1 of burden falls on non-residents for each additional dollar of revenue raised by the locality. \( \alpha = 1 \) of course corresponds to the case where half the locality's output of the traded good is sold to non-residents. If a much higher proportion of the good were exported—e.g., 99 percent—then the marginal export rate would be 99. Clearly, the burden on non-residents per dollar of revenue raised can be made arbitrarily large, within the limits of the example.