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Public facility location and urban spatial structure

Equilibrium and welfare analysis

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We consider a model in which the location of a single public facility is fixed somewhere in an urban area. There are two firms that choose locations; the locations of households, competing for space in the land market, are also endogenous. The analysis examines the nature of the spatial equilibrium and shows that different kinds of equilibria can emerge depending on the parameters of the model. The welfare implications of changes in transportation cost and the location of the public facility are studied, and appear to be non-standard for some equilibrium configurations.

1. Introduction

Many public sector activities occur at specific locations in urban areas, and many of them are fixed in particular locations for long periods of time. Examples abound: major transportation nodes (such as train stations,

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airports, or harbor facilities), city parks, schools, museums, hospitals, and government administrative buildings are frequently kept in fixed locations for decades at a time or even longer. Clearly, it is often of value to firms or individuals to be located in close proximity to such facilities because it is necessary or desirable to take advantage of the services that they provide. Indeed, the most casual observation suggests that the location of public facilities can have a substantial impact on private sector locational choices. This raises issues that are of importance both for positive economics and for policy evaluation.

From the viewpoint of the positive economics of urban spatial structure, one would like to know exactly how the location of public facilities interacts with the locational choices of households and firms. It is natural to conjecture that the presence of a public facility would serve as an agglomeration point for private economic activity, leading to the clustering of firms and households around the facility. But locational problems inherently involve strategic interdependence, making them notoriously difficult to analyze. We know from prior research, such as that of Fujita and Thisse (1986), that the spatial equilibrium of the private sector considered in isolation exhibits certain tendencies for dispersion: in particular, when one takes into account the strategic interactions of neighboring firms engaged in Hotelling-type spatial competition, they may not find it advantageous to agglomerate. Would the presence of a public facility of some kind change the nature of the spatial interaction of firms sufficiently to draw them together into a closer spatial relationship?

The effect of public facility location on private sector behavior is also important from the normative viewpoint, sometimes in ways that are obvious and sometimes in ways that are more subtle. The location of a public facility is itself a policy decision, and one which is often of intense interest to at least some parties – landowners, developers, persons interested in public investment as an instrument for dealing with urban blight, etc. As is always true in the economic evaluation of public policy, it is necessary to understand the effect of the policy on private sector behavior if one is to develop a comprehensive accounting of its social benefits and costs. In the present instance it is apparent that a proper analysis of the implications of public facility location requires consideration of the effect of this decision on the spatial structure in an urban area. Moreover, because the presence of public facilities may influence the nature of the spatial structure in an urban area, they may also influence the evaluation of other policies whose impact is likely to be sensitive to this structure. In particular, a full evaluation of a transportation improvement project (e.g. road improvements, introduction or expansion of public transportation) requires that the effect of the project on urban structure be taken into account.

Despite the obvious potential importance of public facility location for urban spatial structure, these issues – perhaps because of the analytical complexity involved – have attracted surprisingly little attention in the literature.[1] The objective of the present paper is to present a model which permits us to address the above issues. We begin with a framework rather similar to that found in Fujita and Thisse (1986). In this model, as in Hotelling (1929), two firms (e.g. supermarkets selling private goods on a large scale) must locate in an urban area in which there is a continuum of consumers. But, unlike Hotelling, the Fujita–Thisse model allows the locations of households to be endogenously determined: households choose locations by purchasing land in a competitive market, conditional on the locations of firms. The model that we analyze in this paper preserves these basic features of the Fujita–Thisse model, but it adds one more element: the presence of a public facility to which households wish to have access. We imagine that firms make their locational decisions taking the location of the public facility as given. Thus, we think of the public facility location decision as being more or less permanent in nature; the facility provides a sort of spatial reference point for subsequent private locational decisions.

Our model allows for two qualitatively rather different possibilities with respect to public facility location. One possibility is that the public facility is located somewhere in a large 'featureless plain', such that land of homogeneous quality is available, at a constant cost, at considerable distance from the facility. This means that natural topographical boundaries play no role in the analysis. The second possibility is that the facility is located near the edge of the land available for urban use. We have in mind here primarily the possibility that the urban area is on the coast or shore of some body of water. This is of course a situation of considerable practical importance. While such natural features are generally irrelevant in standard monocentric city models, they are important in our model because, in conjunction with

[1] In some sense, the literature of local public finance revolves around the spatial variation of the level of public good provision. However, spatial aspects of local public goods are seldom explicitly considered; rather, somewhat in the spirit of international trade, the basic spatial structure is taken as exogenously given in the form of a set of political entities with predetermined boundaries. A few authors, such as Polinsky and Shavell (1976), Kanemoto (1980), Brueckner (1979), and Hochman (1982a, b), have analyzed the implications of spatial variation in public service provision within an urban area. With few exceptions [e.g. Hochman (1982b)], most studies in this vein assume that public services are distributed smoothly over space, rather than being provided at just one or a few discrete entities in the urban area; the latter approach, however, is more natural if one is interested in public facility location. Moreover, this literature does not consider the impact of public facility location on the interdependent locational choices of firms and households, restricting attention instead to the effect of public service provision on spatial structure in the residential sector of an urban area. For further references to this literature, see Wildasin (1986, 1987). Hansen et al. (1983) and ReVelle (1987) survey a number of studies dealing explicitly with planning and policy issues associated with public facility location. Private sector locational decisions are generally taken as exogenous in this literature, however, whereas the endogeneity of such behavior is essential for the issues that we wish to investigate.

the public facility, they result in spatial asymmetries that have a significant impact on the locational interactions between firms.

Section 2 of the paper describes the basic framework for the analysis, introducing the notation and discussing somewhat informally the determination of equilibrium. In section 3 we present the main results on the existence and characterization of spatial equilibrium in the model. We show that the qualitative properties of the spatial equilibrium depend on several parameters of the model: the level of transportation costs (relative to income), the cost of travel to and from the public facility relative to the cost of travel to private firms, and whether or not the public facility is located far from or close to any natural boundary of the urban area. There is definitely a tendency for the public facility to draw the private firms together, at least by comparison with the Fujita–Thisse case where there is no public facility. However, the spatial equilibrium is by no means necessarily fully agglomerated in the presence of a public facility. We show, in fact, that a range of outcomes is possible. Depending on the values of critical parameters, the firms might locate at extreme ends of the urban area, they might locate together, or they might choose intermediate locations.

There are many policies that could affect the basic parameters of the model. Of course, the location of the public facility itself is a basic policy decision. Furthermore, transportation costs depend heavily on a host of public policies, including investment in transportation infrastructure, pricing of public transportation, and policies affecting the price of fuels. The analysis of section 3 shows that these parameters affect the nature of the equilibrium spatial structure for the region. Section 4 builds on this analysis to explore the welfare implications of changes in transportation costs and public facility location. There are three groups of agents who may be affected by such parametric changes: consumers, the owners of firms, and the owners of land. In general, the welfare of all three groups depends on the equilibrium spatial configuration of the urban area. We find that changes in transportation costs and public facility location often cannot be Pareto-ranked, and we identify the gainers and losers. We show that the magnitude and sometimes even the direction of the welfare effect of a parametric change on a particular group may depend on the precise nature of the spatial equilibrium, thus highlighting the importance of spatial considerations for policy evaluation. One interesting result that emerges from the analysis is that a reduction in per unit transportation costs, even if it could be obtained at zero resource cost, could reduce aggregate real income. This is because it changes the equilibrium spatial structure and hence the pattern of travel in the metropolitan area. By the same token, a transportation toll could be introduced which, in addition to raising revenue for the public sector, would induce a change in equilibrium spatial structure that would also raise private sector aggregate real income. Results of this type show that policy analysis is highly sensitive to the underlying equilibrium model of locational choice. In particular, models that assume that locations are invariant to such policies can give quite misleading results.

Section 5 concludes the main discussion with some summary remarks and a discussion of some of the many ways that our analysis might be extended. An appendix contains the proofs of most of the main results of the paper.

2. The model

As indicated above, the model that will be studied here is similar to that found in Fujita and Thisse (1986) in a number of respects, differing most importantly by the inclusion of a public facility. The basic set-up is as follows.

First, there is a continuum of identical households who must locate in a given region; the size of the total population is given and equal to N. The region, represented by a bounded interval $[0,\ell]$, consists of land of homogeneous quality (except for the presence of public and private facilities at some points, as determined below). We identify locations within it with points in the interval $[0,\ell]$.

Households consume three different types of goods. There is a private good, sold at a given, fixed price (which is taken as numéraire), land, and a public good or service.[2] The private good is considered as essential in that the marginal utility of this good tends to infinity when its consumption goes to zero. The demand for land is perfectly inelastic – i.e. each household consumes a fixed amount of land, the same for all households. Without loss of generality, assume that this demand is such that an interval of the region of length $\alpha \leq \ell$ can accommodate the proportion α/N households. We shall assume that $N<\ell$, so that not all land in the region will be occupied – residential demand will fall short of the available supply. In this case we suppose that the land is allocated to some alternative use, such as agriculture, at which it earns a fixed rent ρ. We refer to the portion of the region that is occupied as the 'urban area'.

Households benefit from the provision of a public good or service in the urban area. We shall assume that households must travel to the place where the public good is provided in order to obtain this benefit. We make the simplest possible assumption in this regard: each household must make exactly one trip to the facility (per unit of time). In order to focus the

[2]The assumption of a common, given price is not innocuous since the same physical goods at different locations are different economic goods which could possibly have different prices. This implies that prices are chosen by some mechanism left outside of the model. Clearly, it would be preferable to deal directly with price competition, but models of spatial competition are often plagued by non-existence of an equilibrium in pure strategies. There could be some hope, however, of solving this problem if goods were differentiated and consumers were heterogeneous in tastes [see Anderson et al. (1992)].

discussion on locational issues, it is assumed that the level of service has already been determined, and so may be ignored for the purposes of our analysis. As examples of the type of facility that we have in mind, one might consider public schools, transportation nodes (such as train stations, seaports, or airports), libraries, parks or other recreational facilities, hospitals, municipal government centers, etc. Each household incurs a travel cost in going to the public facility equal to a positive constant t_y times the distance between the household's residence and the location of the facility. (If desired, t_y could be interpreted as the cost of bringing the service to the household, with increasing cost as a function of distance indicating deterioration of the level or quality of the service. Under this interpretation, the model could describe facilities such as police and fire stations or hospitals with emergency facilities, with the transportation cost t_y representing, for example, the deterioration of response time as the distance to the household being served increases.) The location of the public facility in the interval $[0, \ell]$ is denoted y. For simplicity, it is assumed that the amount of land required by the facility itself is sufficiently small that it can be ignored.

Households obtain the numéraire consumption good (or just 'consumption good') from one of two firms located in the urban area. In this respect the model follows Fujita and Thisse (1986). Each household must travel to one of the firms (per unit of time), incurring a cost in doing so equal to a positive constant t_x times the distance between the household's residence and the firm.[3] A household's consumption of this good will be equal to its income, exogenously given as Y_a, the same for all households, minus transportation costs and land rents. Travel for the purpose of purchasing the consumption good is assumed to be independent of travel to the public facility, so that there is no question of combining travel into multi-purpose trips. If a household is located at some point $\xi \in [0, \ell]$, if it travels to firm i located at point $x_i \in [0, \ell]$ to buy the consumption good, and if the land rent at its location ξ is $R(\xi)$, then its purchases of the consumption good will be given by

$$z(\xi) = Y_a - t_x|\xi - x_i| - t_y|\xi - y| - R(\xi). \tag{1}$$

Since the demand for land is perfectly inelastic, and since the level of public service provision is being treated as a constant, the only variable on which a household's utility depends is its consumption of the numéraire commodity. Thus, household behavior can be described simply in terms of maximizing z. An immediate implication is that households will buy the consumption good from the nearer firm. If the two firms are equidistant (for

[3]This cost depends only upon distance; in particular it does not depend on the quantity of the consumption good purchased from the firm [see Stahl (1987) for a discussion of this assumption].

example, when they are located at the same point), we assume that households choose randomly between them, and that each firm will obtain a one-half share of all such customers.

The two firms in this model are profit-maximizers. Each sells the numéraire output to households and incurs constant and identical marginal costs of production. Without loss of generality, we can assume that this marginal cost is zero. Each firm must choose a location in the urban area, and, as just described, it will then serve all of those households for whom it is the closer supplier plus one-half of those households for whom it and the other firm are equally distant. The firms are assumed to require negligible amounts of land for their establishments. They are owned by agents other than the households living in the urban area.

Land itself is owned by rent-maximizing absentee landlords who rent their land to the highest bidder. Landowners are assumed to behave competitively.[4]

An equilibrium in this model is the outcome of two stages or levels of private sector decision-making, conditional on the location of the public facility. In the first stage, the two firms choose their locations. In the second stage, households choose locations, bidding for land and determining an equilibrium land rent profile.

It should be emphasized that the public facility plays a very different role in the model from the private firms, even though households face similar requirements for travel to both. The crucial distinction between them is that the locations of the private firms are determined by independent profit-maximizing behavior, whereas this is not true of the public facility. While profit maximization is a reasonable decision rule for private sector decision-makers, the same is not true for the location of public facilities. Therefore, we treat the public facility location as parametrically fixed throughout the analysis, whereas the locations of private firms are driven by profit maximization and are endogenously determined. Furthermore, the sequential structure that we have assumed is very appropriate for many public facilities. The locations of many important public facilities are fixed for long periods of time. There is therefore much interest in examining the equilibrium private sector response to long-term public facility locational choices. Our model enables us to undertake this type of analysis.

To define an equilibrium precisely, it is helpful to describe both of these stages of decision-making in detail, working from the second stage back to the first. Thus, when considering household behavior, we take the location of the public facility y, and the locations (x_1, x_2) of the two firms as already determined. A household at any given location will buy the consumption

[4]The assumption of absentee landlords, like the assumption of absentee owners of the firms, is made in order to avoid the technical complications that arise as rents and profits, by affecting incomes, feed back into the determination of consumer demands.

good from the nearer firm, as noted above. For any given structure of land prices in urban area, households will prefer those locations that provide the highest level of utility, i.e. those at which z as given by (1) is maximized. In equilibrium, therefore, the usual spatial arbitrage argument implies all locations that are occupied by households must have rents that allow a common level of utility to be achieved – i.e. in equilibrium, z must be the same for all households. This implies that the equilibrium land rent structure must mirror the structure of transportation costs – where transportation costs are low, more intense competition for land will drive up land prices, and conversely where transportation costs are high. All occupied locations must have transportation costs no higher than all unoccupied locations.

To state these equilibrium conditions more precisely, define

$$t(\xi; x_1, x_2, y) \equiv \min_{\langle x_1, x_2 \rangle} \{t_x|\xi - x_1| + t_y|\xi - y|, t_x|\xi - x_2| + t_y|\xi - y|\};$$

this is the transportation cost incurred by a household located at ξ given the location of the firms and the public facility. The last three arguments may be suppressed when there is no ambiguity. Note that $t(\xi)$ is piecewise linear in ξ and continuous. Define

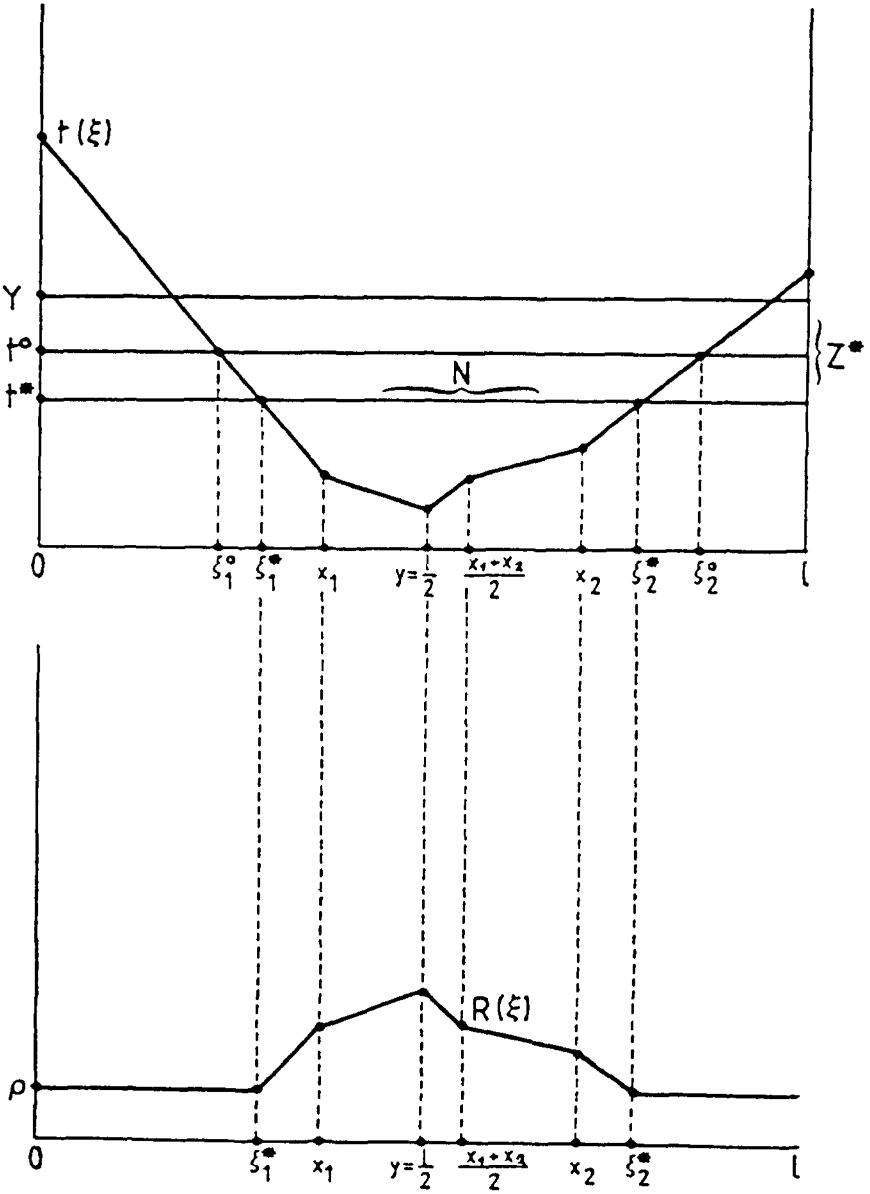
$$M(t) \equiv \mu(\xi \in [0, \ell] \mid t(\xi) \leqq t),$$

where μ denotes Lebesgue measure; thus $M(t)$ is the measure ('length') of all those locations in the region that have transportation costs no greater than t. It is obvious that $M(t)$ is monotonically increasing in t, that $M(0) = 0$, and that M has a maximum of ℓ that is attained at some value of $t_{\max} \leqq (t_x + t_y)\ell$, that is, no location can entail a transportation cost higher than the cost of traveling the entire length of the urban area both to a firm and to the public facility. Finally, define the critical value t^* such that

$$t^* = \inf\{t: M(t) \geqq N\}.$$

Thus, the set of points with transportation costs less than or equal to t^* is large enough to accommodate the whole population; for any smaller value of t, this is not the case.

To illustrate, consider fig. 1. It is assumed that the public facility is located at the center of the region (so that $y = \ell/2$) and that the firms are positioned at some $x_1 < y < x_2$ such that $|y - x_i| < N/2$ for both i, with $y - x_1 < x_2 - y$. Fig. 1 assumes that $t_y > t_x$. Total transportation costs at each point in the urban area are shown by the piecewise-linear curve $t(\xi)$. A household located at any location $\xi \in [0, (x_1 + x_2)/2]$ (resp. in $((x_1 + x_2)/2, \ell])$ will minimize transport costs by buying from firm 1 (resp. firm 2). At locations outside the



t(ξ)
Y
t°
t*
N
Z*
0
ξ°1
ξ*1
x1
y=l/2
(x1+x2)/2
x2
ξ*2
ξ°2
l
R(ξ)
ρ
0
ξ*1
x1
y=l/2
(x1+x2)/2
x2
ξ*2
l

Fig. 1

interval $[x_1, x_2]$, movements away from the center cause transportation costs to rise at the rate $t_x + t_y$. Because the cost of travel to the public facility is assumed to exceed the cost of travel to the private firms, $t(\xi)$ falls in the interval $[x_1, y]$ as ξ increases: $t'(\xi) = -(t_y - t_x) < 0$. Similarly, in the interval $((x_1 + x_2)/2, x_2]$, $t(\xi)$ rises as ξ increases: $t'(\xi) = t_y - t_x > 0$. Finally, in the interval $(y, (x_1 + x_2)/2]$, households buy from firm 1 and $t'(\xi) = t_x + t_y$. Thus, the $t(\xi)$ function takes on an overall kinked-U shape with a minimum at $\xi = y$.

For any arbitrary value of t, say t^0, we can find a pair of points (ξ_1^0, ξ_2^0) such that $M(t) = \xi_2^0 - \xi_1^0$. The figure illustrates the case where $N < \xi_2^0 - \xi_1^0$. Thus, there must be some lower value of t, say t^*, such that $M(t^*) = N$ and that determines the interval $[\xi_1^*, \xi_2^*]$. These critical locations determine the equilibrium boundaries of the urban and agricultural areas. All households will live within these boundaries because to live further away would entail incurring higher transportation costs with no lower land rents than can be obtained at the boundaries. The lower panel of fig. 1 shows the equilibrium land rent profile. At the boundaries of the urban area, land rent drops to the agricultural rent ρ. At locations closer to y, land rents rise, reflecting (and exactly negating) the transportation cost saving that households at those locations enjoy. Total land rents in excess of the agricultural rent are given by the area under the $R(\xi)$ curve and between the two boundary points ξ_1^* and ξ_2^*. Total transportation costs are given by the area under the $t(\xi)$ curve between the boundaries ξ_1^* and ξ_2^*.

Fig. 2 illustrates some of the complexities that can arise in a less well-behaved case. Here, it is again assumed that $y = \ell/2$. However, let us now suppose that $t_x = t_y$ and that $x_1 = x_1^0$, $x_2 = x_2^0$ such that $x_2^0 - y > y - x_1^0 > N$. Any household at a location $\xi \in [x_1^0, y]$ will minimize transport cost by buying from firm 1; since $t_x = t_y$, $t(\xi)$ is constant over this interval. Since firm 2 is more distant from y than firm 1, $t(x_2^0) > t(x_1^0)$. Any household buying from firm 2 and located in the interval $[y, x_2^0]$ will bear a transport cost equal to $t(x_2^0)$, again because $t_x = t_y$. Households located just to the right of y will find it cost-minimizing to buy from firm 1, but $t(\xi)$ rises at the rate $t_x + t_y$ in this area, until the cost of buying from firm 1 is equal to $t(x_2^0)$ at $\xi = (x_1 + x_2)/2$.

In this example, clearly $t^* = t(x_1^0)$, and $M(t^*) > N$. Thus, no households will locate outside of the interval $[x_1^0, y]$ (and hence firm 2 will have no customers). The interval $[x_1^0, y]$ is larger than necessary to accommodate the whole population and, since all locations in the interval have identical transportation costs, the assignment of households to particular locations in the interval is indeterminate. Since there is an excess supply of locations at which transportation cost is minimized, equilibrium land rents at occupied locations are just equal to the reservation agricultural rent ρ. In this case the equilibrium land rent gradient (not illustrated) is perfectly flat, with $R(\xi) = \rho$

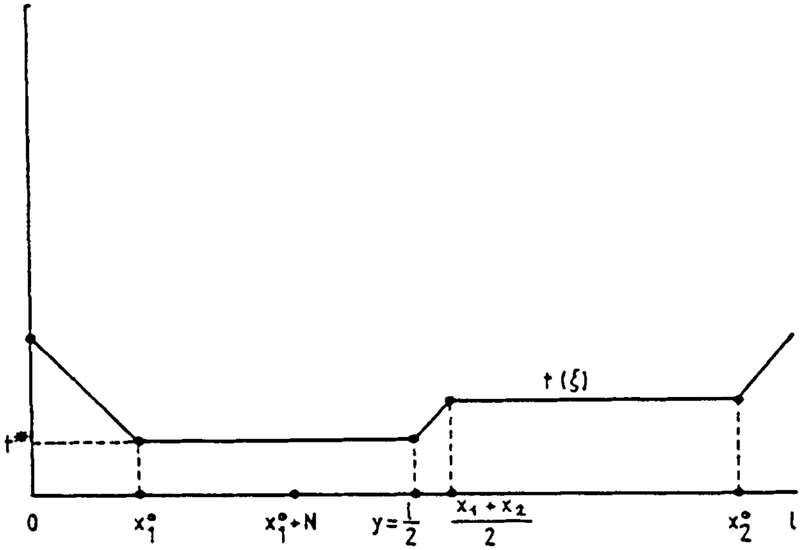


Fig. 2

for all ξ. As we shall see, however, situations such as this will not arise in equilibrium because firms will have incentives not to choose locations in the manner shown there.

In any event, there will necessarily be some border between urban and agricultural land use, regardless of the precise locations of the firms. At such a border, the equilibrium land rent will be equal to ρ, and it will rise to $\rho+t^*$ at the location that minimizes transportation costs. Quite generally, then,

$$R(\xi)=\rho+t^*-t(\xi), \quad \forall\,\xi\in[0,\ell] \quad \text{such that} \quad t(\xi)\leqq t^*. \tag{2}$$

Now define $Y=Y_a-\rho$ and note from (1) and (2) that the equilibrium level of consumption of the numéraire good will be

$$z^*\equiv Y-t^*, \tag{3}$$

the same for all households. It is apparent from (2), and important for the analysis, to note that z^* depends on the locations of the firms, x_1 and x_2, and on the location of the public facility. In particular, z^* depends negatively on x_i if an increase in x_i raises the maximum transportation cost t^* incurred by households, and conversely if it lowers t^*.

So far we have discussed the household locational equilibrium that is achieved conditional on the location of the public facility and on the locational choices of the firms. How do the firms decide where to locate?

A fundamental assumption is that the firms achieve a Nash equilibrium in locations, and that they take into account the effect of their locational choices on the equilibrium location pattern of households. The assumption makes the model suitable for depicting the long-run equilibrium of the two firms, where no adjustments to their decisions will occur that have not been correctly anticipated.[5]

Consider, then, firm i's choice of its location, x_i, taking the location of the other firm, x_j, as given. A necessary condition for profit-maximizing location is that a small change in location cannot raise profits – or, equivalently, revenues, given the assumption of zero marginal production cost. As shown above, all households consume the same amount z^* of the numéraire good in equilibrium. Hence, a firm's revenue will equal z^* times the number of households that buy from it. For firm i, the number of consumers that it sells to is given by

$$\begin{aligned} S_i \equiv \mu(\{\xi \in [0,\ell] \quad &\text{such that} \quad t(\xi) \leqq t^* \quad \text{and} \quad |\xi - x_i| < |\xi - x_j|, i \neq j\}) \\ + \tfrac{1}{2}\mu(\{\xi \in [0,\ell] \quad &\text{such that} \quad t(\xi) \leqq t^* \quad \text{and} \quad |\xi - x_i| = |\xi - x_j|, i \neq j\}). \end{aligned}$$

The value S_i measures the size of firm i's *market area*, that is, the size of the set of households served by the firm. Thus, the total revenue of firm i is given by

$$Q_i = z^* S_i. \tag{4}$$

In fig. 1, S_1 is the size of the interval $[\xi_1,(x_1+x_2)/2]$, which is simply $[(x_1+x_2)/2] - \xi_1$ in this case. This example illustrates a common occurrence, namely the division of the customers between the two firms at the midpoint of the segment defined by their respective locations.

Firm i's objective is to maximize Q_i by the choice of x_i, anticipating firm

[5]This approach to firm and household locational choice is analogous to the Cournot–Walras general equilibrium models pioneered by Gabszewicz and Vial (1972). In these models, firms select quantities, and prices are then established at the Walrasian equilibrium of the corresponding exchange economy. Firms are thus able to determine the 'objective' demand functions relating the quantities they supply to the corresponding equilibrium prices. Using these demand functions, firms compete in quantities, generating a Cournot–Nash equilibrium [see Hart (1985) and Bonanno (1990) for two recent surveys of related literature]. The locations of firms in our model correspond to outputs in Gabszewicz and Vial, while the residential equilibrium, which is influenced by the locations chosen by firms, corresponds to the competitive equilibrium of the exchange economy in the Cournot–Walras models. (One difficulty encountered in the Cournot–Walras models is the non-uniqueness of the competitive equilibrium. This difficulty does not arise here since the residential equilibrium associated with any firm configuration can be shown to be unique [see Fujita (1989, section 3.3)].)

j's choice x_j and taking y as given. By the definitions of z^* and S_i, it is clear that both of these determinants of Q_i may depend on x_i. The effect of x_i on z^* is referred to as the *consumption effect*, and the effect of x_i on S_i is referred to as the *market area* effect.[6] These effects must work in opposite directions at the revenue-maximizing location for the firm.

The key question, then, is how firms make their locational choices, taking the consumption and market areas effects into account. Conditional on the location of the public facility, does there exist a Nash equilibrium in firm locations? The task of the next section of the paper is to determine the conditions under which such equilibria exist, and to characterize their qualitative features.

3. Equilibrium urban structure

We turn now to a detailed analysis of firm locational choice. As a preliminary remark, we note that while it is conceivable that the firms would locate in such a way that their market areas do not touch or overlap, it is not profit-maximizing for them to do so.[7] By contrast, in Fujita and Thisse (1986), firms may space themselves in the region in such a way that each serves its own disjoint interval on the line. This does not happen in the present context because of the presence of the public facility, and the desirability to firms of locating closer to it, other things the same, so as to reduce transportation costs for customers and thus be able to sell higher outputs and to expand market area. Thus, *the public facility provides an additional dimension to the process of interaction between firms and yields at least a minimal incentive for agglomeration in the urban area.*

To analyze further the locational equilibrium of the two firms, consider informally how this might depend on the location of the public facility. Intuitively, if the public facility is located at the center of the region ($y=\ell/2$), one would expect the firms to locate symmetrically about (or perhaps at) that point. On the other hand, the facility might be located at one extreme end of the region (e.g. $y=\ell$). This might be the case, for example, if the city is located on a body of water, and the public facility is a port (or, perhaps, a railroad terminus situated at the waterfront, etc.). It is obvious then that the two firms must both locate on the same side of (or perhaps at the same point as) the public facility. One should expect, therefore, that the qualitative nature of the spatial equilibrium may depend on the location of the public facility. Accordingly, the results that follow are conditional on the value of y.

[6]These terms are defined exactly as in Fujita and Thisse (1986). Indeed, the analysis of that paper would apply to the model presented here if we were to assume that $t_v=0$ and that transportation costs to the private firms are lump-sum.

[7]Details of the arguments appear in Thisse and Wildasin (1990).

They also depend on the nature of transportation costs, and in particular on the relative sizes of t_x and t_y.

It will be convenient to introduce some terms to describe various equilibrium outcomes that can arise in our model. If it should happen that the two firms locate together, and at the same location as the public facility, we shall say that the equilibrium is *fully agglomerated*. If the two firms locate together, but at a location different from the public facility, we shall say that the equilibrium is *partially agglomerated*. If the two firms locate at opposite ends of the urban area, so that all households (and the public facility) are located between them, the equilibrium is *fully dispersed*, and finally if the firms are located apart from each other (and from the public facility), but if they are at locations in the interior of the urban area, we say that the equilibrium is *partially dispersed*.

Our first result concerns the case where the public facility is near the center of the region and where travel to the public facility is at least as costly as travel to the private firms:

Theorem 1. Suppose that $y \in [N/2, \ell-(N/2)]$, *and that* $t_x \leqq t_y$. *Then there exists an equilibrium if and only if* $t_y \leqq 2Y/N$. *If this condition is satisfied, then:*

(a) *if* $t_y \leqq 2Y/N \leqq [(t_x+t_y)^2/t_y]-t_x$, *the firms will be fully dispersed in a symmetric equilibrium about* y *such that*

$$x_1^* = y - \frac{N}{2}, \qquad x_2^* = y + \frac{N}{2};$$

(b) *if* $[(t_x+t_y)^2/t_y]-t_x < 2Y/N < (t_x+t_y)^2/t_y$, *the firms will be partially dispersed in a symmetric equilibrium about* y; *this equilibrium is characterized by*

$$x_1^* = y + \frac{Y}{t_x} - \frac{(t_x+t_y)^2}{t_x t_y}\frac{N}{2}, \qquad x_2^* = y - \frac{Y}{t_x} + \frac{(t_x+t_y)^2}{t_x t_y}\frac{N}{2};$$

(c) *if* $(t_x+t_y)^2/t_y \leqq 2Y/N$, *the firms will be in a fully agglomerated equilibrium, i.e.*

$$x_1^* = x_2^* = y.$$

Proof. See the appendix.

This result shows that there are several possible equilibrium configurations of firms in the model, ranging from the case where they are situated at extreme opposite ends of the market, on the fringe of the urban area, to the case where they cluster together with the public facility at the very center of the urban area. The statement of the results makes clear that which of these outcomes occurs depends critically on the relationship between income and transportation costs. In particular, there is a tendency for firms to move closer together as income rises to transportation costs. In any case, however, the equilibrium will be symmetric. We return to these matters of interpretation later on in the paper.

Consider now the case where $t_y < t_x$, still assuming that the public facility is located away from the boundaries of the region. We have:

Theorem 2. Suppose that $y \in [N/2, \ell - (N/2)]$ *and that* $t_y < t_x$. *Then there exists an equilibrium if and only if* $(t_x + t_y)^2/t_y - (t_x + t_y)/2 \leqq 2Y/N$. *If this condition is satisfied, then*

(a) *if* $(t_x + t_y)^2/t_y - (t_x + t_y)/2 \leqq 2Y/N < (t_x + t_y)^2/t_y$, *then the firms will be partially dispersed in a symmetric equilibrium about* y; *this equilibrium is characterized by*

$$x_1^* = y + \frac{Y}{t_x} - \frac{(t_x + t_y)^2}{t_x t_y}\frac{N}{2}, \qquad x_2^* = y - \frac{Y}{t_x} + \frac{(t_x + t_y)^2}{t_x t_y}\frac{N}{2};$$

(b) *if* $(t_x + t_y)^2/t_y \leqq 2Y/N$, *the firms will be in a fully agglomerated equilibrium, i.e.*

$$x_1^* = x_2^* = y.$$

Proof. The proof is similar to that for Theorem 1 and is omitted for brevity. Details can be found in Thisse and Wildasin (1990).

We now examine what happens when the public facility is located nearer to the extremities of the region. The next result refers, for concreteness, to the case where y is close to ℓ. The results are essentially identical in the case where the facility is near 0, and this case is therefore not discussed explicitly. We have the following result describing the equilibrium:

Theorem 3. Suppose that $y > \ell - (N/2)$. *Then there exists an equilibrium if and only if* $3t_x + 2t_y(y - [\ell - N])/N \leqq 2Y/N$. *If this condition is satisfied, then both firms locate together in a partially agglomerated equilibrium at the point* $\ell - (N/2)$.

Proof. See the appendix.

According to Theorem 3, when the public facility is near the edge of the region, the range of possible equilibria is reduced to just one possible type.

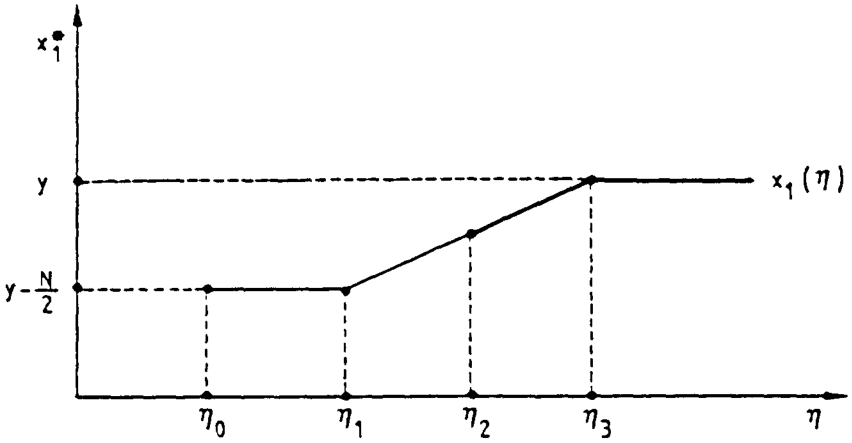


Fig. 3

When an equilibrium exists, the two firms must be located together at the center of the urban area – and separated from the public facility.

The implications of our analysis of equilibrium urban structure become clear or if we compare the results of Theorems 1, 2, and 3. First, Theorems 1 and 2 have shown that *the relationship between income and transportation costs is an important determinant of the existence and properties of equilibria.* Fig. 3 illustrates this relationship by displaying the findings of Theorems 1 and 2. These theorems apply to situations in which the public facility is confined to a relatively central location $y \in [N/2, \ell-(N/2)]$. Let $2Y/N$, denoted by η in the diagram, be measured on the horizontal axis, while the vertical axis measures the location of the left-most firm in the urban area, which we take to be firm 1, relative to the location of the public facility y. Let t_x and t_y be given, and consider the nature of the spatial equilibria for the economy at different values of η.

First, there are minimum values for η that must be met if an equilibrium is to exist at all, and the minimum depends on whether $t_x \leqq t_y$ (the case of Theorem 1) or whether $t_x > t_y$ (the case of Theorem 2). Given that firms are choosing profit-maximizing locations, positive consumption of the private good, and hence the existence of equilibrium, only becomes possible when η exceeds the critical values specified in Theorems 1 and 2. In the first case, an equilibrium exists if and only if $\eta \geqq \eta_0 = t_y$. This is a basic feasibility constraint for a fully-dispersed equilibrium. Households located at the extremities of the urban area incur no cost of travel to the private firms in such an equilibrium, but must travel all the way to the urban center to have access to the public facility. This cost is $t_y(N/2)$, and the condition $\eta \geqq t_y$ ensures that income is sufficiently high that all households can afford this cost of transportation and still have positive consumption of the private good. When η is between this level and a higher level $\eta_1=[(t_x+t_y)^2/t_y]-t_x$, there is an equilibrium in which the two firms are located at the extreme ends of the urban area, with all households residing between them. The location of firm 1 for values of η in this range is indicated by the horizontal segment of the $x_1(\eta)$ curve. By contrast, in the case where $t_x>t_y$, no equilibrium exists for values of η in this range, and so, in particular, there are no fully-dispersed equilibria when transportation costs to the public facility are less than the cost of traveling to the private firms.

As the value of η rises, the equilibria in the case where $t_x \leqq t_y$ change from fully-dispersed to partially-dispersed, with the equilibrium locations of the firms drawing closer to the public facility at y, and to each other, as η increases. Once η rises to a sufficiently high level, denoted η_2 in the figure, equilibria exist not just for the case where $t_x \leqq t_y$, but for the case where $t_x>t_y$ as well. It is interesting to note that this critical value of η is determined not by the condition that households have adequate resources to cover their transportation costs, but by the nature of the game-theoretic interaction between the firms. For smaller values of η, there is simply no Nash equilibrium for the firms. However, once η is sufficiently large, there exist partially-dispersed equilibria. The location of firm 1 depends on the relationship between income and transportation costs in exactly the same way as for the case $t_x \leqq t_y$, so the curve $x_1(\eta)$, for values of $\eta \geqq \eta_2$, shows the location of firm 1 regardless of the relative sizes of t_x and t_y. The line is heavier to the right of η_2 to indicate that it refers there to both cases.

As η rises still further, the equilibrium gradually moves to a fully-agglomerated equilibrium; once η exceeds the critical value $\eta_3=(t_x+t_y)^2/t_y$, the equilibrium becomes $x_1=x_2=y$, as shown by the horizontal segment of the $x_1(\eta)$ curve at points to the right of η_3.

Hence, whatever the relative values of t_x and t_y, *as income increases relative to transport costs, the private sector tends to become more and more spatially concentrated* ($x_2^*-x_1^*$ is a non-increasing function of Y). The urban structure evolves, as Y increases, from a more or less decentralized pattern to the pattern of a monocentric city where both public and private activities are agglomerated. The reason for this change in urban structure can be traced to the fact that, all other things the same, individual consumption of the private good increases with income [see (3)]. Hence, the market area effect, which acts to draw the firms together, is strengthened: when household consumption is greater, the capture of additional customers yields a greater benefit to firms than when the level of consumption of each household is smaller. Firms therefore have stronger incentives to move to the center of the market in order to attract a larger clientele. When Y is sufficiently high, the two firms find it profitable to locate at the center of the urban area in the same place as the public facility.

In addition to the importance of income and transportation costs, *the location of the public facility itself can have a major impact on spatial equilibria.* This can be seen by comparing the results of Theorems 1 and 2 with Theorem 3.

To make this comparison, recall the condition for the existence of equilibrium when $y \in (\ell-(N/2), \ell]$ – i.e. that $3t_x+2t_y(y-[\ell-N])/N \leqq \eta = 2Y/N$. The critical value of η for the existence of equilibrium thus depends on the precise value of y. It ranges over the interval $(\eta', \eta'']$, where

$$\eta' = 3t_x + t_y,$$

corresponding to $y = \ell-(N/2)$, and

$$\eta'' = 3t_x + 2t_y,$$

corresponding to $y = \ell$. Note that

$$\eta' - \eta_3 = \frac{t_x(t_y - t_x)}{t_y}, \qquad \eta'' - \eta_3 = \frac{t_x(t_y - t_x)}{t_y} + t_y. \tag{5}$$

The difference in eq. (5) depends on the size of t_x relative to t_y. Consider first the case where $t_x \leqq t_y$. According to (5), it follows that whenever η is sufficiently high to allow an equilibrium to exist for a value of $y > \ell-(N/2)$, an equilibrium would also exist if the public facility were more centrally located ($y \in [N/2, \ell-(N/2)]$). By Theorems 1 and 2, this would be an agglomerated equilibrium, with $x_1 = x_2 = y$. On the other hand, the converse is not true. That is, for a given structure of transportation costs, there exist values of η for which equilibria exist when $y \leqq \ell-(N/2)$, but do not exist when $y > \ell-(N/2)$. In particular this is true for all values of η in the interval $[\eta_0, \eta']$. Thus, suppose that we start with $y \leqq \ell-(N/2)$ in any fully-dispersed or partially-dispersed equilibrium, or even in an agglomerated equilibrium if $\eta_3 \leqq \eta < \eta'$. If we consider a sequence of locations for the public facility that approach and then exceed $\ell-(N/2)$, there will be a corresponding sequence of spatial equilibria up to the point where $y = \ell-(N/2)$; however, no equilibria would exist for values of y greater than this.

In the case where $t_y < t_x$, matters are a bit more complex. It is no longer the case that η' or η'' bear any necessary relationship to η_3 (the critical point for complete agglomeration when $y \in [N/2, \ell-(N/2)]$), or even η_2 (the critical point for existence of equilibria – partially agglomerated ones – when $t_y < t_x$ and $y \in [N/2, \ell-(N/2)]$). Thus, suppose we start with a value of $y > \ell-(N/2)$, for which an equilibrium exists (say $\eta > \eta''$). If $\eta \in [\eta_2, \eta_3)$, then if the public facility is moved to points left of $\ell-(N/2)$, there will be a discontinuous shift from a partially dispersed equilibrium to an agglomerated equilibrium. If $\eta < \eta_2$, there will again be a discontinuous change, but now from an agglomerated equilibrium to a situation where no equilibrium exists.

Speaking somewhat loosely, then, if $t_x \leqq t_y$, an equilibrium is 'more likely' to exist when the public facility is not too close to the boundaries of the region. Specifically, starting with a dispersed equilibrium, and with the public facility near the center of the region, a displacement of the facility to a location near the edge of the region would destroy the existence of an equilibrium. On the other hand, if $t_y < t_x$, there may exist no equilibrium if the public facility is too close to the center of the region, whereas a partially agglomerated equilibrium can be achieved if the facility is near the boundary. Alternatively, transportation costs and incomes might be such that there exists a partially dispersed equilibrium when the public facility is not near the boundary, but when the facility is located close to the boundary the equilibrium becomes partially agglomerated. In most cases, *there is a discontinuous change in the nature of the equilibrium of the system as the location of the public facility crosses the critical value* $y = \ell - (N/2)$.

4. Policy evaluation of transportation improvements and public facility location

We now turn to an analysis of the welfare effects of changes in the parameters of the model, corresponding to a variety of possible policy changes. We have seen that the equilibrium of the system depends on income, transportation costs, and the location of the public facility. If these parameters change, the welfare of three groups will in general be affected: consumers, owners of firms, and landowners.

We will use welfare indicators for each group that are denominated in units of the numéraire commodity. The measurement of welfare for the owners of firms and land is straightforward: we use the levels of profits and land rents. For consumers, matters are only slightly more complex. Suppose that there is an underlying preference structure defined over consumption of the private good and the public good that is provided by the public facility. Since the level of public good is taken as given in our analysis, utility will vary only with private good consumption. Recall that locational arbitrage results in equal levels of private good consumption, and thus equal utility, for all households. For any household, the change in welfare resulting from some parametric change in the model can thus be measured by the change in z^*, as defined in (3). Equipped with these welfare or real income indicators, we can analyze both the incidence or distributional impact of policy changes and their effect on overall efficiency. The latter is indicated simply by the aggregate change in real income, obtained by adding up the real income changes of consumers, firms, and landowners.

4.1. Policies leading to a change in transportation costs

Many public policies affect the cost of transportation. Although transportation costs are represented in monetary terms within our model, they should be interpreted broadly to include monetary equivalents of the non-pecuniary costs of transportation, such as travel time, comfort, safety, and frequency of service (for buses, trams, trains, or other public transportation). Given this interpretation, one can see that public investments in highway improvements and improved traffic control result in reduced transportation costs. The same is true of capacity expansion or quality improvements in public transportation systems. This can take many forms, such as adding new lines, increasing the numbers and maintenance of buses, or improved security on a subway system. Some tax policies, such as gasoline taxes, can also affect travel costs. The tax treatment of the energy sector of the economy, and energy policy in general, also affect transportation costs, as do highway and bridge tolls and the pricing of public transportation.

These policies affect welfare through two different channels. First, they affect welfare by changing transportation costs, represented in our model by the parameters t_x and t_y. Secondly, they affect the overall government budget through their impact on public expenditure or revenue. In many cases, the budgetary calculations will be relatively straightforward – requiring, for instance, a measure of the outlays required for some transportation project. The benefits of a reduction in transportation costs are more difficult to assess, however. A change in per unit transportation costs does not merely alter the cost of a *fixed* pattern of travel. The amount of travel itself is *endogenously* determined by the locational decisions of households and firms. As travel costs change, the equilibrium spatial structure also changes, and an accurate welfare analysis of transportation policies must take this equilibrium response into account.

Formally, this analysis requires a comparative statics investigation of the equilibrium spatial structure. Our model contains two transportation cost parameters, t_x and t_y, either or both of which might change, giving rise to several cases that would need to be studied in a comprehensive treatment. For brevity, we restrict attention to the simple case where the costs of transportation to the private firms and the public facility are equal, i.e. $t_x = t_y = t$.[8] Under this simplifying assumption, there is only one transportation cost parameter to consider. For this version of the model, note the following:

Corollary to Theorem 1. *Let $t_x = t_y = t$. Suppose that $y \in [N/2, \ell - (N/2)]$. Then there exists an equilibrium if and only if $t \leqq 2Y/N$. The equilibrium is*

[8]This involves an abuse of notation since t is now a per unit cost of transportation rather than the transportation cost function $t(\cdot)$ defined earlier. No confusion should result, however.

(a) *fully dispersed if* $\frac{2}{3}(Y/N) \leqq t \leqq 2Y/N$;
(b) *partially dispersed if* $\frac{1}{2}(Y/N) < t < \frac{2}{3}(Y/N)$;
(c) *fully agglomerated if* $t \leqq \frac{1}{2}(Y/N)$.

We can now describe how consumer welfare, profits, land rents, and aggregate real income change as t varies, recognizing that the nature of the spatial equilibrium changes at the critical values $t = \frac{1}{2}(Y/N)$ and $t = \frac{2}{3}(Y/N)$, the limits of the interval in which the equilibrium is partially dispersed.

Theorem 4. *Let* $t_x = t_y = t$, *and suppose that* $y \in [N/2, \ell - (N/2)]$.

(a) *The welfare of consumers, and the profits of firms, are globally maximized when* $t = 0$. *This occurs in a fully-agglomerated equilibrium. They decline linearly in* t *until* $t = \frac{1}{2}(Y/N)$, *where they reach a local minimum, then rise linearly with* t *over the range of partially-dispersed equilibria, reaching a local maximum at* $t = \frac{2}{3}(Y/N)$, *at which point the equilibrium is fully dispersed. Consumer welfare and profits then decline linearly with further increases in* t, *reaching a global minimum of* 0 *at* $t = (2Y/N)$.

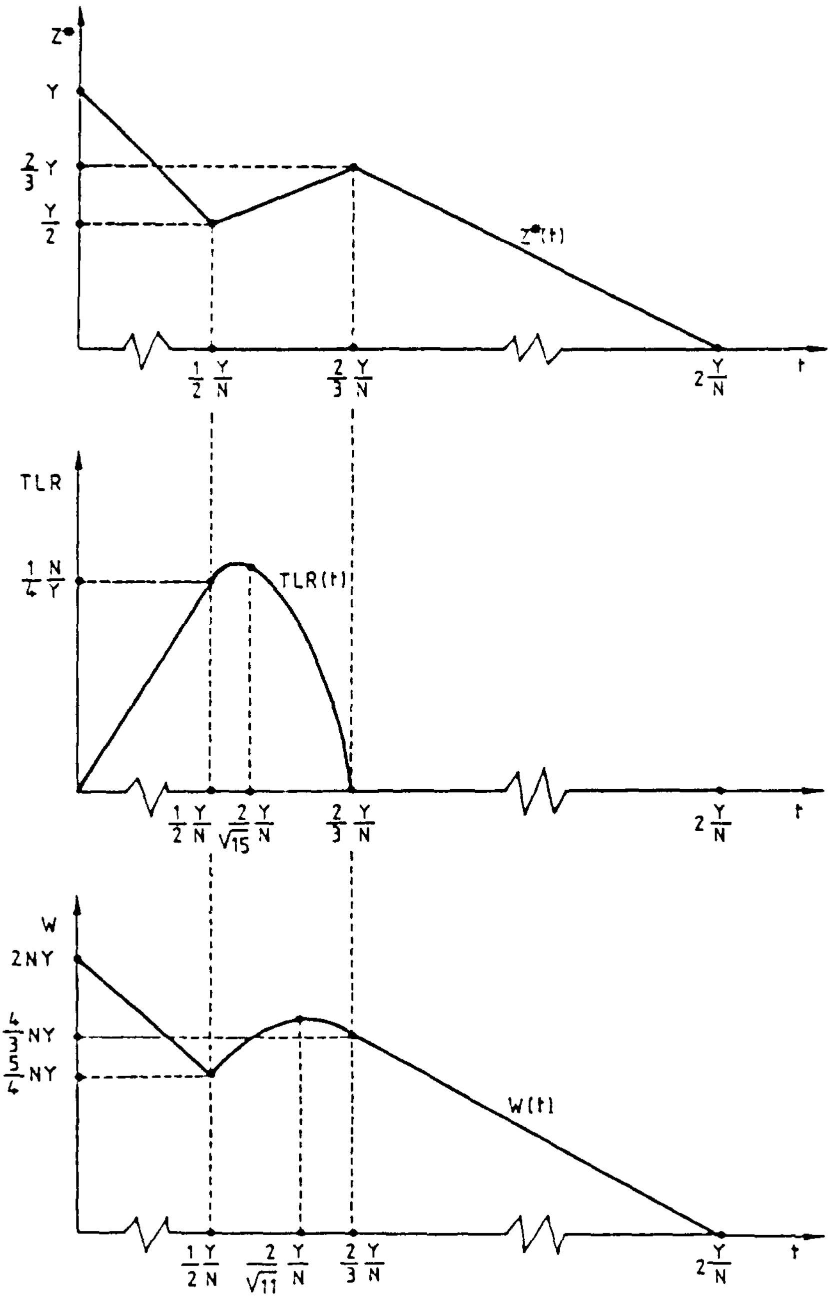
(b) *Land rents are* 0 *when* $t = 0$, *and rise linearly with* t *over the range of fully-agglomerated equilibria. Over the range of partially-dispersed equilibria, land rents are a concave function of* t, *reaching a global maximum in the interior of the interval* $[\frac{1}{2}(Y/N), \frac{2}{3}(Y/N)]$, *and dropping to* 0 *at its right endpoint, at which point the equilibrium becomes fully dispersed. Land rents are* 0 *for all higher values of* t, *i.e. over the entire range of fully-dispersed equilibria.*

(c) *Aggregate real income (the sum of consumer real incomes, profits, and land rents) is globally maximized at* $t = 0$. *It declines linearly in* t *over the range of fully-agglomerated equilibria, reaching a local minimum at* $t = \frac{1}{2}(Y/N)$. *Aggregate real income is a concave function of* t *over the range of partially-dispersed equilibria, reaching a local maximum at a value of* t *in the interior of the interval* $[\frac{1}{2}(Y/N), \frac{2}{3}(Y/N)]$. *It then declines linearly over the range of fully-dispersed equilibria, reaching a global minimum of* 0 *at* $t = 2Y/N$.

Proof. See the appendix.

These results are illustrated in fig. 4. Note that the maximum of land rents occurs at $t = 2(\sqrt{15}/15)(Y/N)$, which is less than $2(\sqrt{11}/11)(Y/N)$, the value at which aggregate real income is maximized. Several aspects of this analysis are of interest.

First, since the model permits us to do a global analysis of the effects of changes in transportation costs, we are in an unusually good position to see some of the limitations of local or marginal analysis. In particular, consumer welfare is not monotonic in t. While it seems reasonable enough that consumer welfare and profits are maximized when $t = 0$, it is more surprising



Z*
Y
$\frac{2}{3}$Y
$\frac{Y}{2}$
Z*(t)
$\frac{1}{2}\frac{Y}{N}$
$\frac{2}{3}\frac{Y}{N}$
$2\frac{Y}{N}$
t
TLR
$\frac{1}{4}\frac{N}{Y}$
TLR(t)
$\frac{1}{2}\frac{Y}{N}$
$\frac{2}{\sqrt{15}}\frac{Y}{N}$
$\frac{2}{3}\frac{Y}{N}$
$2\frac{Y}{N}$
t
W
2NY
$\frac{4}{3}$NY
$\frac{5}{4}$NY
W(t)
$\frac{1}{2}\frac{Y}{N}$
$\frac{2}{\sqrt{11}}\frac{Y}{N}$
$\frac{2}{3}\frac{Y}{N}$
$2\frac{Y}{N}$
t

Fig. 4

to find that these welfare measures reach a local minimum as t rises, and that they are then *increasing* in t as the equilibrium becomes more and more dispersed. Once a fully-dispersed equilibrium is attained, they reach a local maximum and then fall. Land rents are better behaved, in that they rise monotonically with transportation costs and then fall monotonically. However, the non-monotonic behavior of consumer welfare and profits dominates land rents quantitatively, so that aggregate real income is also non-monotonic in t. This means, of course, that *changes in transportation costs that yield small improvements* – from the viewpoint of consumers, firms, or aggregate welfare – *may actually move the system farther from a global optimum.*

Related to the above point is the fact that the qualitative welfare impact of a change in transportation costs can be quite different in different equilibrium regimes. For consumers and firms, a small increase in transportation costs is harmful if the system is in either a fully agglomerated or a fully dispersed equilibrium; if the equilibrium is partially dispersed, however, *an increase in transportation costs is welfare-improving*. Thus, quite aside from their direct budgetary costs, 'improvements' in the transportation system may be undesirable. Even though (or because) they lower the cost per unit distance travelled, they may generate *negative* aggregate benefits. If it seems paradoxical that firms or consumers could gain from increases in transportation costs, recall that changes in these costs do not simply make it more costly to carry out a given pattern of travel. Changes in transportation costs also change the equilibrium location of the firms, and thus the structure of the spatial equilibrium itself. This changes the pattern of travel, and it also affects a third group, the owners of land. Thus, one cannot expect a priori that an increase in transportation costs will have a negative impact on any particular group or even on aggregate real income. The results described in Theorem 4 take all of these equilibrium adjustments into account.

This is one obvious and important way in which our model leads to conclusions quite different from those found in standard monocentric city models. In those models, it is conventional simply to *assume* that all business activity occurs at the city center, and that all households must commute to the CBD. When this is assumed, travel patterns are essentially fixed, so that any reductions in unit travel costs are bound to be welfare-improving. Indeed, our model replicates this finding when transportation costs are sufficiently low. The Corollary to Theorem 1 shows that the equilibrium spatial structure is fully agglomerated for low values of t. The fully-agglomerated equilibria of our model are like a monocentric city model, and as part (c) of Theorem 4 shows, transportation improvements always have positive (gross) benefits in such equilibria. However, monocentricity is just a possible *equilibrium* feature of our model, not an a priori requirement, and for some parameter values the model generates partially dispersed rather than fully-agglomerated equilibria. The welfare analysis of transportation 'improvements' is quite different in this case. Here, a small reduction in transportation costs reduces the dispersion of the firms in the urban area. This implies that the total amount of travel undertaken by households must increase. Despite the reduction in per unit transportation costs, the extra quantity of travel can actually make households worse off. Note that a finding of this type can only occur in a model which does not impose a priori restrictions on the location of firms.

Note also that the presence of the public facility in the present model plays a crucial role in the analysis of changes in transportation costs, even though the location of the public facility is held fixed in this analysis. To see this clearly, consider what happens when t changes in a model without such a facility – that is, when the model reduces to the case analyzed in Fujita and Thisse (1986). In that case, provided that the amount of land in the region is sufficiently large, the market areas of the two firms are disjoint. As t changes, therefore, the spatial equilibrium of the model does not change: neither the households nor the firms change locations in the face of higher transportation costs. Thus, with a fixed location pattern and a fixed volume of travel, a reduction in t must simply lower aggregate transportation costs and land rents while raising consumer welfare and profits. Aggregate real income is monotonically falling in t. By contrast, when a public facility is present, the nature of the equilibrium spatial structure is very different. The firms' market areas always touch, and the firms' locations can change with changes in t. The foregoing discussion has shown that this can have major consequences for welfare analysis and policy evaluation. Thus, even when the location of the public facility is held fixed, the mere presence of the facility changes the analysis of transportation improvements in qualitatively important ways.

We conclude this discussion by recalling that a perennial question in the literature on benefit–cost analysis of transportation improvements is whether changes in land rents are a good indicator of the overall benefits of the project, whether including land rents along with other direct measures of benefits results in double-counting, etc.[9] In the present model, several points are clear. First, aggregate real income is maximized when $t=0$, and it is diminishing in t for values close to 0. The same is true of consumer welfare and profits. By contrast, land rents are *minimized* at $t=0$, and they are *increasing* in t for values close to 0. Thus, *when equilibria are fully agglomerated, changes in land rents are inversely related to the change in aggregate real income from transportation improvements*, or to the gains to consumers and firms.

[9] Mohring (1961) provides an early treatment of this issue. See also, for example, Pines and Weiss (1976, 1982) and Wheaton (1977). Additional references to the related literature on air pollution and property values and benefit capitalization for local public goods can be found in Wildasin (1986, 1987).

For values of $t \in [\frac{1}{2}(Y/N), \frac{2}{3}(Y/N)]$, the spatial equilibrium will be partially dispersed. Here, land rents attain a maximum at $t = 2\sqrt{15}/15$, while aggregate real income is locally maximized at a higher value of $t = 2\sqrt{11}/11$. In this range of t values, it happens at least sometimes that rents and aggregate real income move together with respect to t. But they need not, and, at least in this particular model, land rent maximization leads to a level of t that is lower than would be optimal from the viewpoint of aggregate welfare.

4.2. Changes in public facility location

Let us now analyze the effect of a change in y on consumer welfare and profits when the public facility is located near to the boundary of the region (i.e. $y \in (\ell - (N/2), \ell])$.[10] As shown in the proof of Theorem 3,

$$z^* = Y - t(\ell - N)$$

$$= Y - t_x\left(\left[\ell - \frac{N}{2}\right] - [\ell - N]\right) - t_y(y - [\ell - N]),$$

so that

$$\frac{\partial z^*}{\partial y} = -t_y.$$

Thus, if y increases by some discrete amount, say from y to y', consumer welfare and profits both fall by the amount $-N\Delta z^* = Nt_y(y' - y)$. Welfare for these groups reaches a minimum at $y = \ell$.

The change in land rents is easily calculated using a simple diagram. The solid line in fig. 5 illustrates the equilibrium land rent gradient when the public facility is located at y; it is labelled R_y. It starts at the value ρ at the urban boundary at $\ell - N$, then rises at rate $t_x + t_y$ up to the point $\ell - (N/2)$. Between $\ell - (N/2)$ and y, it rises at the rate $t_y - t_x$, and beyond y it falls at the rate $t_x + t_y$. This pattern of rents simply reflects the structure of transportation costs for households at different locations. (It is shown in the proof of Theorem 3 that urban rents always exceed ρ at the point ℓ.)

If y now shifts to y', it is clear that the rent profile shifts upward in the interval $[y, \ell]$, as shown by the broken line labelled $R_{y'}$. The area between the two profiles is the total change in land rents, which is given by

[10]If the facility is not located near the edge of the region, a small change in its location has no real effects. It simply displaces the entire equilibrium structure of the urban area with no effects on consumer welfare, profits, or aggregate land rents. Thus, the discussion here focuses on the more interesting case where the facility is near the edge of the region.

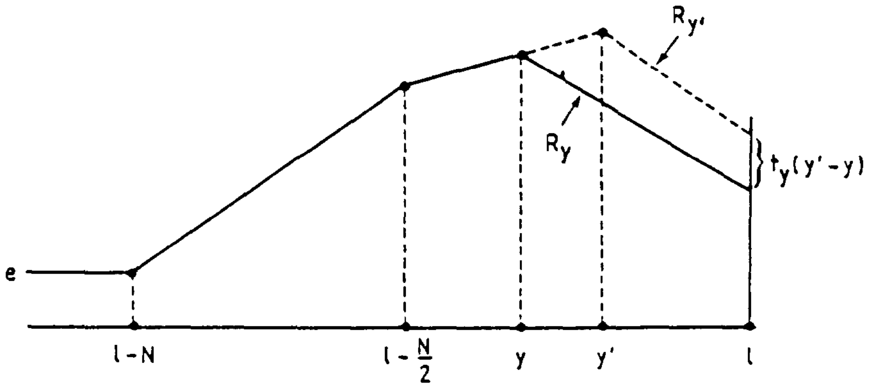


Fig. 5

$$R_{y'}-R_y=\tfrac{1}{2}t_y(y'-y)^2+t_y(y'-y)(\ell-y').$$

This is obviously positive. Land rents are maximized when $y=\ell$.

The effect of public facility location on aggregate real income is obtained by adding up welfare changes across groups. With some manipulation of the above results, one finds that

$$2N\Delta z^*+(R_{y'}-R_y)=t_y(y'-y)\left[\ell-\frac{y'+y}{2}-2N\right]$$

$$\leqq t_y(y'-y)\left[\frac{N}{2}-2N\right]<0, \tag{6}$$

where the first inequality follows from the fact that $\ell-(N/2)\leqq y$. Thus, the loss in real income to households and firms exceeds the gain in real income to the landowners. Indeed, the loss to either the household or the firms alone is $N\Delta z^*$, and one can see that either of these taken by itself is greater than the gain of the landowners.[11] Thus, unless a differentially high weight is placed on real income gains to landowners relative to those of consumers and firms, the movement of the facility towards a boundary of the region lowers social welfare.

To summarize:

Theorem 5. *Let $y\in(\ell-(N/2),\ell]$. An increase in y lowers consumer welfare*

[11]If we compare the loss to the households with the gain to the landlords, the bracketed expression in (6) becomes $(N/2)-N$, which is still negative.

and profits and raises land rents. It also lowers the sum of the changes in real incomes of customers, owners of firms, and owners to land.

Note that in this case, as in the preceding analysis of changes in transportation costs, land rent changes do not provide a good indicator of the 'social benefits' of the location of the public facility.

5. Conclusion

Much of the literature of public economics focuses on the *level* of provision of public goods and services. This is clearly a matter of great importance. However, many public services also have a spatial dimension, and when they do, their *location* becomes a policy question that also deserves careful attention. To date, however, there has been little discussion in the literature of either the positive or the normative economics of public facility location. Such work as has been done has focused mainly on the case of the monocentric city, where the spatial structure of private sector economic activity is often taken as exogenous to the model. The monocentric city model is very useful, but it does abstract from the complexity of spatial interactions by imposing exogenous constraints on location and travel patterns. The present paper, by contrast, begins with a model in which private firms may but *need not* locate together. A monocentric or agglomerated locational equilibrium can sometimes occur, but other spatial structures can also occur, depending on the parameters of the model. In the spirit of Hotelling-type models, our model captures some of the inherently strategic features of spatial equilibrium for competing firms. As in the analysis of Fujita and Thisse (1986), our model blends elements of pure spatial equilibrium models of the Hotelling type with traditional urban economics models by incorporating endogenous locational choice by households competing for scarce space in a land market. Land rents play a crucial role in equilibrating the locational choices of households in this model.

We have seen that the presence of a public facility can have quite important effects on the interactions between private firms. Depending on the parameters of the model (income, transportation costs, and location of the public facility), the public facility may serve as an agglomeration point for private firms, it may serve as the center of a dispersed spatial configuration for the firms, or it may induce a clustering of the firms in a location different from that of the public facility itself. Changes in the location of the public facility change the nature of the strategic spatial interactions between the firms and can change the qualitative properties of the equilibrium spatial structure of the system.

The literature on benefit–cost analysis has repeatedly shown that the evaluation of the welfare effects of public policy is sensitive to the structure of the economy in which policy interventions occur. The present analysis confirms this general proposition by showing how the effects of changes in transportation costs (for instance as a result of investment in transportation improvements) and public facility location depend on and interact with the spatial structure of an urban area. Depending on the parameters of the model, the distribution and aggregate value of the gains and losses from such policy changes may vary widely. For example, consumer welfare is not monotonic in transportation costs, so that policies that lower costs (such as public investment in transportation improvements, a reduction in taxes in gasoline, or subsidies to public transportation) may cause consumers to gain or lose, depending on the nature of the initial spatial equilibrium of the system. The analysis also shows that land rents may provide a very inaccurate indication of the aggregate gains from policy changes. It is quite possible, for example, for landowners to gain from policy changes which cause losses to consumers and firms, and for the losses to the latter to be larger in magnitude than the gains to the former.

In summary, public facility location warrants attention both because the location of a public facility is an important public policy question in its own right and because public facilities affect urban spatial structure and are therefore important for the analysis of other public policies. Explicit consideration of the endogenous locational choices of private firms and households has led to quite different sorts of results than those that would emerge from a typical monocentric city model. However, the model that we have used is certainly very simple in many respects, so the specific results of the analysis should be seen as suggestive rather than definitive. It would certainly be of interest to consider what happens when various simplifying assumptions are relaxed. For example, different assumptions might be considered concerning transportation costs, the distribution of income among households, the structure of consumer preferences, the number of firms, and so on. Consideration of these questions is beyond the scope of the present paper, however, and we leave them open for future study.

Appendix

This appendix presents the proofs of several of the main results in the paper.[12]

Proof of Theorem 1. We must show that there exist (x_1^*, x_2^*) such that x_i^* maximizes $Q_i(x_i, x_j^*)$ under the stated conditions, and that (x_1^*, x_2^*) have the properties given in (a), (b), and (c). The first step is to derive expressions for

[12]The following proofs omit certain detailed calculations because they are routine and tedious. The authors will be pleased to provide further details on request.

the Q_i's. For the purposes of the proof, let $\ell = 1$. We assume that $y = \frac{1}{2}$ to begin with; the generalization to other values of y is straightforward and is left to the end.

To understand the profit-maximization problems of the firms, we proceed in a series of steps. First, suppose that firm 2 locates at some point in the interval $[\frac{1}{2}, \frac{1}{2} + (N/2)]$. Conditional on this, firm 1 could locate anywhere, but we shall show later that it will not be profitable to locate outside of the interval $[\frac{1}{2} - (N/2), \frac{1}{2}]$. The most interesting aspects of the locational choice of the firm concern the tradeoffs within this interval, so we focus on those first.

Suppose, then, that $x_1 \in [\frac{1}{2} = (N/2), \frac{1}{2}]$ and $x_2 \in [\frac{1}{2}, \frac{1}{2} + (N/2)]$. Let ξ_1 and ξ_2 denote, respectively, the minimum and maximum values of $\xi \in [0, 1]$ such that $t(\xi; x_1, x_2, y) \leqq t^*$. Then, we have $\xi_1 \leqq x_1 \leqq \frac{1}{2}$ and $\xi_2 \geqq x_2 \geqq \frac{1}{2}$. From (2),

$$R(\xi_1) = \rho + t^* - t(\xi_1) = \rho + t^* - t_x(x_1 - \xi_1) - t_y(\tfrac{1}{2} - \xi_1),$$

$$R(\xi_2) = \rho + t^* - t(\xi_2) = \rho + t^* - t_x(\xi_2 - x_2) - t_y(\xi_2 - \tfrac{1}{2}).$$

Equilibrium in the land market requires that $R(\xi_1) = R(\xi_2) = \rho$. Also, $\xi_2 - \xi_1 = N$ since all households must locate somewhere. These two equations can be solved for

$$\xi_1 = \frac{t_y + t_x(x_1 + x_2) - (t_x + t_y)N}{2(t_x + t_y)}, \tag{A.1}$$

$$\xi_2 = \frac{t_y + t_x(x_1 + x_2) + (t_x + t_y)N}{2(t_x + t_y)}. \tag{A.2}$$

Since $t((\xi_i)) = t^*$, we can compute z^* from (3):

$$z^* = Y - \frac{t_x(x_1 - x_2)}{2} - \frac{(t_x + t_y)N}{2}. \tag{A.3}$$

Firm 1 has a market area equal to the interval $[\xi_1, (x_1 + x_2)/2]$; this is true even if either x_1 or x_2, or both, are located at $\frac{1}{2}$. Ignoring the question of the non-negativity of z^* for the moment, it follows that

$$Q_1 = z^* \left(\frac{x_1 + x_2}{2} - \xi_1 \right).$$

Making a substitution from (A.3) and from (A.1), Q_1 can be expressed in terms of x_1, x_2, and the parameters of the problem. The derivative of Q_1 is easily calculated:

$$\frac{\partial Q_1}{\partial x_1} = \frac{t_x t_y}{2(t_x+t_y)}\left(\frac{Y}{t_x} - \frac{(t_x+t_y)^2}{t_x t_y}\frac{N}{2} + \frac{1}{2} - x_1\right).$$

It is obvious from this expression that $\partial^2 Q_1/\partial x_1^2 < 0$. Therefore, maximization of Q_1 subject to $\frac{1}{2}-(N/2) \leqq x_1 \leqq \frac{1}{2}$ implies that

$$x_1^* = \begin{cases} \dfrac{1}{2} - \dfrac{N}{2}, & \text{if } \dfrac{2Y}{N} \leqq \dfrac{(t_x+t_y)^2}{t_y} - t_x; \\ \dfrac{1}{2}, & \text{if } \dfrac{(t_x+t_y)^2}{t_y} \leqq \dfrac{2Y}{N}; \\ \dfrac{1}{2} + \dfrac{Y}{t_x} - \dfrac{(t_x+t_y)^2}{t_x t_y}\dfrac{N}{2}, & \text{otherwise.} \end{cases} \tag{A.4}$$

It is important to note that this value of x_1 is determined independently of the precise value of x_2, i.e. the profit-maximizing location for firm 1 in the interval $[\frac{1}{2}-(N/2), \frac{1}{2}]$, conditional on firm 2 locating in the interval $[\frac{1}{2}, \frac{1}{2}+(N/2)]$, is determined uniquely and independently of the exact locational choice of firm 2. In this restricted sense, the firm's decision is a dominant strategy. This restricted dominant strategy property simplifies the analysis considerably.

Consider now the problem facing firm 2. By symmetric reasoning, assuming that $\frac{1}{2}-(N/2) \leqq x_1 \leqq \frac{1}{2}$, the profit-maximizing location for firm 2 subject to $x_2 \in [\frac{1}{2}, \frac{1}{2}+(N/2)]$ is given by

$$x_2^* = \begin{cases} \dfrac{1}{2}, & \text{if } \dfrac{(t_x+t_y)^2}{t_y} \leqq \dfrac{2Y}{N}; \\ \dfrac{1}{2} + \dfrac{N}{2}, & \text{if } \dfrac{2Y}{N} \leqq \dfrac{(t_x+t_y)^2}{t_y} - t_x \\ \dfrac{1}{2} - \dfrac{Y}{t_x} + \dfrac{(t_x+t_y)^2}{t_x t_y}\dfrac{N}{2}, & \text{otherwise.} \end{cases} \tag{A.5}$$

These choices, too, are independent of the precise value of x_1. Note the symmetry with (A.4).

Now, for the values of x_1 and x_2 given in (A.4) and (A.5) to be equilibria, it must be the case that $z^* \geqq 0$, as has been assumed so far. Substituting from

(A.4) and (A.5) into (A.3), we find that $z^* \geqq 0$ if and only if $t_y \leqq 2Y/N$, as required in the statement of the theorem. (Of course, it still remains to show that x_1^* and x_2^* are global profit maxima, and not just local.)

Finally, it is necessary to verify that the constraints $x_1 \in [\frac{1}{2}-(N/2), \frac{1}{2}]$ and $x_2 \in [\frac{1}{2}, \frac{1}{2}+(N/2)]$ are not binding for the profit-maximization problems of the two firms, provided that the other is located in its interval. A sketch of the argument will suffice. Suppose, first, that $(t_x+t_y)^2/t_y \leqq 2Y/N$, and that $x_2^* = \frac{1}{2}$. We have shown that $x_1 = \frac{1}{2}$ is the profit-maximizing choice for firm 1 subject to $x_1 \in [\frac{1}{2}-(N/2), \frac{1}{2}]$. But a move to points to the left of $\frac{1}{2}-(N/2)$ would not be profitable because this entails a negative consumption effect, a negative market area effect, or both. Similarly, a move by firm 1 to a point in the interval $(\frac{1}{2}, 1]$ can be ruled out by symmetry. Next, suppose that $t_y \leqq 2Y/N \leqq [(t_x+t_y)^2/t_y]-t_x$, and that $x_2^* = \frac{1}{2}+(N/2)$. A move by firm 1 to the left of $\frac{1}{2}-(N/2)$ would have negative effects on both consumption and market areas, as would a move to the right of x_2^*. Finally, given that $x_2^* = \frac{1}{2}+(N/2)$, Q_1 is continuously differentiable in the interval $(\frac{1}{2}, \frac{1}{2}+(N/2))$. Therefore, $\partial Q_1/\partial x_1$ is negative for all $x_1 \in (\frac{1}{2}-(N/2), \frac{1}{2}+(N/2))$, not just in the interval $(\frac{1}{2}-(N/2), \frac{1}{2})$ as previously shown. The last case to consider is where $[(t_x+t_y)^2/t_y]-t_x < 2Y/N < (t_x+t_y)^2/t_y$. Here again, however, the argument is essentially the same: for all those values of x_1 for which Q_1 can be calculated as above, which includes in particular all points in the interval $(\frac{1}{2}-(N/2), x_2^*)$, the concavity of the profit function ensures that the value for x_1 given in (A.4) is most profitable. For values of x_1 less than $\frac{1}{2}-(N/2)$, negative consumption and market area effects reduce profits; and the same is true for values of $x_1 > x_2^*$.

This completes the proof for the case $y = \frac{1}{2}$. Noting that in all cases the interval from $\frac{1}{2}-(N/2)$ to $\frac{1}{2}+(N/2)$ is occupied, it then becomes clear that the location of the public facility y can be varied within the range $[N/2, \ell-(N/2)]$ without changing the nature of the equilibrium. This simply displaces the entire equilibrium spatial structure in step with y, and only changes the amount of vacant land on each side of the urban area. Q.E.D.

Proof of Theorem 3. It will be shown below that the only candidates for equilibria are those such that the urban area consists of the interval $[\ell-N, \ell]$, and such that maximum transportation costs are incurred at the left-most boundary, i.e. $t^* = t(\ell-N) > t(\xi)$ for all $\xi \in (N-\ell, \ell]$. Therefore, we begin by assuming these conditions to be true.

Now suppose that firm 2 is located at some given location $x_2 \in (\ell-N, \ell-(N/2)]$; this implies $x_2 < y$. Consider the profit-maximizing location for firm 1 conditional on x_2. If $x_1 < x_2$, then $z^* = Y - t^* = Y - t(\ell-N) = Y - t_x(x_1 - [\ell-N]) - t_y(y-[\ell-N])$, and the market area of firm 1 is the interval $[\ell-N, (x_1+x_2)/2]$. The profit function for firm 1 is then $Q_1(x_1, x_2) = z^*((x_1+x_2)/2 - [\ell-N])$, and we can compute

$$\frac{\partial Q_1}{\partial x_1} = -t_x x_1 + \tfrac{1}{2}(Y + 3t_x[\ell - N] - t_y(y - [\ell - N]) - t_x x_2). \tag{A.6}$$

Note that $\partial^2 Q_1/\partial x_1^2 < 0$ and that $\partial^2 Q_2/\partial x_1\,\partial x_2 < 0$.

One can see from (A.6) that $\partial Q_1/\partial x_1 > 0$ for all value of $x_1 \in [\ell - N, x_2]$, and for all $x_2 \in [\ell - N, \ell - (N/2)]$, if and only if

$$\frac{2Y}{N} \geqq 3t_x + 2t_y \frac{(y - [\ell - N])}{N}. \tag{A.7}$$

When (A.7) holds, firm 1 would not wish to locate at any point to the left of x_2.

On the other hand, if firm 1 moves to the point x_2, it obtains half of the market share. If $x_2 < \ell - (N/2)$, this means that Q_1 jumps up when $x_1 = x_2$. If $x_2 = \ell - (N/2)$, Q_1 is continuous from the left (in x_1) at $x_1 = x_2$. In either case, it remains true that Q_1 is maximized with respect to x_1, over the interval $[\ell - N, x_2]$, at the point $x_1 = x_2$.

Consider now points $x_1 \in (x_2, y)$. At such points, given our assumption that $t(\ell - N) = t^*$, x^* will depend on x_2 but not on x_1. With z^* independent of x_1, firm 1 will maximize profits by maximizing market area, i.e. $\ell - [(x_1 + x_2)/2]$, which means that $\partial Q_1/\partial x_1 < 0$ in this range. Thus, firm 1 will not wish to locate at any point in the interval (x_2, y). One can similarly verify that it is not profit-maximizing for firm 1 to locate in the interval $[y, \ell]$.

The fact that Q_1 is diminishing in x_1 over the interval $(x_2, \ell]$ does not necessarily mean, however, that x_2 is a profit-maximizing location for firm 1. In particular, as firm 1 approaches x_2 from the right, its market area approaches $\ell - x_2$, which exceeds $N/2$ if $x_2 < \ell - (N/2)$. But if firm 1 actually moves to x_2, its market share drops to exactly $N/2$.

Putting the above results together, we find that if (A.7) holds, and if $x_2 < \ell - (N/2)$, then Q_1 as a function of x_1 is increasing over the interval $[\ell - N, \ell - (N/2))$; it then jumps up discontinuously at $\ell - (N/2)$; it then jumps up discontinuously again and declines monotonically over the interval $(\ell - (N/2), \ell]$. Since this interval is left-open, there is therefore no profit-maximizing location for firm 1 when $x_2 < \ell - (N/2)$. Heuristically speaking, however, firm 1 would like to locate 'just to the right' of firm 1.

If $x_2 = \ell - (N/2)$, however, Q_1 is continuous in x_1 over the entire interval $[\ell - N, \ell]$, since the market area of firm 1 in the interval $(x_2, \ell]$ is $\ell - [(x_1 + x_2)/2] = (\ell/2) + (N/4) - (x_1/2)$, which approaches $N/2$ as x_1 approaches x_2 from the right. Thus, when $x_2 = \ell - (N/2)$, it is profit-maximizing for firm 1 also to locate at $\ell - (N/2)$. Symmetrically, it is profit-maximizing for firm 2 to locate at $\ell - (N/2)$ when firm 1 locates there. It is also obvious that $t(\ell-N)>t(\ell)$ when the firms locate at $\ell-(N/2)$. Furthermore, given these values of x_1 and x_2,

$$z^*=\frac{N}{2}\left(\frac{2Y}{N}-t_x-2t_y\left[\frac{y-(\ell-N)}{N}\right]\right).$$

This is positive if (A.7) is satisfied. Given (A.7), then, $x_1=x_2=\ell-(N/2)$ is a Nash equilibrium.

If (A.7) is not satisfied, then Q_1 is maximized at some point to the left of x_2 when $x_2=\ell-(N/2)$. Consider the locational choice of firm 2, however, conditional on $x_1<\ell-(N/2)$. By reasoning similar to that already given above, Q_2 as a function of x_2 is discontinuous at the point $x_2=x_1$. Firm 2 would like to locate 'just to the right' of firm 1. This discontinuity of the profit functions at all points other than $\ell-(N/2)$ precludes the existence of equilibrium for any set of parameter values not satisfying (A.7).

It remains to verify that no equilibrium (x_1^*,x_2^*) is possible such that $t(\ell)\geqq t(\ell-N)$. Suppose the contrary. We know that $R(\ell-N)=\rho$ as a condition of land market equilibrium. Thus $t(\ell)\geqq t(\ell-N)$ means that $R(\ell)\leqq R(\ell-N)$; but since $R(\xi)\geqq\rho$ for all ξ in the urban area, this means in fact that $R(\ell)=\rho$ and $t(\ell)=t(\ell-N)=t^*$. In turn, this implies that

$$z^*=Y-t(\ell)=Y-t(l-N).$$

If $x_1^*=x_2^*$, then it must be the case that $x_1^*=x_2^*=\ell-(N/2)$; if not, each firm would have an incentive to move 'just' to the right or left of its competitor in order to capture a larger market share. That is, the profit functions have discontinuities unless both firms are located at $\ell-(N/2)$. However, $x_1^*=x_2^*=\ell-(N/2)$ is also not possible in equilibrium, because then $t(\ell-N)>t(\ell)$, contrary to assumption.

Thus, if there is an equilibrium, it must involve $x_1^*\neq x_2^*$, say $x_1^*<x_2^*$, and $Q_1=z^*([(x_1^*+x_2^*)/2]-[N-\ell])$, $Q_2=z^*(\ell-[(x_1^*+x_2^*)/2])$. For equilibrium, it must be the case that

$$\frac{\partial Q_1(x_1^*,x_2^*)}{\partial x_1}=\frac{\partial Q_2(x_1^*,x_2^*)}{\partial x_2}=0. \tag{A.8}$$

However,

$$\frac{\partial z^*}{\partial x_1}=-t_x=-\frac{\partial z^*}{\partial x_2}$$

and

$$\frac{\partial\left(\frac{x_1^*+x_2^*}{2}-[N-\ell]\right)}{\partial x_1}=\frac{1}{2}=-\frac{\partial\left(\ell-\frac{x_1^*+x_2^*}{2}\right)}{\partial x_2}.$$

Hence, (A.8) implies that

$$\frac{x_1^*+x_2^*}{2}-(N-\ell)=\ell-\frac{x_1^*+x_2^*}{2},$$

i.e. the market areas of the two firms are equal. This means that the firms are symmetrically located in the urban area, equidistant from the market center at $(x_1^*+x_2^*)/2=N/2$. However, this contradicts the hypothesis that $t(\ell)=t(\ell-N)$, since $t(\ell)<t(\ell-N)$ for any symmetric locations of the firms. Q.E.D.

Proof of Theorem 4. The proof consists of explicit calculations of z^*, total land rents, denoted TLR, and aggregate real income, denoted by W, as functions of the transportation cost parameter t. Each equilibrium regime must be examined separately.

Fully-agglomerated equilibria. When $t\leq\frac{1}{2}(Y/N)$, the equilibrium is fully agglomerated. In such an equilibrium, households at the boundary of the urban area travel the distance $N/2$ both to go to the private firms and to go to the public facility, so that

$$z^*(t)=Y-tN,$$

which is maximized at $t=0$ and falls linearly to $Y/2$ at $t=\frac{1}{2}(Y/N)$.

One can verify geometrically that total land rents in a fully agglomerated equilibrium are given by

$$TLR(t)=\tfrac{1}{2}tN^2,$$

which rises from 0 to $\frac{1}{4}NY$ as t rises from 0 to $\frac{1}{2}(Y/N)$. Aggregate real income is given by

$$W(t)=2Nz^*(t)+TLR(t)=2NY-\tfrac{3}{2}tN^2.$$

This function attains a maximum value of $2NY$ at $t=0$, and falls to $\frac{5}{4}NY$ at $t=\frac{1}{2}(Y/N)$.

Fully-dispersed equilibria. If $t\in[\frac{2}{3}(Y/N),2(Y/N)]$, the equilibrium will be fully dispersed. Noting that transportation costs are identical at all locations, total land rents are 0, and the level of private good consumption is

$$z^*(t)=Y-t\frac{N}{2}.$$

Then z^* is decreasing in t, with a maximum of $\frac{2}{3}Y$ at $t=\frac{2}{3}(Y/N)$ and a minimum of 0 at $t=2(Y/N)$. Aggregate real income is just

$$W(t)=2Nz^*=2NY-tN^2,$$

which attains a maximum of $\frac{4}{3}NY$ when $t=\frac{2}{3}(Y/N)$.

Partially-dispersed equilibria. Now suppose that $t\in[\frac{1}{2}(Y/N),\frac{2}{3}(Y/N)]$. Substituting for the equilibrium values of x_1 and x_2 from Theorem 1, part (b) into (A.3),

$$z^*=tN,$$

which is increasing in t, achieving a minimum of $Y/2$ at $t=\frac{1}{2}(Y/N)$ and a maximum of $\frac{2}{3}Y$ at $t=\frac{2}{3}(Y/N)$.

To calculate total land rents, note that the land rent gradient rises at the rate $2t$ over the interval $[\frac{1}{2}-(N/2),x_1]$, reaches a maximum of $2t(x_1-[\frac{1}{2}-(N/2)])$ over the interval from x_1 to x_2, and then falls at the rate $2t$ from x_2 to $\frac{1}{2}+(N/2)$. Total land rents are therefore given by $2t(x_1-[\frac{1}{2}-(N/2)])\cdot(x_2-[\frac{1}{2}-(N/2)])$ or, using the expressions for x_1 and x_2 given in Theorem 1(b),

$$TLR(t)=(2Y-3tN)\cdot\left(\frac{5}{2}N-\frac{Y}{t}\right)=8NY-\frac{15}{2}tN^2-2\frac{Y^2}{t}.$$

It follows that TLR is concave in t and direct calculations show that it is maximized at $t=2(\sqrt{15}/15)(Y/N)$. At $t=\frac{1}{2}(Y/N)$, $TLR=NY/4$, while $TLR=0$ at $t=\frac{2}{3}(Y/N)$.

Aggregate real income is

$$W(t)=2Nz^*(t)+TLR(t)$$

$$=8NY-\frac{11}{2}tN^2-\frac{2Y^2}{t}.$$

W is concave in t and attains its maximum value of $8-2\sqrt{11}\approx 1.37$ at $t=2(\sqrt{11}/11)(Y/N)$. Finally, $W(\frac{1}{2}(Y/N))=\frac{5}{4}NY$ and $W(\frac{2}{3}(Y/N))=\frac{4}{3}NY$. Q.E.D.

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