Pareto-Efficient International Taxation

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This paper analyzes Pareto-efficient international tax regimes. Because every country faces its own national budget constraint, the Diamond-Mirrlees production-efficiency theorem, which underlies key tenets of policy advice in international taxation—the desirability of destination basis for commodity taxation, of the residence principle for capital income taxation, and of free trade—does not apply. The paper establishes conditions—relating to the availability of explicit or implicit devices for reallocating tax revenues across countries—under which production efficiency is nevertheless desirable, and characterizes the precise ways in which Pareto-efficient international taxation may require violation of established tenets. (*JEL* H2, F0, H7)

A key task in the theory of public finance is to characterize the set of Pareto-efficient tax structures. Starting with the analysis of Pareto-efficient commodity taxation by Richard G. Harris (1979) and continuing through a wide range of papers inspired by Joseph E. Stiglitz' (1982) treatment of nonlinear taxation, this task has received considerable attention in the context of closed economies comprising several types of person. Yet the international variant of this general task—characterizing the set of tax-spending policies that are Pareto efficient in terms of countries' distinct national interests—appears not to have been addressed. A clear understanding of Pareto efficiency in international taxation is not only important from a normative viewpoint, it is also of critical importance in defining the potential scope for internationally coordinated policy changes that are mutually beneficial for all participating countries. Unexploited Pareto improvements represent missed opportunities for coordination of policy, and Pareto-inefficient policies may not be sustainable in a world where countries enjoy autonomy in policy-making. The absence of systematic analyses of Pareto-efficient international taxation is especially striking given both the evident practical importance of understanding the conditions that must be satisfied for international tax arrangements to be Pareto efficient and the rapid growth of policy and analytical interest in issues of international tax competition and coordination over the last decade or so. Indeed the literature appears to have arrived at some articles of faith as to proper practice in international taxation without even considering whether or not they are consistent with the minimal requirement of Pareto efficiency. The purpose of this paper is to argue that there is a fundamental intellectual gap to be filled in developing principles for Pareto efficiency in international taxation—and hence in thinking about issues of international tax coordination—and to go some way towards doing so.

The strange neglect of this issue[1] may reflect

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[1] Exceptions—from which this paper arises—are Wildasin (1977) and Keen and Stephen Smith (1996). Liam Ebrill and Steven M. Slutsky (1989, 1990) address some issues of regulatory design in hierarchical industrial structures that have similarities with the question addressed here. More recently, the importance of the distinctness of national budget constraints has been recognized by Charles Blackorby

a view that the fundamental lessons of many-person tax theory—including in particular the Diamond-Mirrlees theorem on the desirability of production efficiency—can be translated directly into many-country contexts by the simple device of thinking of countries as people. But that is not so. There is a fundamental difference between tax design in a many-country world and in a single country. In the latter case, there is naturally only a single government budget constraint to consider. In many-country settings, in contrast, each government will have its own distinct revenue constraint: that, indeed, is close to being a definition of an independent sovereign state. The countries of the world do not pool their tax revenues.

The implications for tax analysis of this obvious observation are profound. Consider, in particular, the classic Diamond-Mirrlees theorem (Peter A. Diamond and James A. Mirrlees, 1971, Theorem 4): that any Pareto-efficient tax structure is characterized by production efficiency so long as any pure profits are taxed at 100 percent and there are no restrictions on the distorting tax instruments that can be deployed. This has proved one of the most powerful results of optimal tax theory.[2] But the proof of the theorem presumes there to be only a single government budget constraint. Thus, the Diamond-Mirrlees (DM) theorem simply does not apply in international settings.[3]

This has potentially profound implications. Consider, for instance, what are perhaps the three central tenets in the normative theory of international taxation:

- The destination principle for commodity taxation (according to which commodities are taxed according to where they are consumed) is superior to the origin principle (under which they are taxed according to where produced);
- The residence principle for capital income taxation (under which taxation is by the country in which the investor resides) is preferable to the source principle (taxation by the country in which the income arises); and
- Free trade is better than restricted trade.

Of course it is well known that these are not universal truths. There are many circumstances in which one or all of these claims is incorrect. Origin taxation can be superior to destination, for instance, if taxes are set noncooperatively (Ben Lockwood, 1993) or in the presence of imperfect competition (Keen and Sajal Lahiri, 1998); an element of source taxation may be desirable when rents cannot be fully taxed (Harry Huizinga and Søren Bo Nielsen, 1997; Keen and Hannu Piekkola, 1997); and there are familiar arguments that can, in principle, be used to justify protectionist trade policies—administrative constraints on the ability of governments to collect taxes on domestic transactions (Shantayanan Devarajan et al., 1996), market imperfections and externalities (such as environmental considerations or Adam Smith's infant industry argument and its modern descendants) can all provide a theoretical rationale for trade interventions. Nevertheless, these three tenets are widely accepted as central benchmarks by which much policy advice in the area of international taxation is framed: the perceived preferability of destination taxation, for example, has been a powerful consideration in the policy advice given to the CIS countries and in discussions of the definitive VAT regime for the European Union. And much discussion of capital income tax coordination begins with the view that the issue has substance only because residence taxation is administratively infeasible (Vito Tanzi, 1995). But the interest the profession attaches to these "exceptions to the rule" is in itself testimony to the central role played by these basic tenets.

The important and troubling common feature of these tenets, for present purposes, is that such intellectual appeal as they possess seems—

and Craig Brett (1999) and Christos Kotsogiannis and Moltos Makris (1999). Jeremy Edwards (2002) explores further a number of the results derived in this paper.

[2] For example, it has provided much of the intellectual basis for replacing the cascading turnover taxes previously found in many developing and transition economies by the VAT, which does not tax intermediate transactions.

[3] The setting of the DM theorem also precludes an important feature of reality in international taxation by presuming that all consumers face the same prices, a point stressed by Stefan Homburg (1998). But in this respect the problem of international tax design is evidently *less* constrained than the regular DM one, so that one would not on this account expect the desirability of production efficiency to be overturned; and Proposition 2 below verifies that it is not.

especially in the first two cases—to derive largely from the view, sometimes made explicit,[4] that they are essentially just applications of the DM theorem on production efficiency. The destination principle, for instance, treats all firms equally, irrespective of their location, and so will ensure (in a competitive market, and in the absence of other distortions) that all firms have the same marginal cost of production in equilibrium, and hence that the global allocation of production is efficient. Under the origin principle, in contrast, arbitrage by consumers will ensure that consumer prices are equated across countries (so that there is exchange efficiency); but then firms in different locations will face different net prices if they face different origin-based taxes, and the allocation of production will be inefficient. Very similar arguments apply to the comparison between residence and source taxes: under the latter, since consumers in all countries will require the same net of tax return on their investments, pretax rates will differ across countries in the presence of unequal source-based taxes; residence taxation, in contrast, leaves intertemporal consumer prices differing across countries, but pretax returns equated. And free trade ensures production efficiency—in the absence of origin-based taxes—by ensuring that producers in all countries face the same prices.[5]

Appealing as these arguments appear to be, the distinctness of national revenue constraints that is of the essence in the international context means that the DM theorem cannot be invoked as formal justification for them.[6] This observation thus removes what appears to be a central intellectual underpinning of these ideas: contrary to apparently widespread belief, none of them can be rationalized by appealing to the Diamond-Mirrlees theorem. Consequently, none is as trustworthy as has been widely believed.

The task of characterizing Pareto-efficient international tax structures thus remains both open and pressing. Is production efficiency in the allocation of the world's resources desirable even when distinct countries face distinct revenue constraints? If not, what features do Pareto-efficient international tax structures possess? These are the questions to which this the paper is addressed.

Section I below presents a theoretical framework within which the problem of Pareto-efficient taxation in an international setting can be investigated. This framework is essentially the Arrow-Debreu model of competitive general equilibrium used by DM but recast in an international context. Section II presents the principal results, and Section III takes up the (harder) task of developing the intuition behind them. Section IV discusses the policy implications, and Section V concludes.

I. The Model

The framework within which we address these issues is a standard competitive trade model, augmented to allow countries to deploy both destination-based commodity taxes and tariffs, and—crucially—to recognize distinct revenue constraints for the distinct countries. This provides a very general setting; we shall spell out the ways in which it encompasses all three of the specific questions of principle referred to above.

The world consists of S countries. In each there are $L \equiv T + N$ commodities: the first T are tradable, the rest—including, in particular, internationally immobile factors like labor, as well as nontraded consumption goods like housing—are not. There is in each country s a single representative consumer with preferences described by an expenditure function $e^s(\mathbf{q}^s, \mathbf{g}^s, u^s)$ defined over the L-vector[7] of consumer prices $\mathbf{q}^s$, an L-vector of publicly provided goods $\mathbf{g}^s$, and utility u^s.[8] Consumer prices are

[4] See, for example, Alberto Giovannini (1989) and Keen (1996).

[5] The link between free trade and the production efficiency theorem is well known: see DM (1971, pp. 25–26) and many others, such as Partha Dasgupta and Stiglitz (1974), and Nicolas Stern (1987).

[6] Of course the conditions of the DM theorem are far from trivial even in a closed economy. The problem raised here, however, is of a quite different kind: it is that the economic environment presumed in the proof of the DM theorem is inherently inapposite in international settings.

[7] All vectors are column vectors, and a prime indicates transposition. Superscripts refer to countries.

[8] The assumption of a representative household in each country means that none of our results hinge on the unavailability of policy instruments to achieve desired distributions

partitioned in obvious notation between those relating to tradables and nontradables as $\mathbf{q}^s \equiv (\mathbf{q}_\mathbf{T}^{s\prime}, \mathbf{q}_\mathbf{N}^{s\prime})'$. Producer prices, similarly, are $\mathbf{p}^s \equiv (\mathbf{p}_\mathbf{T}^{s\prime}, \mathbf{p}_\mathbf{N}^{s\prime})'$. We abstract from issues related to public production by supposing that the public provision $\mathbf{g}^{s\prime} \equiv (\mathbf{g}_\mathbf{T}^{s\prime}, \mathbf{g}_\mathbf{N}^{s\prime})'$ simply arises from public purchases of that amount.[9] Destination-based commodity taxes in country s are $\mathbf{t}^s \equiv \mathbf{q}^s - \mathbf{p}^s$. World prices of the T tradable goods, which are of course common to all countries, are given by the T-vector $\boldsymbol{\omega}$. Tariffs are $\boldsymbol{\tau}^s \equiv \mathbf{p}_\mathbf{T}^s - \boldsymbol{\omega}$; thus $\tau_i^s > 0$ means an import tariff if i is imported, and an export subsidy if it is exported. Note too—a point of some importance for later intuition—that while world prices $\boldsymbol{\omega}$ are something of a fiction in the sense that no private agent may trade at them, they do matter for the revenues that national governments collect.

To isolate the implications for the desirability of production efficiency of the distinctness of national revenue constraints, we assume throughout that all other conditions of the DM theorem are satisfied: in particular, there are no constraints on the distortionary taxes that may be deployed, and each country taxes pure profits (which arise, as in the present model, in the presence of decreasing returns to scale) at 100 percent. If either of these conditions fails, it is well known that production inefficiency may be desirable even in the single-country DM framework: see David Newbery (1986) on the former point and Dasgupta and Stiglitz (1972) and Mirrlees (1972) on the latter. The assumption of 100-percent taxation of pure profits can be shown to imply, in the present context, that Pareto efficiency for the world as a whole requires production efficiency *within* each country: the proof, omitted here for the sake of brevity, can be found in Wildasin (1977). Given this, the production technology in each country can be described by a profit function $\pi^s(\mathbf{p}^s)$. Attention thus focuses on the possibility that production inefficiency may be desirable not within particular countries—that can be, and is, ruled out—but between them, consequent upon differences in the producer prices that they face.

With full taxation of pure profits, consumers have no lump-sum income. Their budget constraints are thus

$$(1) \qquad e^s(\mathbf{q}^s, \mathbf{g}^s, u^s) = 0, \quad s = 1, \dots, S.$$

The revenue constraint in each country requires that

$$(2) \quad (\boldsymbol{\omega}, \mathbf{p}_\mathbf{N}^s)' \cdot \mathbf{g}^s = \pi^s(\mathbf{p}^s) + (\mathbf{q}^s - \mathbf{p}^s)' \cdot (\mathbf{e}_q^s(\mathbf{q}^s, \mathbf{g}^s, u^s) + (\mathbf{p}_\mathbf{T}^s - \boldsymbol{\omega})' \cdot \{\mathbf{e}_\mathrm{T}^s(\mathbf{q}^s, \mathbf{g}^s, u^s) - \boldsymbol{\pi}_\mathrm{T}^s(\mathbf{p}^s)\} + \boldsymbol{\omega}' \cdot \boldsymbol{\alpha}^s$$

where $\mathbf{e}_\mathbf{T}^s \equiv \partial e^s/\partial \mathbf{q}_\mathbf{T}$, $\boldsymbol{\pi}_\mathbf{T} \equiv \partial \pi^s/\partial \mathbf{p}_\mathbf{T}$ (a notational convention for price derivatives that we shall use throughout) and $\boldsymbol{\alpha}^s$ denotes a T-vector of unrequited transfers received by country s. For reasons that will become apparent, we assume that such transfers are feasible only among a subset comprising the first $\bar{S}$ countries (thus $\sum_{s=1}^{\bar{S}} \boldsymbol{\alpha}^s = \mathbf{0}_T$, where $\mathbf{0}_k$ denotes the k-vector of zeros); this may be an empty set, or contain all countries, or anything in between. Equation (2) shows that the total value of government inputs (on the left) equals the revenue derived from taxing profits (the first term on the right-hand side), from taxation of consumer purchases (the second), from tariffs on trade with the rest of the world (the third), and from international transfers (the fourth).

Market-clearing for tradables requires that

$$(3) \quad \sum_{s=1}^{S} \{\mathbf{e}_\mathrm{T}^s(\mathbf{q}^s, \mathbf{g}^s, u^s) + \mathbf{g}_\mathrm{T}^s - \boldsymbol{\pi}_\mathrm{T}^s(\mathbf{p}^s)\} = \mathbf{0}_T$$

and for nontradables that

$$(4) \quad \mathbf{e}_\mathrm{N}^s(\mathbf{q}^s, \mathbf{g}^s, u^s) + \mathbf{g}_\mathrm{N}^s - \boldsymbol{\pi}_\mathrm{N}^s(\mathbf{p}^s) = \mathbf{0}_N, \qquad s = 1, \dots, S.$$

The homogeneity properties of the functions in (1)–(4) in the variables $\mathbf{q}^s$, each of the $\mathbf{p}^s$, and in $\boldsymbol{\omega}$ imply that, without loss of generality, we

of income within countries. The immobility of labor is implicit in this assumption; the issues that we analyze would continue to arise, however, in models with labor mobility.

[9] The utility function is thus defined over inputs to the public sector production process, the technology through which these inputs produce public goods and services valued by the consumer being subsumed within the form of the expenditure function.

can take the first tradable as numeraire, bearing no tax or tariff in any country: that is, in obvious notation, $q_1^s = p_1^s = \omega_1 = 1$, $\forall s$. To focus on tax rather than spending issues, we assume that the government purchases only the numeraire commodity, and that the public use of this good does not affect the compensated demands for any good other than the numeraire (see Wildasin, 1979). Thus (2) reduces to[10]

$$(5)\quad g^s = \pi^s(\mathbf{p}^s) + (\mathbf{q}^s - \mathbf{p}^s)' \cdot \mathbf{e}_q^s(\mathbf{q}^s, u^s) + (\mathbf{p}_{\mathbf{T}}^s - \omega)' \cdot \{\mathbf{e}_{\mathbf{T}}^s(\mathbf{q}^s, u^s) - \boldsymbol{\pi}_{\mathbf{T}}^s(\mathbf{p}^s)\} + \alpha^s$$

and, using Walras' Law to drop the market-clearing condition for good 1, (3) becomes

$$(6)\quad \sum_{s=1}^{S} \{\mathbf{e}_{\mathbf{T}}^s(\mathbf{q}^s, u^s) - \boldsymbol{\pi}_{\mathbf{T}}^s(\mathbf{p}^s)\} = \mathbf{0}_{T-1},$$

Equilibrium is thus described by the system (1), (4) (with $\mathbf{g}_{\mathbf{N}}^s = \mathbf{0}_N$), (5), and (6).

This framework is very general—sufficiently so, in particular, to encompass the three issues raised in the introduction. Destination-based commodity taxes and tariffs appear directly as $\mathbf{t}$ and $\boldsymbol{\tau}$ respectively; origin-based taxes appear implicitly, being equivalent to destination-based consumption taxes and export taxes/import subsidies levied at the same rate.

Despite its outwardly atemporal form, the model also encompasses the issues of capital income taxation raised in the introduction. This is by the simple device of taking commodities to be distinguished not only (or perhaps at all) by their physical characteristics but also by the date at which they are available, along the lines of, for example, Gérard Debreu (1959, pp. 33–34). The consumer prices $\mathbf{q}$ are then present value prices that reflect the path of net interest rates faced by the consumer; the producer prices $\mathbf{p}$ are the present value prices faced by firms.[11] Under residence-based taxation, consumers pay tax on their capital income, whichever country it arises in, at the rate specified by the country in which they live; assuming these rates to be different, this translates into cross-country difference in consumer prices. Producers in the two countries, meanwhile, must then pay the same pretax rate of return if they are to attract any savings, so that producer prices are the same across countries. Translated into the language of the framework above, the situation is thus equivalent to one in which there are destination-based commodity taxes but no tariffs: the outcome is characterized by production efficiency. Under source-based taxation, in contrast, income arising in different countries is taxed (potentially) at different rates. Consumers in different countries are now all in the same tax position, and hence—if they are to attract any savings—firms in different countries will have to earn different pretax returns in order to offer savers the same posttax return as is available elsewhere. A source-based capital income tax is thus exactly analogous to origin-based taxes in the outwardly atemporal model above, and so would appear in the guise of a destination-based

[10] We abuse the notation slightly by henceforth using $\mathbf{q}$ and $\mathbf{p}$ to refer to the $L - 1$ nonnumeraire goods, the subscript $\mathbf{T}$ to refer to the $T - 1$ nonnumeraire tradables, and interpreting α^s as a scalar relating to the transfer of good 1.

[11] It may be helpful to spell this out for the familiar special case of a two-period life-cycle savings model—here assuming no taxes, so as to focus on the analytical structure—with a single good (that can be used for either consumption or production) available in each period and an endowment E of the good in the first period only. Denoting consumption in period i by C_i, preferences are described by the expenditure function

$$e(q_1, q_2, u) = \min_{C_1 C_2}\left\{\rho_1 C_1 + \frac{\rho_2 C_2}{1+R} \,\middle|\, U(C_1, C_2) = u\right\}$$

where the ρ_i denote atemporal prices, $q_1 = \rho_1$, and q_2 is the present value price $\rho_2/(1 + R)$. Firms acquire capital in an amount K in period 1, which produces output $F(K)$ in period 2, with the capital itself (assumed for simplicity not to depreciate) then being sold. So the profit function is

$$\pi(p_1, p_2) = \max_K \left\{\frac{\rho_2(F(K) + K)}{1+R} - \rho_1 K\right\},$$

with, in particular, $p_2 = \rho_2/(1 + R)$. [Note that this gives the familiar equality between the marginal value product of capital $\rho_2 F'(K)$ and the user cost of capital (for this simple case) of $(R - (\rho_2 - \rho_1)/\rho_1)\rho_1$.] Using these forms of the expenditure and profit function (noting that savings are $e - \rho_1 C_1$), the market-clearing condition (3) for period 1 is the equality between global savings and global investment, while that for period 2 is the equality between global consumption and the accumulated value of savings.

commodity tax combined with an export tax/subsidy at the same rate.[12]

The formal analysis, which uses Motzkin's Theorem of the Alternative to characterize a Pareto-efficient policy,[13] is presented in the Appendices. Appendix A presents the necessary conditions for Pareto-efficient taxation. These conditions involve variables z^s, one for each country, that can be interpreted, somewhat loosely, as the negative of the implicit or shadow value (evaluated at the Pareto-efficient allocation being characterized) of the utility of the household residing in each country. It can be shown that each of these z^s's must be strictly negative if every government's budget constraint is binding at an optimum, which we assume to be the case.[14]

II. Pareto-Efficient International Tax Structures

This section seeks to characterize Pareto-efficient international tax structures. Given the central importance that the Diamond-Mirrlees theorem has come to have in policy design and evaluation, we focus in particular on the question of whether or not Pareto-efficient international taxation requires production efficiency in the use and production of mobile goods and factors: that is, on whether Pareto efficiency requires $\mathbf{p}_\mathbf{T}^s = \mathbf{p}_\mathbf{T}$ in all countries. Except where indicated, it is assumed that the lump-sum transfers α^s cannot be deployed: the instruments at the planner's disposal are thus $\mathbf{q}$, $\mathbf{p}$, and $\boldsymbol{\omega}$.

We start with a fairly bald statement of the formal results themselves, and then turn in the next section to the more difficult task of developing the economic intuition behind them.

The following establishes two key features of any Pareto-efficient allocation:

PROPOSITION 1: *At any Pareto-efficient allocation, in every country s:*

(*a*) (*Ramsey rule*) $\mathbf{t}^{s\prime} \cdot \mathbf{e}_{\mathbf{qq}}^s = \theta^s \mathbf{e}_\mathbf{q}^{s\prime}$, *where* $\theta^s \equiv -(1 + e_g^s)/e_g^s$; *and*
(*b*) (*Collinearity of tariff vectors*) $\boldsymbol{\tau}^s = \lambda^s \boldsymbol{\tau}$, *with* $\lambda^s \equiv 1/z^s e_g^s > 0$.

PROOF:
See Appendix B.

Part (a) of Proposition 1 calls for destination-based consumption taxes to be set in accordance with the Ramsey rule, whose optimality in closed economies is familiar.[15]

Part (b) of the proposition is more striking. Remember that producers in country s face prices $\mathbf{p}^s = \boldsymbol{\omega} + \boldsymbol{\tau}^s$. It follows that production will be efficient if and only if the tariff vectors $\boldsymbol{\tau}^s$ are the same for all countries. Since there is in general no reason to suppose that $\boldsymbol{\tau} = 0$, nor that $z^s e_g^s$ takes the same value for all s, the implication is that production efficiency is typically *not* a requirement for Pareto efficiency. Instead, producer prices for tradable goods $\mathbf{p}_\mathbf{T}^s$ must generally differ across countries.

Notice too that any production inefficiency takes a very particular form: the tariff vectors of the various countries are collinear at any Pareto-efficient allocation. That is, all countries set the same *relative* tariff rates, differing only in the *level* at which tariffs are set.

Proposition 1 applies whether or not lump-sum international transfers are possible. However:

PROPOSITION 2: *Suppose that lump-sum transfers can be made among some group of countries. Then any allocation that is Pareto-efficient from the perspective of the world as a whole involves production efficiency within that subgroup.*

[12] While this framework can thus be used to address issues of capital income taxation, it should be noted that it is not the only possible such framework: with one individual in each country, this is effectively the dynastic approach to savings behavior. Alternative approaches, based on overlapping generations or the Blanchard-Yaari model, would require a different structure.

[13] Edwards (2002) provides an analysis of the problem as one of constrained optimization.

[14] The problem of optimal taxation essentially presupposes such a condition. Formally, a sufficient but far from necessary condition for this to be true is that the income elasticity of demand for all taxed commodities is zero.

[15] Note that the absence of supply responses from the rule in (a) reflects the assumed taxation of pure profits at 100 percent (see Dasgupta and Stiglitz, 1972, and Alan J. Auerbach, 1985).

PROOF:

See Appendix C.

If—in the extreme case—transfers can be made between all countries, Pareto efficiency thus requires global production efficiency.

The next result shows, however, that Pareto efficiency may still require global production efficiency even when no international transfers can be made:

PROPOSITION 3: *Pareto-efficient international taxation requires global production efficiency if the matrix* **M** *of net import vectors*[16] *is of maximal column rank (i.e., of rank* $S - 1$*).*

PROOF:

See Appendix D.

Propositions 2 and 3 do not, of course, imply that there exist Pareto-efficient allocations marked by production inefficiency, merely that such allocations can exist only if the conditions of those propositions fail. It can be shown by example, however, that there *do* exist such allocations:[17] that is, the possibility that Pareto efficiency in international taxation will require production inefficiency is a real one.

III. Interpretation and Intuition

The central implication of the results in the preceding section is that the principles at the heart of most policy advice on international tax design are generally not appropriate: there are circumstances in which the world must depart from production efficiency in the allocation of its resources in order to attain a preferred point on its second-best utility possibility frontier, or to ensure a Pareto gain from a desirable reform of domestic taxation within one country.

Proposition 1 gives a conceptually simple scheme for attaining constrained Pareto efficiency in international taxation. Loosely speaking, moving along the world's second-best utility possibility frontier requires that commodity taxes be set at all times according to the Ramsey rule and that all countries set the same relative tariff rates but scaled up or down according to the implicit social weight attached to each. Thus production efficiency loses its primacy in international settings. It might conceivably have been optimal, for instance, to retain production efficiency but move around the utility possibility frontier by restructuring commodity taxes so as to trigger redistribution through changes in world prices; but, in general, it is not. Even when all other conditions of the DM theorem are met, the distinctness of national budget constraints—unless it can be overcome by interjurisdictional transfers, explicit (as in Proposition 2) or implicit (which, as we shall see, is the import of Proposition 3)—means that in international settings production efficiency ceases to be a prerequisite for Pareto-efficient taxation.

To see the precise economic meaning and intuition behind the formal results of the previous section, it is helpful to start with Proposition 2. Take first the case in which lump-sum transfers are possible between all countries. The result that global production efficiency is then a prerequisite for Pareto efficiency is essentially a trivial extension of the DM theorem. When lump-sum transfers among countries are feasible, their separate government budget constraints are, in effect, merged into a single global government budget constraint. In this case, the only difference between the optimal taxation problem with many countries that we analyze here and the standard DM optimal tax model is that, in the international tax version of the problem, consumers in different countries may potentially face different prices, corresponding to their respective national tax systems, whereas in the traditional model all consumers face the same taxes. And, in general, different preference structures mean that the Ramsey-optimal tax structures will indeed involve taxing any given commodity at different rates in different countries. Aside from this distinction, however, the international taxation problem is fundamentally no different from the standard DM problem when lump-sum transfers between countries are possible. In this case,

[16] This is the matrix **M** defined in (A13) below, whose typical column is the $(T - 1)$-vector of imports of country s, with the element corresponding to the numeraire removed.

[17] Any such example is bound to be cumbersome—as will become clear shortly, it must have at least three countries—so that details are omitted. The example is available on request.

intuition suggests, and Proposition 2 confirms, that international trade should not be the subject of fiscal interventions that distort the global pattern of production.

Essentially the same logic applies when transfers are feasible only among some subset of countries, as for instance when they are members of some federation with access to horizontal transfers among the lower-level jurisdictions. They then form a single DM economy, and Pareto efficiency requires production efficiency in their *internal* allocation of resources. And internal production efficiency requires—for the reason spelled out after Proposition 1(b)—that all members of the federation set the same tariff vector. Thus free trade within federations, combined with the application of a common external tariff, is a precondition for Pareto efficiency for the world as a whole. Simply put, global Pareto efficiency requires that each federation form a customs union—a point which, like much else in this area, has often been taken for granted without any proper analysis.

At first sight, Proposition 3 appears to bear little relation to Proposition 2. The rank condition is simply that the net import vectors have as much linear independence as possible (given that market-clearing requires them to add to zero). As there are S such vectors, this requires that the rank of the matrix $\mathbf{M}$ be $S - 1$. This in turn requires that $\mathbf{M}$ have at least as many rows as it has columns, and hence[18] that $T \geq S$. Very loosely speaking (in taking maximal rank for granted), production efficiency is desirable if there are at least as many tradable goods as there are countries. To take one simple case, consider a world with just two countries. The possibility of international trade presupposes the existence of at least two traded goods. By Proposition 3, production efficiency must therefore characterize any Pareto-efficient tax structure in a two-country world, even if lump-sum international transfers are not possible.

To see the connection between Propositions 2 and 3, it is only necessary to note that trade policy—taxes and subsidies on trade flows—can effectively transfer fiscal resources among countries in a lump-sum fashion, provided that there are at least as many traded goods as countries and that the import/export vectors of all countries are linearly independent. This is easiest to see in the case of two countries and two traded goods, with country A importing the nonnumeraire traded good 2. For suppose that A imposes an import tariff on 2 while country B imposes an export subsidy on 2 of exactly the same magnitude (so that $\tau_2^B = \tau_2^A > 0$). The effect of these trade policies is to transfer net fiscal resources from country B to country A, with no departure from globally efficient production. Offsetting taxes and subsidies on internationally traded goods shift fiscal revenues among countries but do not distort production. Clearly, then, in the case of two countries with two traded goods, the ability to undertake direct lump-sum international transfers is superfluous: any desired transfers between the two countries can just as well be achieved by the use of offsetting trade policies, in which the country that collects a tax gains revenue at the expense of the country that pays a subsidy.

The equivalence between trade interventions and lump-sum transfers obviously continues to hold if there are two countries and three or more traded goods; in this case, tariffs/subsidies for all but one traded good can be set equal to zero, while instituting offsetting taxes/subsidies on the remaining traded good. More generally, if there are three or more countries and if the number of traded goods is at least as large as the number of countries, and if the net import vectors are linearly independent, then controlling the tariffs and subsidies of each country appropriately provides enough degrees of freedom to achieve any desired pattern of international fiscal transfers. More formally, note from (5) that a perturbation of world prices $\mathbf{d}\boldsymbol{\omega}$, holding constant all consumer and producer prices—and hence also all relevant behavioral decisions—implies an effective pattern of international transfers given by the S-vector $\mathbf{d}\boldsymbol{\alpha}^* = \mathbf{M}' \cdot \mathbf{d}\boldsymbol{\omega}$, with (denoting the vector of ones by $\boldsymbol{\iota}$) the market-clearing condition (6) implying $\boldsymbol{\iota}'\mathbf{d}\boldsymbol{\alpha}^* = 0$. If $\mathbf{M}$ has its maximal possible column rank of $S - 1$, then by manipulating world prices it is possible to achieve any de facto pattern of international lump-sum redistribution

[18] Recall that $\mathbf{M}$ relates only to nonnumeraire tradables, so has $T - 1$ rows.

that might be required.[19] With nondistorting redistribution possible by this indirect route, the potential rationale for using distorting tariffs to redistribute tax revenues across countries vanishes. Production efficiency is again desirable.

What, then, of the case for production inefficiency if the conditions of Propositions 2 and 3 are not satisfied? This takes us back to Proposition 1 and the general characterization provided there, which is valid for any number of goods and countries.

Proposition 1(a)—the Ramsey rule—is evident enough. Proposition 1(b) implies that any trade interventions that occur with Pareto-efficient policies will leave global production efficiency undisturbed provided that $z^s e_g^s$, which is strictly positive, takes the same value for all countries s. With z_s interpreted as the (negative of the) social weight attached to the welfare of country s, this term can be thought of as the shadow value of fiscal resources in country s: the lower the value of tax revenue in country s, the lower will be the marginal valuation of public spending, $-e_g$, weighted by the multiplier z^s. Proposition 1(b) indicates that, relative to a situation in which all countries set equal and offsetting tariffs and export subsidies (so that production is efficient), a country that is disfavored in the Pareto-efficient allocation (i.e., one that is "fiscally rich," in the sense of having a low shadow value of government revenue) taxes imports or subsidizes exports more heavily.

As we have already seen, these fiscal interventions in the markets for imports and exports need not necessarily lead to production inefficiency—not if the number of traded goods is larger than the number of countries. But suppose the contrary; for example, let there be three countries and two traded goods, such that countries A and B import commodity 2, which is exported by country C. Suppose, further, an initial situation in which country A is very "fiscally poor" in the sense that government revenue there is highly valued (the value of $z^A e_g^A$ is very high), that the other importing country, B, has a lower shadow value of government revenue (the value of $z^B e_g^B$ is not so high), and that the shadow value of government revenue in the exporting country C (the value of $z^C e_g^C$) is even lower. If lump-sum international transfers were possible, Pareto-efficient taxation would require that country C transfer fiscal resources to country A; depending on the exact shadow values, country B might also receive some transfers from country C or perhaps it too would pay some transfers to country A. If such lump-sum transfers are not possible, what sorts of trade interventions might achieve similar results?

Following the logic of the two-country case, the "fiscally rich" or "disfavored" country C could subsidize its exports of good 2 so as to transfer fiscal resources abroad. If both countries A and B were to impose offsetting import tariffs, they would then both be the beneficiaries of fiscal transfers from country C and production efficiency would be preserved. However, these policies are ineffective in targeting transfers from country C, with a low shadow value of revenue, to country A, the country with the highest shadow value, since some of the subsidy paid by country C accrues to the government of country B. Instead, says Proposition 1(b), country B, with its lower shadow value of government revenue, should impose a higher tariff on imports than country A. This would have the effects of increasing the volume of trade and the volume of fiscal transfers between countries C and A and of reducing the volume of trade and transfers between C and B, thus directing country C's subsidies more effectively toward the "favored" country A. While differentiating the tariff rates of countries A and B serves this useful purpose, it also results in production inefficiency: the producer price of good 2 will be higher in the country (B) with the higher tariff, as compared with the other countries. This is the source of the production inefficiency in the general case: the production efficiency is a price worth paying to achieve a desired redistribution of tax revenue.

It now remains only to explain the collinearity between tariff vectors that Proposition 1(b) shows to be required for Pareto-efficient international taxation. For this, imagine a small change in consumer prices q in countries A and B which has the sole effect of increasing A's imports of good j by an amount $dm_j^A > 0$ while reducing B's imports of j by the same amount,

[19] In a similar spirit, Arja H. Turunen-Red and Alan D. Woodland (1996, Theorem 2) show that adjustment of tariffs/subsidies on traded goods can be used to redistribute the efficiency gains from reforms of quotas on traded goods.

$dm_j^B = -dm_j^A$. World prices and producer prices in all countries remain unchanged; consumer prices have changed only in countries A and B, and as an envelope property these changes have negligible welfare effects. Thus the only welfare effects are those arising from the impact on tariff revenues in A and B. These give a net welfare gain of $(z^A e_g^A \tau_j^A - z^B e_g^B \tau_j^B) dm_j^A$, so that at an optimum it must be the case that

$$\frac{\tau_j^A}{\tau_j^B} = \frac{z^B e_g^B}{z^A e_g^A}. \tag{7}$$

But this same argument can be applied to any good k, and to any pair of countries, so that (7) implies

$$\frac{\tau_j^A}{\tau_k^A} = \frac{\tau_j^B}{\tau_k^B}, \quad \forall j, k, A, B \tag{8}$$

which gives the collinearity of tariff vectors. Loosely speaking, tariffs and subsidies on traded goods serve as devices for transferring revenues between countries. Effective deployment of these instruments requires simply scaling up or down a common vector of tariffs: from the perspective of reallocating tax revenues, there is no point in varying tariff rates across countries in light of heterogeneities in their preferences or production patterns.

IV. Implications

What does all this imply for international tax policy design and, in particular, for the three tenets highlighted in the introduction?[20]

In the absence of explicit international transfers, Pareto-efficient international taxation generally involves the use of both destination-based consumption taxes and tariffs. (Or, if tariffs for some reason cannot be deployed, the use of origin-based taxes/production subsidies to achieve, in combination with consumption taxes, the same effect.) These tariffs, however, do not necessarily induce any production inefficiency. Their key role is to redistribute tax revenues across countries so as to ensure that desirable reforms of domestic taxation actually turn to all countries' benefit. When there are more tradable goods than countries (and there is enough linear independence in their import vectors), this redistributional role can be achieved without inducing any production inefficiency. When this condition is not met, however, the use of tariffs may nevertheless serve to ensure a Pareto gain from domestic tax reforms even though it entails some production inefficiency.

Applying the analysis to issues of capital income (by reinterpreting the model along the lines described towards the end of Section I), the implication is that Pareto efficiency typically requires both residence- and source-based taxes to be deployed. The reason for using the potentially distorting source-based taxes is essentially the same as that for using tariffs in the discussion above: to expand the set of Pareto-improving reforms by providing a way of reallocating tax revenues across countries. To see this most clearly, note that residence and source-based taxes can be used to replicate taxes on capital exports or imports. The combination of a residence-based tax and a source-based subsidy at the same rate, for instance, is equivalent to a tax on capital exports. Such taxes on capital exports and imports can then be used to move tax revenue between countries in just the same way as described for tariffs above. The use of a source-based tax in addition to a residence-based one thus signals, in effect, the taxation or subsidization of capital movements. The precise significance of such taxes on capital movements then turns on the relative numbers of goods (now distinguished also by date) and countries: with fewer countries than goods, these taxes are set so as to redistribute revenue without distorting production; with more countries than goods, they induce a production inefficiency.

The likelihood that Pareto-efficient international taxation will require production inefficiency thus hinges in this analysis upon the relative numbers of goods and countries. With about 200 countries in the world and over 5,000 commodities distinguished in the Harmonized Tariff System, it would seem that the most empirically relevant case is that in which tariffs can fulfill their role of rechanneling revenues without any need for production inefficiency.

[20] See also the discussion of these and related results in Edwards (2002).

This would also seem to be true a fortiori in the context of capital income taxation, since goods are then also distinguished by the date at which they are available. Matters are not quite this simple, however. For it is clear that what really determines the scope for reallocating tax revenues is not the number of commodities in itself but the number of distinct tariff rates that can be applied to them, and this is invariably a much smaller number, commonly in the order of zero to ten. A full treatment of this issue would thus need to recognize that tax and tariff rates are commonly imposed at the same rate on a range of goods (and, moreover, that this constraint in itself may imply that Pareto efficiency requires production inefficiency (Newbery, 1986). The important point for present purposes is that it is not as obvious as it may seem that the empirically relevant case is that in which production inefficiency can be ruled out.

It may seem far-fetched to think of tariffs and source-based capital income taxes being used to redistribute resources between countries. Certainly lump-sum transfers are at least as efficient (and more transparent) as a means of doing so. But it is important to recognize that there are, nevertheless, cases in which tax instruments are used in precisely this way. In Brazil, for instance, sales from richer provinces to poorer are subject to particularly low rates of VAT; through the crediting mechanism of the VAT, the (deliberate) effect of this—in relation to transactions between registered traders—is to shift revenue from richer provinces to poorer. Perhaps the most striking and important example, however, is in the international tax treatment of capital income. Many capital exporting countries—including the United States and the United Kingdom—levy tax on a residence basis but give a credit against that liability for taxes paid in the source country (with the credit limited to the residence-country liability, so that foreign tax is never refunded). Assuming the foreign tax rate to be no higher than the domestic—and it is a primary concern in many developing countries to ensure that this is the case—the effect of these arrangements is to leave decisions as to the location of investment undistorted but to transfer revenue from the residence country to the source country. Nor are the sums involved insignificant: on some accounts, the amount that the United States transfers to the exchequers of foreign countries in this way exceeds the amount of explicit U.S. foreign aid.

One other aspect of experience with systems of foreign tax crediting is also of interest here. In itself, the credit is not an especially well-targeted means of reallocating revenue, since it extends to investments in rich countries as much as to those in poorer ones. To sharpen the targeting of the benefits conveyed by the foreign tax credit, a number of countries—though not the United States—adopt the practice of "tax-sparing," by which they agree to disregard the reduction in source-country tax liabilities implied by investment incentives offered there (such as tax holidays), the point being that such reductions in source-country liability would otherwise be exactly undone by an increased residence tax liability. By preserving the benefits of source-country tax incentives, tax sparing distorts location decisions[21] and so induces a production inefficiency, with the residence country effectively subsidizing investments abroad. The situation is directly analogous to the three-country example of the previous section, with a production inefficiency accepted as the price of channeling the resources made available under its foreign tax credit by the "rich" country most effectively to the "poorest" country, bypassing to some degree the "middle income" country.

This is not to suggest that such institutions as the foreign tax credit and tax sparing—let alone tariffs—should be seen simply as means that nations have found to put themselves at a preferred point on the world's utility possibility frontier: there is clearly much more to them than that. The point is simply that they may indeed have such a role when, for some reason, explicit sharing of tax revenues is difficult; and that even when they distort, these devices may have a role to play in fully coordinated outcomes.

The analysis here also has implications for the internal organization of federations. It points, in particular, to a close link between the extent of horizontal transfers across the constituent states of a federation—serving to consolidate their otherwise distinct revenue constraints into one—and the desirability of internal ar-

[21] James R. Hines, Jr. (1998) finds evidence that tax sparing does indeed affect investment decisions.

rangements conducive to production efficiency in the allocation of the federation's resources. Where horizontal transfers are weak or nonexistent, Pareto efficiency within the federation may, for example, require the states to adopt measures, such as taxes on trade between them, that actually interfere with the functioning of the internal market. Conversely, enforcing the three tenets within federations which—as in the European Union—have only nascent internal redistribution might actually leave all member states worse off than they need be.

Not least, the present results also have implications for the modeling of international tax issues. Public finance theorists, like trade theorists, make much use of two-country models. Proposition 3 warns, however, that these models are inherently and seriously misleading: for since trade balance requires that there then be at least two traded goods, production efficiency will always be desirable in such settings. Indeed it is the need to think beyond the usual 2×2 framework that makes interpreting some of the results here so hard. The simplest case in which Proposition 1(b) on the collinearity of tariff vectors has force, for instance, is that in which there are three goods and four countries.[22] Unfortunately, coming to terms with this complexity is essential if one is to build a coherent framework for international tax analysis.

V. Summary and Conclusion

The analysis here has shown that Pareto-efficient international taxation may require production inefficiency in the allocation of the world's resources: tariffs and other policies that distort world production patterns may actually make all countries better off. This means that the three principles of international taxation identified at the outset are not, in general, valid guides to optimal policy. Source-based capital taxes, origin-based consumption taxes, and tariffs on international trade flows, even though they may distort production, can nonetheless be Pareto-improving and/or part of a Pareto-efficient tax structure. Of course, these results in no way suggest that policies that depart in arbitrary ways from traditional principles are Pareto-improving, and one must beware the danger that—as Francis Y. Edgeworth (1908, p. 555) feared in respect of Charles F. Bickerdike's (1906) discovery of the optimal tariff—these arguments might afford "... unscrupulous advocates of vulgar Protection a peculiarly specious pretext for introducing the thin end of the fiscal wedge." Indeed, the analysis has identified important cases where policies that cause production inefficiency are definitely *not* Pareto efficient.

The key consideration behind the potential for mutual benefit from production inefficiency is the existence of multiple government budget constraints, implying a potential gain from transferring revenues, directly or indirectly, among countries. When this can be done directly, through lump-sum intergovernmental transfers, the separate budget constraints facing different governments are, in effect, merged, and the standard production-efficiency theorem applies. Even if direct intergovernmental transfers are not possible, an appropriately designed system of trade interventions, generally involving offsetting taxes and subsidies in different countries, may achieve the desired intergovernmental redistribution of resources without inducing production inefficiency. For this purpose, it is necessary that there be sufficient degrees of freedom: specifically, at least as many traded commodities as there are countries. Outside these cases, however, pursuit of conventional wisdom in designing international tax arrangements may leave all countries worse off than they need be. The attempt to implement this conventional wisdom may therefore encounter resistance. And, by the same token, the expansion of the range of international policy coordination to include intergovernmental transfers or careful coordination of trade interventions that achieve desired transfers can open up opportunities for mutually beneficial elimination of tariffs that interfere with global production efficiency.

[22] At least three goods are needed because, by normalization, good 1 bears no tariff and collinearity of the tariff on a single good is vacuous; and then with less than four countries, Proposition 2 implies that tariffs are the same in all countries.

APPENDIX A: NECESSARY CONDITIONS FOR PARETO EFFICIENCY

The analysis proceeds by first perturbing the equilibrium conditions and then applying a theorem of the alternative to characterize Pareto-efficient structures.

For the perturbation, setting $e_u^s = 1$ for all s (without loss of generality) one finds from (1) that

$$du^s = -\mathbf{e}_{\mathbf{q}}^{s\prime} \cdot \mathbf{dq}^s - e_g^s dg^s. \tag{A1}$$

Perturbing (5), using (4), and substituting for dg^s in (A1) gives

$$a_u^s du^s = \mathbf{a}_{\mathbf{q}}^{s\prime} \cdot \mathbf{dq}^s + \mathbf{a}_{\mathbf{p}}^{s\prime} \cdot \mathbf{dp}^s + e_g^s \mathbf{m}^{s\prime} \cdot \mathbf{d\boldsymbol{\omega}} + e_g^s d\alpha^s \tag{A2}$$

where $\mathbf{m}^s \equiv \mathbf{e}_{\mathbf{T}}^s - \boldsymbol{\pi}_{\mathbf{T}}^s$ denotes the $(T-1)$-vector of imports of country s,

$$a_u^s \equiv 1 + e_g^s(\mathbf{t}^{s\prime} \cdot \mathbf{e}_{\mathbf{q}u} + \boldsymbol{\tau}^{s\prime} \cdot \mathbf{e}_{\mathbf{T}u}) \tag{A3}$$

$$\mathbf{a}_{\mathbf{q}}^{s\prime} \equiv -[\mathbf{e}_{\mathbf{q}}^{s\prime} + e_g^s(\mathbf{e}_{\mathbf{q}}^{s\prime} + \mathbf{t}^{s\prime} \cdot \mathbf{e}_{\mathbf{qq}}^s + \boldsymbol{\tau}^{s\prime} \cdot \mathbf{e}_{T\mathbf{q}}^s)] \tag{A4}$$

$$\mathbf{a}_{\mathbf{p}}^{s\prime} \equiv e_g^s \boldsymbol{\tau}^{s\prime} \cdot \boldsymbol{\pi}_{\mathbf{Tp}}^s \tag{A5}$$

and we partition the $(L-1) \times (L-1)$ matrix of compensated price effects as

$$\mathbf{e}_{\mathbf{qq}}^s \equiv \begin{bmatrix} \mathbf{e}_{\mathbf{Tq}}^s \\ \mathbf{e}_{\mathbf{Nq}}^s \end{bmatrix} \tag{A6}$$

where $\mathbf{e}_{\mathbf{Tq}}$ is $(T-1) \times (L-1)$. Perturbing (6) and (4) and stacking the result with (A2), one arrives at the system

$$\mathbf{J}_u \cdot \mathbf{du} = \mathbf{J}_{\mathbf{q}} \cdot \mathbf{dq} + \mathbf{J}_{\mathbf{p}} \cdot \mathbf{dp} + \mathbf{J}_{\omega} \cdot \mathbf{d\boldsymbol{\omega}} + \mathbf{J}_{\alpha} \cdot \mathbf{d\boldsymbol{\alpha}} \tag{A7}$$

where, denoting by $\mathbf{I}_k$ the k-dimensional identity matrix and by $\mathbf{0}_{a,b}$ the $a \times b$ matrix of zeros,

$$\mathbf{J}_u \equiv \begin{bmatrix} a_u^1 & 0 & \cdots & 0 \\ 0 & a_u^2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & a_u^S \\ \mathbf{e}_{\mathbf{Tu}}^1 & \mathbf{e}_{\mathbf{Tu}}^2 & \cdots & \mathbf{e}_{\mathbf{Tu}}^S \\ \mathbf{e}_{\mathbf{Nu}}^1 & 0 & \cdots & 0 \\ 0 & \mathbf{e}_{\mathbf{Nu}}^2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \mathbf{e}_{\mathbf{Nu}}^S \end{bmatrix} \quad \mathbf{du} \equiv \begin{bmatrix} du^1 \\ du^2 \\ \vdots \\ du^S \end{bmatrix} \tag{A8}$$

$$\mathbf{J}_{\mathbf{q}} \equiv \begin{bmatrix} \mathbf{a}_{\mathbf{q}}^{1\prime} & 0 & \cdots & 0 \\ 0 & \mathbf{a}_{\mathbf{q}}^{2\prime} & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \mathbf{a}_{\mathbf{q}}^{S\prime} \\ \mathbf{e}_{\mathbf{Tq}}^1 & \mathbf{e}_{\mathbf{Tq}}^2 & \cdots & \mathbf{e}_{\mathbf{Tq}}^S \\ \mathbf{e}_{\mathbf{Nq}}^1 & 0 & \cdots & 0 \\ 0 & \mathbf{e}_{\mathbf{Nq}}^2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \mathbf{e}_{\mathbf{Nq}}^S \end{bmatrix} \quad \mathbf{dq} \equiv \begin{bmatrix} \mathbf{dq}^1 \\ \mathbf{dq}^2 \\ \vdots \\ \mathbf{dq}^S \end{bmatrix} \tag{A9}$$

(A10)
$$\mathbf{J_p} \equiv \begin{bmatrix} \mathbf{a_p^{1\prime}} & 0 & \cdots & 0 \\ 0 & \mathbf{a_p^{2\prime}} & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \mathbf{a_p^{S\prime}} \\ -\boldsymbol{\pi}_{\mathbf{Tp}}^1 & -\boldsymbol{\pi}_{\mathbf{Tp}}^2 & \cdots & -\boldsymbol{\pi}_{\mathbf{Tp}}^S \\ -\boldsymbol{\pi}_{\mathbf{Np}}^1 & 0 & \cdots & 0 \\ 0 & -\boldsymbol{\pi}_{\mathbf{Np}}^2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & -\boldsymbol{\pi}_{\mathbf{Np}}^S \end{bmatrix} \quad \mathbf{dp} \equiv \begin{bmatrix} \mathbf{dp}^1 \\ \mathbf{dp}^2 \\ \vdots \\ \mathbf{dp}^S \end{bmatrix}$$

(A11)
$$\mathbf{J}_\omega \equiv \begin{bmatrix} e_g^1 & 0 & \cdots & 0 \\ 0 & e_g^2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & e_g^S \\ & \mathbf{0}_{T-1+S(1+N)} & & \end{bmatrix} \mathbf{M}' \quad \mathbf{d}\boldsymbol{\omega} \equiv \begin{bmatrix} \mathbf{d}\omega_2 \\ \mathbf{d}\omega_3 \\ \vdots \\ \mathbf{d}\omega_T \end{bmatrix}$$

(A12)
$$\mathbf{J}_\alpha \equiv \begin{bmatrix} e_g^1 & e_g^1 & \cdots & e_g^1 \\ -e_g^2 & 0 & \cdots & 0 \\ 0 & -e_g^3 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & -e_g^{\bar{S}} \\ & \mathbf{0}_{S-\bar{S}+T-1+SN,\bar{S}-1} & & \end{bmatrix} \quad \mathbf{d}\bar{\alpha} \equiv \begin{bmatrix} d\alpha^2 \\ d\alpha^3 \\ \vdots \\ d\alpha^{\bar{S}} \end{bmatrix},$$

where we have also defined the $(T-1) \times S$ matrix of net import vectors

(A13)
$$\mathbf{M} \equiv [\mathbf{e}_\mathbf{T}^1 - \boldsymbol{\pi}_\mathbf{T}^1, \mathbf{e}_\mathbf{T}^2 - \boldsymbol{\pi}_\mathbf{T}^2, \ldots, \mathbf{e}_\mathbf{T}^S - \boldsymbol{\pi}_\mathbf{T}^S].$$

In arriving at (A12), use has been made of the restriction that $\sum_{s=1}^{\bar{S}} \alpha^s = 0$.

By Motzkin's theorem of the alternative,[23] an initial equilibrium is either Pareto inefficient in the weak sense that[24] $\mathbf{du} > \mathbf{0}_S$ for some perturbation ($\mathbf{dq}$, $\mathbf{dp}$, $\mathbf{d}\boldsymbol{\omega}$, $\mathbf{d}\boldsymbol{\alpha}$) satisfying (A7) or there exists some $(S + T - 1 + SN)$-vector $\mathbf{y}$ such that $\mathbf{y}' \cdot \mathbf{J}_u \ll \mathbf{0}$ and $\mathbf{y}' \cdot \mathbf{J_k} = 0$ for each of the matrices $\mathbf{J_k}$, $\mathbf{k} = \mathbf{q}, \mathbf{p}, \boldsymbol{\omega}$ and, when transfers are allowed, for $\mathbf{k} = \boldsymbol{\alpha}$. It proves helpful to partition $\mathbf{y} = (\mathbf{z}', \mathbf{x}', \mathbf{x}^{1\prime}, \ldots, \mathbf{x}^{S\prime})'$ where $\mathbf{z}$ is an S-vector, $\mathbf{x}$ a $(T-1)$-vector and the $\mathbf{x}^s$ are all N-vectors.

Note that the claim in the text that $\mathbf{z} \ll \mathbf{0}$ when all income effects are zero for non-numeraire commodities follows immediately from (A3) and the condition of the alternative that $\mathbf{y}' \cdot \mathbf{J}_u \ll \mathbf{0}$.

Appendix B: Proof of Proposition 1

Supposing the initial allocation to be Pareto efficient, the conditions $\mathbf{y}' \cdot \mathbf{J_q} = \mathbf{0}$ and $\mathbf{y}' \cdot \mathbf{J_p} = \mathbf{0}$ imply respectively that

(B1)
$$z^s \mathbf{a_q^{s\prime}} + \mathbf{x}^{s*\prime} \cdot \mathbf{e_{qq}^s} = \mathbf{0}, \quad s = 1, \ldots, S$$

[23] Olvi L. Mangasarian (1969, p. 34).

[24] The notation $\mathbf{k} \gg 0$ means that all elements of the vector $\mathbf{k}$ are strictly positive; $\mathbf{k} > 0$ that all are nonnegative and at least one strictly positive.

and

$$z^s \mathbf{a}_{\mathbf{p}}^{s\prime} - \mathbf{x}^{s*\prime} \cdot \boldsymbol{\pi}_{\mathbf{pp}}^s = \mathbf{0}, \quad s = 1, \dots, S \tag{B2}$$

where $\mathbf{x}^{s*} \equiv (\mathbf{x}', \mathbf{x}^{s\prime})'$. Using the definition in (A5) and assuming there to be sufficient substitution in production that $\boldsymbol{\pi}_{\mathbf{pp}}$ is nonsingular, (B2) gives

$$\mathbf{x}_s^{*\prime} = z^s e_g^s \boldsymbol{\tau}^{s\prime} \cdot \boldsymbol{\pi}_{\mathbf{Tp}}^s (\boldsymbol{\pi}_{\mathbf{pp}}^s)^{-1}. \tag{B3}$$

Using in (B1) both (B3) and the definition of $\mathbf{a}_{\mathbf{q}}^s$ in (A4) one finds

$$\mathbf{e}_{\mathbf{q}}^{s\prime} + e_g^s(\mathbf{e}_{\mathbf{q}}^{s\prime} + \mathbf{t}^{s\prime} \cdot \mathbf{e}_{\mathbf{qq}}^s) = e_g^s \boldsymbol{\tau}^{s\prime} \cdot [\boldsymbol{\pi}_{\mathbf{Tp}}^s (\boldsymbol{\pi}_{\mathbf{pp}}^s)^{-1} \mathbf{e}_{\mathbf{qq}}^s - \mathbf{e}_{\mathbf{Tq}}^s] \tag{B4}$$

$$= 0, \tag{B5}$$

the second step following on noting that the partitioned form in (A6) implies that

$$\boldsymbol{\pi}_{\mathbf{Tp}}^s (\boldsymbol{\pi}_{\mathbf{pp}}^s)^{-1} \mathbf{e}_{\mathbf{qq}}^s = [\mathbf{I}_{T-1} | \mathbf{0}_{T-1,N}] \mathbf{e}_{\mathbf{qq}}^s = \mathbf{e}_{\mathbf{Tq}}^s. \tag{B6}$$

Part (a) follows from (B5). Part (b) follows on using (B6) to write (B3) as $(\mathbf{x}', \mathbf{x}^{s\prime})' = z^s e_g^s \boldsymbol{\tau}^{s\prime} \cdot [\mathbf{I}_{T-1} | \mathbf{0}_{T-1,N}]$.

Appendix C: Proof of Proposition 2

When explicit lump-sum transfers can be deployed, the alternative requires that $\mathbf{y}' \cdot \mathbf{J}_{\boldsymbol{\alpha}} = 0$. From the definition in (A12), this implies that $[z^1 e_g^1 - z^2 e_g^2, \dots, z^1 e_g^1 - z^{\bar{S}} e_g^{\bar{S}}] = \mathbf{0}_{\bar{S}-1}'$. Thus $z^s e_g^s = \kappa$ for all $s \in \bar{S}$, and the conclusion follows from Proposition 1(a).

Appendix D: Proof of Proposition 3

Among the conditions for the alternative is that $\mathbf{y}' \cdot \mathbf{J}_{\boldsymbol{\omega}} = \mathbf{0}$. Recalling the definition in (A11), this in turn implies that

$$\mathbf{M} \cdot \mathbf{v} = \mathbf{0}_{T-1} \tag{C1}$$

where $\mathbf{v} \equiv [z_1 e_g^1, \dots, z_S e_g^S]'$. Recall, too, from (6) that market clearing implies $\mathbf{M} \cdot \boldsymbol{\iota}_S = \mathbf{0}_{T-1}$, so that $\mathbf{M}$ has column rank of no more than $S - 1$. If it has precisely that rank, then $\mathbf{v}$ must be collinear with $\boldsymbol{\iota}$, and the result follows.

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