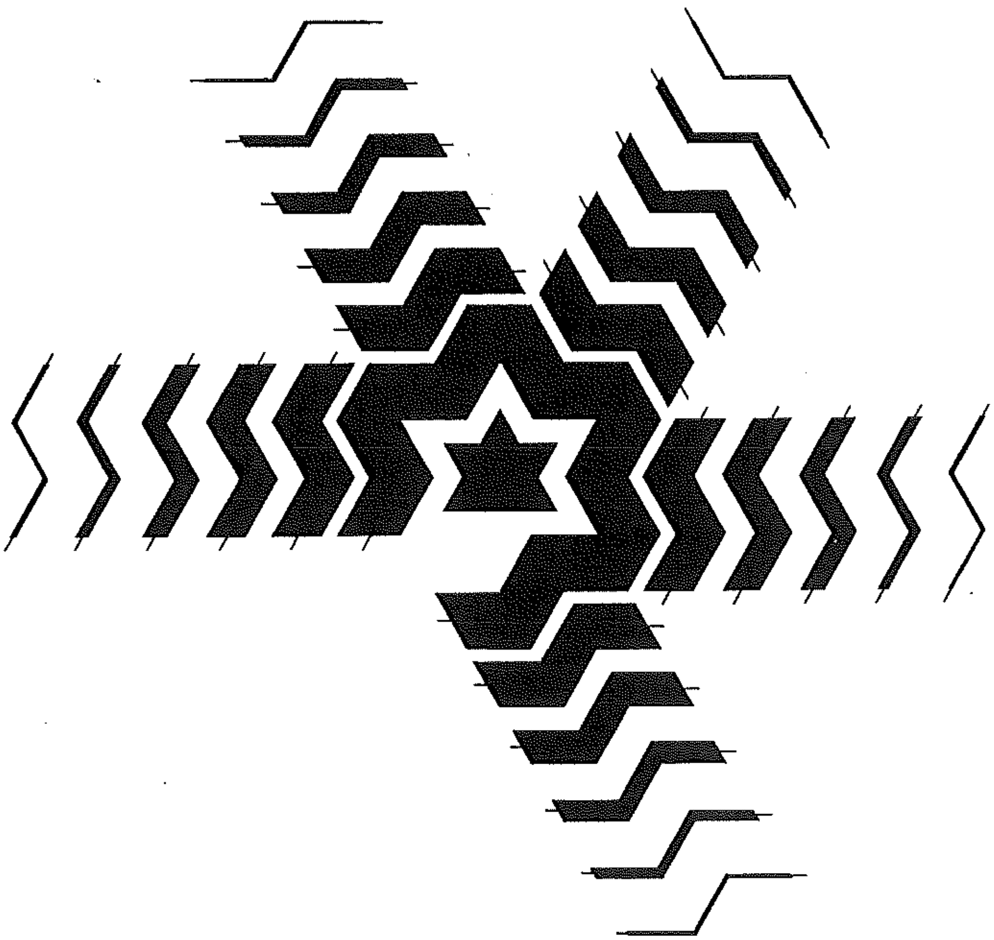
EQUALIZATION PAYMENTS:

Past, Present and Future

Thomas J. Courchene

Ontario Economic Council



A note on the analytics of the RFPS equalization formula

Thomas J. Courchene and David E. Wildasin*

For fifteen years, from 1967 to 1982, Canada's equalization program was geared, in principle at least, to ensuring that provinces had access to the all-province average of per capita revenues for a wide but not encompassing set of provincial revenues. This approach to equalization has come to be known as the representative tax system (RTS) approach. However, since the standard of equalization was the all-province or national average, we shall refer to this system as the representative national-average standard (RNAS). Since April 1982, a new system has prevailed, namely the representative five-province standard (RFPS). As the name suggests, the poorer provinces' revenues are now to be brought up to the level of the five provinces that make up the standard. These are Ontario, Quebec, Manitoba, Saskatchewan, and British Columbia. Excluded from the RFPS are the richest province (Alberta) and the four poorest (the Atlantic Provinces). This switch from the RNAS to the RFPS has already generated a substantial policy literature. The purpose of this paper is more analytical in nature—to focus on the properties of the new RFPS formula both in isolation and in comparison with the RNAS approach.[1]

The first section of the paper presents the basic formulae for the RNAS and RFPS approaches. Following sections deal in turn with the impact on equalization of tax rate changes, tax base changes, and population movements. As one might expect, the results for the RFPS formula depend upon whether a province is part of the group of provinces making up the representative average standard.

* David E. Wildasin is a member of the Department of Economics of Indiana University. This paper was written while both authors were visiting Queen's University.

THE RNAS AND RFPS FORMULAE

The base per capita formulation

In per capita terms, the RNAS formula can be expressed as follows:

$$\frac{E_{ij}}{P_i} = t_{cj}\left(\frac{B_{cj}}{P_c} - \frac{B_{ij}}{P_i}\right), \tag{A.1}$$

where

Eij = equalization to province i from revenue source j,

t_{cj} = the national average (i.e., all-province) tax rate, defined as total revenues for revenue source j (TR_j), divided by the total base for source j (B_{cj}), where subscript c refers to Canada or the all-province total,

$\frac{B_{cj}}{P_c}$ = national average per capita base for revenue category j, and

$\frac{B_{ij}}{P_i}$ = province i's per capita base for source j.

For each province the entitlements are summed over the revenue sources and the total, if positive, is the province's per capita equalization payment. If the total is negative, the equalization payment is set equal to zero (i.e., the program is not an interprovincial revenue-sharing pool).

If one assumes that province i's tax rate is identical to the national average tax rate (i.e., $t_{ij} = t_{cj}$), the intuitive rationale underlying equation A.1 is straightforward—a province will receive in total per capita revenue (i.e., own revenues, $t_{ij} \cdot B_{ij}/P_i$, plus equalization) an amount equal to the product of the national average base and the national average tax rate. In effect, it levies its tax rate against the national average base.

The RFPS formula is similar except that the provinces' overall revenues are brought up to the per capita yield at national average rates in the five provinces constituting the RFPS:

$$\frac{E_{ij}}{P_i} = t_{cj}\left(\frac{B_{Rj}}{P_R} - \frac{B_{ij}}{P_i}\right), \tag{A.1'}$$

where

$\frac{B_{Rj}}{P_R}$ = the per capita base for source j in the RFPS provinces.

The population-share formulation

For certain of the experiments that follow it is more convenient to work with the population-share versions of these formulae. For the RNAS

$$E_{ij} = TR_j\left(\frac{P_i}{P_c} - \frac{B_{ij}}{B_{cj}}\right), \tag{A.2}$$

which follows from rearranging the terms of equation A.1. The intuition here is that, for $t_{ij} = t_{cj}$, a poor province's overall revenues for any revenue source will equal its population share of total revenues (TR_j) for this source.

The corresponding population-share formulation under the RFPS approach is not quite as straightforward:

$$E_{ij} = \frac{B_{Rj}}{B_{cj}} TR_j\left(\frac{P_i}{P_R} - \frac{B_{ij}}{B_{Rj}}\right), \tag{A.2'}$$

where B_{Rj}/B_{cj}, henceforth denoted as β_j, is the share of the total base located in the RFPS provinces.

From equations A.1 and A.1' it is clear that, for any given revenue source, equalization will be higher under the RFPS formulation if the

RFPS per capita base exceeds the RNAS per capita base. Equivalently, and intuitively, the RFPS equalization for poor provinces will be larger if $B_R/B_c > P_R/P_c$; that is, if the five RFPS provinces have a larger share of the tax base than they have of the population.

We now proceed to focus on some of the properties of the formulas.

CHANGING TAX RATES

From equations A.1 and A.1', the impact on per capita equalization of an increase in a province's own tax rate is straightforward: $(B_{ij}/B_{cj})D_{ij}$ for the RNAS and $(B_{ij}/B_{cj})D'_{ij}$ for the RFPS, where D_{ij} and D'_{ij} are, respectively, the fiscal deficiencies (bracketed terms) in equations A.2 and A.2'. In other words, if a province is poor in terms of a particular base (i.e., if D is positive), per capita equalization will increase by an amount equal to this deficiency times the increase in own (and, hence, overall) revenues, B_{ij}, divided by the national base. Equalization per capita for a poor province will also increase if the tax rate increases in *another* province, by $(B_{i*j}/B_{cj})D_{ij}$ for the RNAS and by $(B_{i*j}/B_{cj})D'_{ij}$ for the RFPS (where the asterisk denotes a province other than province *i*). Obviously, under both the RNAS and the RFPS, a small province such as Nova Scotia will benefit more from equalization if a large province such as Ontario (with a larger B_{ij}) increases its tax rate than if Nova Scotia increases its own tax rate.

For this experiment the fact that some provinces are included in the RFPS and some are not is of no consequence. However, this is not the case for the remaining experiments.

CHANGING OWN TAX BASES

The RNAS

Assume that a province's own tax base increases, *ceteris paribus*. Intuitively, this will have two contrary effects on equalization. The province will now be richer, so that equalization will fall on this count. On the other hand, overall revenue will increase, which will increase equalization. As will be seen, the former effect will dominate. Let G_{ij} and G'_{ij} represent the 'gaps' in equations (A.2) and (A.2'); that is,

$$G_{ij} = \frac{P_i}{P_c} - \frac{B_{ij}}{B_{cj}}$$

and

$$G'_{ij} = \frac{P_i}{P_R} - \frac{B_{ij}}{B_{Rj}} .$$

Assuming once again that $t_{ij} = t_{cj}$, the effect of a base increase in province i under the RNAS is:

$$\frac{\partial E_{ij}}{\partial B_{ij}} = t_{cj} \left(\frac{P_i}{P_c} - 1 \right) < 0. \tag{A.3}$$

Per capita equalization will fall, as one would expect, and this fall will be larger for more sparsely populated provinces. Of more interest is what happens to the change in the province's overall revenues, including own revenues. Noting that the province's total revenue from tax source j is $R_{ij} = t_{ij} B_{ij} + E_{ij}$, one can easily show that

$$\frac{\partial R_{ij}}{\partial B_{ij}} = t_{cj} \left(\frac{P_i}{P_c} \right). \tag{A.4}$$

In other words, overall, the province's revenues will *increase* by an amount equal to the product of its population share and the tax rate.[2] This is, of course, a desirable property in the sense that one would not want *total* revenues to decrease as a province became richer. However, if the province's tax rate is less than the national average tax rate, it is possible for the province's overall revenues to fall as it becomes richer.[3]

The RFPS

If the province is one of the five provinces making up the RFPS, the result, not surprisingly, is virtually identical to that for the RNAS:

$$\frac{\partial R_{ij}}{\partial B_{ij}} = (t_{ij} - t_{cj})(1+ \beta_j G'_{ij}) + t_{cj}\left(\frac{P_i}{P_R}\right). \tag{A.6}$$

(To derive this see equation S.3 in the annex.) With $t_{ij} = t_{cj}$, a province's overall revenues will increase, and this increase will be larger than the RNAS increase since it depends on the province's population share not of P_c but of P_R.

Matters are very different if the province is not part of the five-province standard because an increase in a province's base will not increase the RFPS base; that is, the term B_{ij}/B_{Rj} will be larger. Under these circumstances, overall revenue changes arising from an increase in B_{ij} are

$$\frac{\partial R_{ij}}{\partial B_{ij}} = (t_{ij} - t_{cj})(1+ \beta_j G'_{ij}). \tag{A.7}$$

If $t_{ij} = t_{cj}$, *the loss offset will be 100 per cent*; that is, a province will *not* gain from an increase in its own base. Indeed, *it will lose unless its own tax rate is greater than the national average rate.* This is a rather perverse feature of the RFPS formula, to say the least.[4]

CROSS-BASE CHANGES

Under the RNAS, province i's equalization will rise if some other province's base increases. This is clear from equation A.1. The effect on i's equalization from an increase in i^*'s base is:

$$\frac{\partial E_{ij}}{\partial B_{i^*j}} = (t_{i^*j} - t_{cj})\ G_{ij} + t_{cj}\left(\frac{P_i}{P_c}\right). \tag{A.8}$$

A similar expression occurs under the RFPS approach when province i^* is part of the standard:

$$\frac{\partial E_{ij}}{\partial B_{i*j}} = (t_{i*j} - t_{cj})\ \beta_j G'_{ij} + t_{cj} \left(\frac{P_i}{P_R}\right), \text{ for } i^* \text{ in } R, \tag{A.9}$$

where R is the set of representative provinces.

Once again, the situation is different if the province that increases its tax base is not part of the standard:

$$\frac{\partial E_{ij}}{\partial B_{i*j}} = (t_{i*j} - t_{cj})\ \beta_j G'_{ij}, \text{ for } i^* \text{ not in } R, \tag{A.10}$$

With $t_{i*j} = t_{cj}$, a base increase in a non-RFPS province will have no effect on province i's equalization. This is evident from equation A.1'—a base increase in a non-RFPS province does not affect the bracketed term and will not affect t_{cj} if the base increase is proportional to the increase in provincial revenues. However, if the province has a tax rate less than the national average rate, province *i*'s equalization will *fall*—again, an intuitively perverse result.[5]

INTERPROVINCIAL BASE SHIFTS

What happens when the tax base shifts from one province to another? In order to make this experiment more realistic, it is convenient to visualize it in terms of a shift in a corporation's headquarters from one province to another, which will shift the allocation of profits across provinces.[6] In what follows, we are assuming that the only change is in the base—that is, we are neglecting any accompanying population movements, etc. If the base moves from another province i^* into province i under RNAS, the effect on total revenues is:

$$\frac{\partial R_i}{\partial B_{ij}} - \frac{\partial R_i}{\partial B_{i*j}} = (t_{ij} - t_{cj}) + (t_{ij} - t_{i*j})\, G_{ij}. \tag{A.11}$$

Thus, if tax rates across provinces are identical, this will not result in an increase in province *i*'s total revenues: there is a 100 per cent offset. In other words, the province is fully compensated for a loss in its base to another province and vice versa (recalling that we are neglecting any accompanying population shifts, etc.).

The same basic result holds for province i under the RFPS provided that (a) province i and the other province i^* are both *out* of the standard or (b) province i and the other province i^* are both *in* the standard. However, if the receiving province is in the RFPS (e.g., Manitoba) and the sending province is not (e.g., Nova Scotia), then Manitoba will benefit in terms of overall revenues (assuming all tax rates are identical):

$$\frac{\partial R_i}{\partial B_{ij}} - \frac{\partial R_i}{\partial B_{i^*j}} = (t_{ij} - t_{cj}) + (t_{ij} - t_{i^*j})\, \beta_j G'_{ij} + t_{cj} \left(\frac{P_i}{P_R}\right). \tag{A.12}$$

But if the receiving province is Nova Scotia (not in the RFPS) and the sending province is Manitoba (in the RFPS), Nova Scotia's overall revenues will fall:

$$\frac{\partial R_i}{\partial B_{ij}} - \frac{\partial R_i}{\partial B_{i^*j}} = (t_{ij} - t_{cj}) + (t_{ij} - t_{i^*j})\, \beta_j G'_{ij} - t_{cj} \left(\frac{P_i}{P_R}\right). \tag{A.13}$$

The intuition underlying these last two results can be drawn from equation A.1'. The increase in B_{ij} will be the same in both cases. But in the first case B_{Rj} (the total base in the RFPS) will increase, whereas in the shift from Manitoba to Nova Scotia B_{Rj} will fall.

This is another rather questionable feature of the RFPS. In effect, it means that the equalization compensation that Quebec, for example, will receive from losing a corporate headquarters will be larger if the corporation moves to Ontario than it will be if the corporation moves to Alberta.

POPULATION SHIFTS

Population shifts; no shift in base

There are several experiments one could conduct with respect to population shifts. We shall restrict ourselves only to shifts in population among the provinces (i.e., keeping P_c fixed). Even here, two sorts of experiments are possible: (a) population shifts that do not affect the base and (b) population shifts that do affect the base. The former relate to the resource revenue categories while the latter relate to categories such as personal income taxes and sales taxes, where it is obvious that a shift in population will also affect a province's base.

To deal first with those revenue categories where population movements do not affect the base, it is clear from equation A.2 that an increase in the population of province *i*, *ceteris paribus*, will increase its RNAS equalization. Of more interest is what happens to its per capita equalization. Will it increase as well? The answer is yes: a one-unit shift in population from province i^* to province i will change E_{ij}/P_i according to

$$\frac{\partial (E_{ij}/P_i)}{\partial P_i} - \frac{\partial (E_{ij}/P_i)}{\partial P_{i*}} = \frac{1}{P_i}\left(t_{cj}\frac{B_{ij}}{P_i}\right). \tag{A.14}$$

In effect, per capita equalization increases because the population inflow will generate a proportional increase in equalization *plus* a further amount, as the province is now poorer in terms of per capita base. The term in A.14 represents this second impact; the first is subsumed in the experiment itself, since we are focusing on per capita equalization. Note, however, that if the province in question has a zero base (i.e., $B_{ij} = 0$), per capita equalization will be unchanged; that is, it cannot get 'poorer' in terms of its endowment of this base. For some revenue categories, such as energy revenues, many provinces do indeed have a zero base.

Under the RFPS, equation A.14 also obtains if both the sending and receiving provinces are included in the RFPS or if both are excluded. However, if the population-receiving province is excluded and the sending province is included in the RFPS, the per capita equalization increase will be greater than that in equation A.14 (see equation S.7 in the annex). This occurs because, from equation A.1', not only does B_{ij}/P_i fall, but B_{Rj}/P_R rises, since P_R is now lower.

The opposite occurs if the receiving province is in the RFPS and the sending province is not; that is, one subtracts the above term from the expression in equation A.14. If the province in question has a zero base (e.g., the Atlantic Provinces for the energy revenue sources) then per capita equalization falls, since the term in equation A.14 becomes zero.

Population and base shifts

A population shift will also have an impact on the tax base for revenue sources other than the resource categories, since the migrant will carry with him a bundle of income and spending characteristics. Let this tax base accompanying the migrant be B_{mj} for source j (B_{mj} has the dimensions of dollars of base per person). Under the RNAS scheme we can, from equation A.1, obtain the total derivative relating to a move from province i^* to province i:

$$\frac{d(E_{ij}/P_i)}{dP_i} = \frac{1}{P_i}\left[(t_{ij} - t_{i^*j})\frac{P_i B_{mj} D_{ij}}{B_{cj}} - t_{cj}\left(B_{mj} - \frac{B_{ij}}{P_i}\right)\right]. \quad \text{(A.15)}$$

The first term relates to total revenues that enter the formula. If province i has a higher tax rate than province i^*, then total revenues from this source will be higher (or t_{cj} will increase), which will increase per capita equalization. The second term relates to the impact that the migrant will have on a province's relative base. Assuming that all provinces' tax rates are equal (so that the first term is zero), immigration to province i will increase per capita equalization if $B_{mj} < B_{ij}/P_i$; that is, if the migrant has a lower base for this source than the existing residents.[7] It should be pointed out that equation A.14 is really a special case of equation A.15, corresponding to $B_{mj} = 0$.

As usual, equation A.15 also obtains for the RFPS if sending and receiving provinces are either both in or both out of the standard (except that D_{ij} becomes D'_{ij}). When the sending province is outside the RFPS and the receiving province inside, then the change in per capita equalization becomes:

$$\frac{d(E_{ij}/P_i)}{dP_i} = \textit{equation A.15} + \frac{t_{cj}}{P_R}\left(B_{mj} - \frac{B_{Rj}}{P_R}\right). \quad \text{(A.16)}$$

The intuitive reason for this additional increase in equalization (for $B_{mj} > B_{Rj}/P_R$) is that it enriches the RFPS standard; that is, it increases B_{Rj}/P_R. The result is symmetric in the sense that if the assumptions are reversed (i.e., if the sending province is in and the receiving province out), then the rightmost term in equation A.16 is prefixed by a negative sign. For $B_{mj} > B_{Rj}/P_R$, the migrant's departure from the RFPS lowers the standard and lowers everyone's equalization payment.

Interestingly enough, these results can be generalized to a case in which province i is the third party—that is, in which migration occurs between two other provinces. If all three provinces have identical tax rates, there will be no effect on province i under the RNAS or under the RFPS if both of the other provinces are either in or out of the standard. However, a move of population from Ontario (in the RFPS) to Nova Scotia (outside the representative provinces) would, for $B_{mj} > B_{Rj}/P_R$, lower equalization for Newfoundland. This is clear from equation A.1', since the only variable that would change is B_{Rj}/P_R, which would fall. The amount of this decrease is $t_{cj}/P_R\,(B_{mj} - B_{Rj}/P_R)$, the rightmost term in equation A.16. Similarly, Newfoundland's equalization would increase by this amount if the migration flow went from Nova Scotia to Ontario, again assuming that $B_{mj} > B_{Rj}/P_R$.

CONCLUSION

Canada's new equalization formula incorporates some rather perverse effects in comparison with the old RNAS formula. As detailed above, these are likely to include marginal rates of implicit taxation of over 100 per cent on resource development. In addition, some effects depend sensitively on whether changes in population or economic activity involve representative provinces, nonrepresentative provinces, or both. In our view, the latter aspect of the new formula is particularly troublesome, since it leads to incentives where equalization-receiving provinces are not neutral with respect to development prospects elsewhere in the federation. These differential locational impacts can be significant, a point that has been dealt with elsewhere by Courchene (1983b).

NOTES

1 We do not attempt to take the *financing* of the equalization program into account in our discussion. Rather, we are concerned solely with the implications of the RNAS and RFPS formulas for the interprovincial allocation of equalization flows. The annex to this appendix focuses on the derivations of the main equations that follow.

2 Since the RNAS formula allowed only one-half of energy revenues to enter the formula, it provided an even greater incentive for a poor province to engage in

resource development. Correcting for this requires that one add a term, $\frac{1}{2}t_{cj}$, to equation A.4.

3 The relevant expression is:

$$\frac{\partial R_{ij}}{\partial B_{ij}} = (t_{ij} - t_{cj})(1 + G_{ij}) + t_{cj}\left(\frac{P_i}{P_c}\right), \qquad \text{(A.5)}$$

where the first term is negative if $t_{ij} < t_{cj}$.

4 As noted above, this perversity can arise under the RNAS as well. It is obviously more pronounced, however, for a nonrepresentative province under the RFPS. The fact that under the RFPS resource bases no longer receive special treatment (see note 2 above) accentuates this possibility.

5 As before, this perverse result can occur under the RNAS as well. But because province i does not receive its per capita share of the (representative-rate) revenue flowing from i^*'s extra base, it is more likely to occur under the RFPS. Incidentally, the statement in the text assumes $G'_{ij} > 0$, i.e., that i is a 'have-not' province for base j. The results are reversed if $G'_{ij} < 0$.

6 By agreement among the provinces, the allocation of profits of a multiprovince corporation is determined by the following formula

$$\frac{1}{2}\left(\frac{S_i}{S_c} + \frac{W_i}{W_c}\right) = \text{province's share of profits},$$

where

$$\frac{S_i}{S_c} = \text{the proportion of the corporation's sales in province } i$$

and

$$\frac{W_i}{W_c} = \text{the proportion of the corporation's wage bill in province } i.$$

7 The expression for total revenues (own revenues plus equalization) is:

$$\frac{d(R_{ij}/P_i)}{dP_i} = \frac{1}{P_i}\left[(t_{ij} - t_{i^*j})\frac{P_i B_{mj} D_{ij}}{B_{cj}} + (t_{ij} - t_{cj})\left(B_{mj} - \frac{B_{ij}}{P_i}\right)\right].$$

Thus, if province i has a *lower* tax rate than do other provinces, its *total* per capita revenues will fall if the characteristics of the in-migration are such that

$$B_{mj} > \frac{B_{ij}}{P_i}.$$

Annex

A number of the results in the technical appendix are stated without detailed derivations and/or are presented for special cases (e.g., when a provincial tax rate is assumed to equal the national average). Here we present necessary derivations and more general results. The sections of this supplement are keyed to the sections of the appendix. Equation numbers in the supplement are prefixed by the letter S.

CHANGING TAX RATES

The results cited in the text follow directly from equations A.1 and A.1', noting the definition $t_{cj} = (\Sigma_i t_{ij} B_{ij})/(\Sigma_i B_{ij})$.

CHANGING OWN TAX BASES

The RNAS

From equation A.2 we have

$$\frac{\partial E_{ij}}{\partial B_{ij}} = t_{ij} G_{ij} - \frac{TR_j}{B_{cj}} + \frac{TR_j B_{ij}}{B^2_{cj}} = t_{ij} G_{ij} - t_{cj} + t_{cj} \left(\frac{B_{ij}}{B_{cj}}\right), \qquad \text{(S.1)}$$

from which equation A.3 follows if $t_{ij} = t_{cj}$. The change in revenue from all sources is thus

$$\frac{\partial R_{ij}}{\partial B_{ij}} = \frac{\partial E_{ij}}{\partial B_{ij}} + t_{ij}, \qquad \text{(S.2)}$$

which, from equation S.1 and the definition of G_{ij}, is just equation A.5 of footnote 3. This, of course, reduces to equation A.4 if $t_{ij} = t_{cj}$.

The RFPS

The change in equalization associated with a change in base is computed from equation A.2' and yields, for i belonging to R,

$$\frac{\partial E_{ij}}{\partial B_{ij}} = t_{ij}\beta_j G'_{ij} + \left(\frac{TR_j}{B_{cj}}\right)G'_{ij} - \left(\frac{TR_j B_{Rj}}{B^2_{cj}}\right)G'_{ij} - \frac{TR_j \beta_{Rj}}{B_{Rj}} + TR_j\beta_j\left(\frac{B_{ij}}{B^2_{Rj}}\right)$$

$$= (t_{ij} - t_{cj})\beta_j G'_{ij} + t_{cj}\left(G'_{ij} + \frac{B_{ij}}{B_{Rj}} - 1\right)$$

$$= (t_{ij} - t_{cj})\beta_j G'_{ij} + t_{cj}\left(\frac{P_i}{P_R} - 1\right). \qquad \text{(S.3)}$$

Adding t_{ij} to this expression yields equation A.6.

If province i is not in R, the second and last terms after the first equation in S.3 do not appear, and we find instead that

$$\frac{\partial E_{ij}}{\partial B_{ij}} = (t_{ij} - t_{cj})(\beta_j G'_{ij}) - t_{cj} . \qquad \text{(S.4)}$$

Again, adding t_{ij} to account for the change in own-revenue, we get equation A.7.

CROSS-BASE CHANGES

From A.2 under RNAS we have

$$\frac{\partial E_{ij}}{\partial B_{i^*j}} = t_{i^*j} G_{ij} + \left(\frac{TR_j}{B^2_{cj}}\right) B_{ij}$$

$$= (t_{i*j} - t_{cj})\,G_{ij} + t_{cj}\left(\frac{P_i}{P_c}\right),$$

which is equation A.8. For the RFPS case where i^* belongs to R, we have

$$\frac{\partial E_{ij}}{\partial B_{i*j}} = t_{i*j}\,\beta_j\,G'_{ij} + \left(\frac{TR_j}{B_{cj}}\right)G'_{ij} - TR_j\left(\frac{B_{Rj}}{B^2_{cj}}\right)G'_{ij} + TR_j\,\beta_j\left(\frac{B_{ij}}{B^2_{Rj}}\right)$$

$$= (t_{i*j} - t_{cj})\,\beta_j\,G'_{ij} + t_{cj}\left(G'_{ij} + \frac{B_{ij}}{B_{Rj}}\right), \qquad \text{(S.5)}$$

which reduces to equation A.9. When i^* is not in R, the second and last terms following the first equality in S.5 do not appear, and we get equation A.10.

INTERPROVINCIAL BASE SHIFTS

To get equation A.11, simply subtract equation A.8 from equation A.5 in note 3.

Under the RFPS, if both i and i^* are among the representative five provinces, we can subtract equation A.9 from equation A.6 to obtain

$$\frac{\partial R_{ij}}{\partial B_{ij}} - \frac{\partial R_{ij}}{\partial B_{i*j}} = \left(t_{ij} - t_{cj}\right) + (t_{ij} - t_{i*j})\,\beta_j\,G'_{ij}, \qquad \text{(S.6)}$$

which is quite analogous to equation A.11. The same result holds if neither i nor i^* are in the representative group; in that case, subtracting equation A.10 from equation A.7 yields equation S.6.

To derive equation A.12, subtract A.10 from A.6. To derive equation A.13, subtract A.9 from A.7.

POPULATION SHIFTS

Population shifts; no shift in base

Equation A.14 follows directly from differentiation of equation A.1 with respect to P_i, holding P_c constant. The same result holds under the RFPS if P_R is constant; that is, if either both i and i^* belong to R or neither do. We then have from equation A.1':

$$\frac{\partial(E_{ij}/P_i)}{\partial P_i} - \frac{\partial(E_{ij}/P_i)}{\partial P_{i^*}} = \frac{1}{P_i}\left(t_{cj}\frac{B_{ij}}{P_i}\right) + t_{cj}\left(\frac{B_{Rj}}{P_R^2}\right). \tag{S.7}$$

When i belongs to R and i^* does not, it is obvious that the sign on the second term is reversed.

Population and base shifts

To obtain equation A.15, simply note that $dB_{ij}/dP_i = B_{mj}$ and that $dB_{i^*}/dP_i = -B_{mj}$. Then, from equation A.1,

$$\frac{d(E_{ij}/P_i)}{dP_i} = \left(\frac{dt_{cj}}{dP_i}\right)D_{ij} - t_{cj}\frac{d(B_{ij}/P_i)}{dP_i}$$

$$= (t_{ij} - t_{i^*j})D_{ij}\frac{B_{mj}}{B_{cj}} - \frac{t_{cj}}{P_i}\left(B_{mj} - \frac{B_{ij}}{P_i}\right), \tag{S.8}$$

which yields equation A.15. Since

$$\frac{d(t_{ij}B_{ij}/P_i)}{dP_i} = \frac{t_{ij}}{P_i}\left(B_{mj} - \frac{B_{ij}}{P_i}\right), \tag{S.9}$$

we obtain the expression in note 7.

From equation A.1′, we see that the result is identical under the RFPS if both i and i^* belong to R, since the migration leaves B_{Rj}/P_R unaffected. This is also true if neither i nor i^* are in R. If, however, i is and i^* is not in R, we must add the term

$$t_{cj}\frac{d(B_{Rj}/P_R)}{dP_i} = \frac{t_{cj}}{P_R}\left(B_{mj} - \frac{B_{Rj}}{P_R}\right), \tag{S.10}$$

as indicated in equation A.16, while the sign is reversed if i is not in R and i^* is.