The Political Economy of Social Security
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3. Voting Models of Social Security Determination

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1. Introduction

Government budgetary measures affect economies in a number of ways. In addition to their effect on resource allocation through the price system, they also redistribute income among households in the economy. Historically, this has been the subject matter of tax and expenditure incidence studies and most attention has been devoted to incidence across income groups or across factor incomes. Recently, the pervasiveness and importance of another dimension of budgetary effects, the intergenerational one, has become recognised. Much of this recognition can be attributed to the work of Feldstein in the context of social security (1974) and asset taxation (1977a), but the basic framework for analyzing the intergenerational impact of budgets goes back to Samuelson (1958) and Diamond (1965).

The instruments through which intergenerational transfers of real income are effected are many and have been summarized recently in Kotlikoff (1984). Unfunded (pay-go) social security represents an ongoing transfer from younger generations to older ones. Public debt can be interpreted in precisely the same way. Many types of public expenditures have an intergenerational component, including health, welfare, and education. Overall, tax-financed public expenditures can probably be viewed on net as being transfers from younger to older generations. Finally, changes in the tax structure itself can have important intergenerational impacts. For example, a switch from a tax on consumption to a tax on wages is essentially equivalent to an ongoing redistribution from young to old.

The consequences of the intergenerational transfer component in the budget have been worked out in conventional overlapping generations models. In the absence of an operative bequest motive in the sense of Barro (1974), intergenerational transfers from young to old are predicted to reduce the capital stock of the economy and the level of GNP. In the long run, per capita consumption and utility will fall if the rate of growth of the economy n is less than the rate of return on capital r, and vice versa. Since estimates have tended to support the view that $r > n$ (e.g. Feldstein (1977b)), ongoing transfers from younger to older generations are expected to reduce long-run societal welfare. Indeed, Summers (1981) has calculated that the long-run effects of a switch from income to consumption taxation could be to reduce per capita welfare by as much as 12 percent. Although this is based on best-guess estimates in the context of a simple simulation model, the sheer magnitude of his results are suggestive and dwarf conventional static measures of the welfare gains from tax reform.

For our purposes, the short-run effects of intergenerational transfer schemes are also relevant. The introduction of a scheme to redistribute income from younger to older generations will provide a windfall gain to those older generations alive when the scheme is introduced (with a corresponding windfall loss to those alive if and when the scheme is terminated). An opportunity therefore exists for current generations to improve their welfare at the expense of successive future generations by introducing new schemes, or increasing the size of existing schemes, of transfers from young to old.

The question is: to what extent will current, voting, generations take advantage of this opportunity ? That is the question that the political economy of intergenerational transfers is intended to answer. The literature on this issue is still in its infancy, and has been limited to simple majority voting models. The purpose of this paper is two-fold. First, we present a majority voting model of social security in which capital markets are fully specified, and use it to analyze the effect of demographic change – specifically, changes in the population growth rate – on the equilibrium social security program. Second, since our analytical framework differs in several crucial repects from those that have previously appeared in the literature, we present a critical review and comparison of alternative modelling strategies in this area.

Although we postpone a detailed discussion of the literature until Section 4, we should mention here some of the forerunners of our research. The original contribution in this area was by Aaron (1966), who showed that the median-aged voter may have an incentive to perpetuate an unfunded social security system because of a positive net present value of future benefits less taxes over the remaining lifetime. The first attempt to model this precisely was by Browning (1975). He analyzed the long-run level of pay-go social security in a model of overlapping generations of identical households with given incomes in working years. Assuming that households expect the system they vote for to remain in effect for the rest of their lifetimes, and that forced saving through social security is the only form of saving, he shows that the long-run equilibrium level of social security chosen will be higher than that which maximizes per capita lifetime welfare.

More recently, Hu (1982) has also adopted an overlapping generations framework, but with a number of different institutional and behavioral assumptions. Households live for two working periods and one retirement period, earning given wages in the first two. At the time of a vote, the two younger generations

are assumed to face uncertainty about the level of social security chosen in the future, reflecting uncertainty about future behavior or demographic conditions. In the absence of such uncertainty, voters would prefer either zero social security or an indefinitely large amount, depending on the sign of their wealth effect. The addition of uncertainty serves to generate an internal solution for the social security level preferred by a median voter with a positive wealth effect. Hu shows that if expected future social security levels are positively correlated with currently chosen ones, social security will be lower and savings higher than in the absence of uncertainty.

Each of these approaches, though providing valuable insights, is somewhat incomplete, Browning's in its failure to specify capital markets fully and Hu's in its somewhat arbitrary formulation about the behavior of future voters based on a notion of uncertainty. In Boadway and Wildasin (1987), we have attempted to overcome some of these difficulties by carefully specifying the operation of capital markets and addressing the issue of the intertemporal interdependence of voting behavior among generations. In Section 2, therefore, we begin by summarizing the main features of the model and the results obtained. Section 3 then extends the analysis in that model in a direction which is highly relevant for contemporary social security debates, by studying the effect of demographic changes on the long-run equilibrium choice of social security. Finally, in Section 4, we compare our work with that of earlier contributors to the literature, attempting thereby to identify crucial assumptions and modelling issues. This discussion points the way to a number of directions for future research.

2. Model of Voting for Social Security with Fully-Specified Capital Markets

The model we use is that of Cass and Yaari (1967), also used by Summers (1981). It is a single-sector neo-classical growth model with overlapping generations of households. Households are continuously born at an exponential growth rate of n. A cohort born at time t is age 0 at birth, works for the interval $[0, R]$ at an exogenously-given wage w_s at time s, and is retired in the interval $[R, T]$. Both R and T are given, as is the labor supply in the working period. The only economic decision taken by the household is a savings decision. Savings are done only to convert the stream of earnings into a stream of consumption. There are no bequests. The fixed growth rate n, along with the fixed R and T, ensures a fixed age distribution of the population, although we shall consider the consequences of varying n later. The median-aged person is denoted m. For fixed n, m will also be fixed. On the producer side, producer prices are taken to be fixed. They include the interest rate λ, the wage profile w_s, and the output price, normalized to unity.

In the context of this model the government has only one policy instrument: pay-go social security. Under a social security system, the government taxes all households of working age τ dollars per instant and pays benefits of β to all households in retirement. The tax rate τ is related to the benefit rate β by a simple instantaneous budget constraint:

$$\tau = \eta\beta \tag{1}$$

η is the ratio of the retired population to the working population and is constant [1]. The level of β, and hence τ, is determined by a periodic majority vote. Assuming preferences for β are single-peaked, the level of β will be that preferred by the median voter.

To make the analysis tractable, we assume that when a vote is taken, voters expect the scheme they vote for to remain in effect for the remainder of their lives. The consequences of relaxing this will be discussed later. That being the case, if capital markets were perfect, preferences for β would be determined simply by calculating the present value of future benefits less taxes of various schemes β. Obviously, the net present value of social security is monotonic in β for a person of given age. Furthermore, the net present value will increase with age. In these circumstances, majority voting will yield a corner solution coinciding with the preferences of the median-aged person. If person m obtains a negative net present value for social security, $\beta = 0$ would be chosen [2]. If a positive present value is obtained, β would be set at its maximum permitted level, denoted β_{max}. This is the level of β which would tax away all the wealth of the working generations [3].

There are various ways these extreme outcomes could be avoided. One would be to introduce variable factor prices. For example, with w and r variable, increases in β and τ would reduce the capital stock, increasing r and reducing w and thereby offsetting the wealth effects of the increased β. Making labor supply or R endogenous would have a similar effect. Another way would be to introduce uncertainty as in Hu (1982). Both of these have some attractions as means of obtaining an internal solution, but both have their disadvantages. Making factor prices variable makes the dynamic analysis of voting prohibitively complex and virtually restricts one to steady-state or simulation analysis. The use of uncertainty raises issues discussed further in Section 4.

Partly for reasons of tractability and partly because we believe that evidence and intuition suggests it to be important, we have chosen to induce an internal solution by introducing capital market constraints. In particular, we assume here that households cannot borrow against future social security benefits [4]. This constraint has some appeal in view of the fact that many benefits in retirement are either in-kind or take the form of an annuity.

Given this capital market constraint, we proceed by considering the life-cycle

[1] In particular, $\eta = \int_R^T e^{-ns}\,ds / \int_0^R e^{-ns}\,ds = (e^{-nR} - e^{-nT})/(1 - e^{-nR})$.

[2] Of course, if $\beta < 0$ were permitted, it would be preferred to $\beta = 0$ by person m.

[3] These extreme outcomes were those obtained by Hu (1982) for his case of infrequent voting.

[4] Browning (1975) assumed this in the extreme by allowing social security to be the only form of saving. We have treated in detail elsewhere the general case in which households can borrow neither against future retirement benefits nor against future wages. The qualitative results obtained are similar to those reported here. See Boadway and Wildasin (1987).

savings decision of the household for a given level of β. Next, we consider the preferences of households over β, and use that to characterize majority voting equilibria.

2.1. Household Savings Behavior

The problem is a simple extension of the conventional life-cycle savings problem with the addition of a borrowing constraint. For a person aged t the problem is:

$$\text{Max} \int_t^T e^{-\delta(s-t)} u(c_s)\,ds \tag{2}$$

subject to

$$\dot{A}_s = w_s - \tau + rA_s - c_s, \qquad t \le s \le R$$

$$\dot{A}_s = \beta + rA_s - c_s, \qquad R < s \le T$$

$$A_s \ge 0, \qquad R \le s \le T$$

where A_s is wealth at time s and c_s is consumption. The initial condition is

$$A_t = \bar{A}_t(\beta^0)$$

where β^0 is the level of social security that existed up to age t. It is relevant for subsequent analysis to note that

$$\partial \bar{A}_t / \partial \beta_0 < 0$$

is an implication of this theory. It is useful to think of this life-cycle problem as coinciding with the time of a vote on β so β may be unequal to β^0.

The solution to this problem can be shown to yield the following first-order conditions (see Boadway and Wildasin (1987)):

$$e^{-\delta(s-t)} u'(c_s) - \lambda_s = 0 \tag{3}$$

$$\dot{\lambda}_s = -\lambda_s r - \mu_s \tag{4}$$

$$\mu_s A_s = 0, \qquad R \le s \le T \tag{5}$$

where γ_s is the multiplier in the assset accumulation constraint, and hence is the marginal utility of wealth, and μ_s is the multiplier on the non-negative asset constraint. This solution has a straightforward interpretation. For simplicity, consider the case of a constant elasticity of marginal utility $\varepsilon = -u'' c_s / u'$.

Suppose the borrowing constraint is binding and that $r > \delta$. Then, it will be the case that $A_R = 0 = A_T$, but $A_s > 0$ everywhere else. This implies that $\mu_R \ne 0$

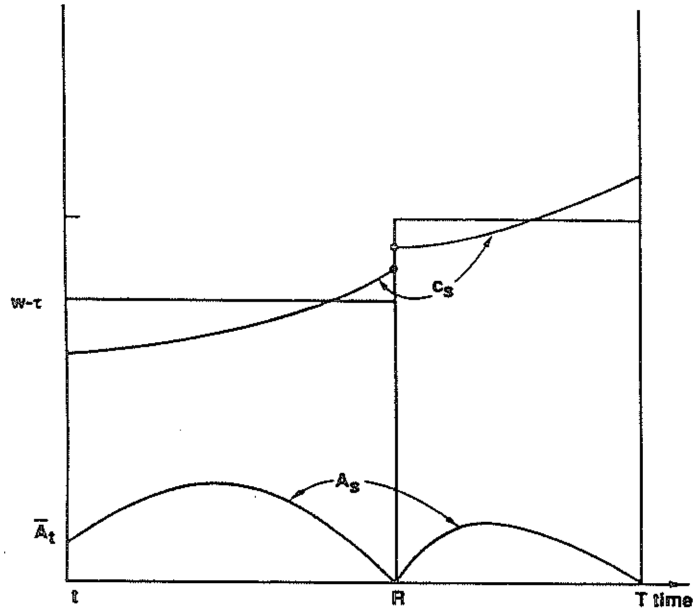

Figure 1. Household life cycle behavior.

and that we can divide the consumer's consumption path into two segments $[t, R]$ and $[R, T]$. Over each segment the following equation of motion applies:

$$\dot{c}_s/c_s = (r - \delta)/\varepsilon$$

with c_t and c_R at the beginning of the segment being such that $A_R = 0 = A_T$ at the end. At the point R, consumption jumps discontinuously. Equivalently, over the two segments, the marginal value of wealth follows:

$$\dot{\lambda}_s/\lambda_s = -r. \tag{7}$$

At R, λ_R jumps discontinuously downward. Figure 1 illustrates the consumption path followed by the liquidity – constrained household.

2.2. Voter Preferences over β

To analyze voting behavior requires knowledge of how household welfare varies with β. Continue with the person aged t analyzed above. Given w_s, r, and the demographic variables, the path of consumption can be viewed as a function of the social security levels β and β^0, denoted $c_s(\beta;\ \beta^0)$. The remaining lifetime utility of the household can be written:

$$V(\beta;\ \beta^0) = \int_t^T e^{-\delta(s-t)} u\left[c_s(\beta;\ \beta^0)\right] \mathrm{d}s \tag{8}$$

The effect of β on lifetime utility can be obtained by differentiating (8) with respect to β and using (3):

$$V_t'(\beta;\ \beta^0) = \int_t^T \lambda_s \frac{\partial c_s}{\partial \beta} \mathrm{d}s \tag{9}$$

Furthermore, using (7) and the consumer's wealth constraint over the two segments of his life cycle (working and retirement), (9) can be rewritten in the following form [5]:

$$V_t'(\beta;\ \beta^0) = \lambda_t\left(e^{-r(R-t)}\Delta_{R,T} - \eta\Delta_{t,R}\right) - \delta_R \Delta_{R,T} \tag{10}$$

where

$$\Delta_{xy} \equiv \int_x^y e^{-r(s-x)} \mathrm{d}s$$

and δ_R is the amount by which λ_R drops discontinuously, and so is an indication of the tightness of the borrowing constraint.

From (9) and (10) we can derive two important results. First, differentiating (9) with respect to β again yields the result that $V_t''(\beta;\ \beta^0) < 0$ [6]. This implies that preferences over β are *single-peaked* so that the majority voting equilibrium will coincide with that preferred by the median voter.

The second result involves interpretation of (10). The first term is the marginal value of social security wealth to the household, since the term in brackets is the net present value of an additional dollar of social security $d\beta$. The second term is the marginal cost imposed by the borrowing constraint distortion. For small values of β, social security provides only a wealth effect, which we take to be positive for the median voter. As β increases the constraint eventually binds ($\delta_R > 0$) and the second term offsets the first. It can be shown that for the constant elasticity utility function we are using here, V_t' will become negative at some $\beta < \beta_{max}$ (for all persons of age $t < R$) [7]. Thus, under our borrowing constraint, if the wealth effect is positive, preferences will be single peaked at

[5] For example, over the interval $[R, T]$,

$$\int_R^T \lambda_s \frac{\partial c_s}{\partial \beta} \mathrm{d}s = \lambda_R \int_R^T e^{-r(R-t)} \frac{\partial c_s}{\partial \beta} \mathrm{d}s = \lambda_R \int_R^T e^{-r(s-R)} \mathrm{d}s$$

by the budget constraint in $[R, T]$. Defining $\delta_R = e^{-r(R-t)}\lambda_t - \lambda_R$ yields the desired result.

[6] The differentiation yields

$$V_t'' = \int_t^T \lambda_s \frac{\partial^2 c_s}{\partial \beta_2} \mathrm{d}s + \int_t^T e^{-\delta(s-t)} u'' \left(\frac{\partial c_s}{\partial \beta}\right)^2 \mathrm{d}s$$

The second term is negative when $u'' < 0$. The first term is zero by the household's budget constraint.

some $0 < \beta < \beta_{max}$. For young persons for whom the wealth effect may be negative, the peak would occur at $\beta = 0$. For retired persons, the peak is at β_{max}. In our discussion, we focus on the interesting case where the median voter has a positive wealth effect and is of age $t < R$ [8].

The above results are for a person aged t who has lived under a regime β^0. As can be seen, the most desired level of β, denoted β_t^*, depends on β^0. Specifically, β_t^*, will be implicitly determined by:

$$V_t'(\beta_t^*, \beta^0) = 0. \tag{11}$$

Differentiating this expression yields:

$$\frac{\partial \beta_t^*}{\partial \beta^0} = \frac{\partial V_t'/\partial \beta^0}{V_t''} = \frac{-\eta \Delta_{t,R}}{V_t''} \frac{\partial \lambda_t}{\partial \beta^0} < 0 \tag{12}$$

where the second equality comes from differentiating (10) recognizing that changes in β^0 do not affect consumption in retirement. The inequality results from the fact that $V_t'' < 0$ from above, and $\partial \lambda_t / \partial \beta^0 < 0$ since a higher β^0 implies a lower $\bar{A}_t$. Therefore, we can think of the functional relationship $\beta^*(\beta^0)$ as being decreasing in β^0.

2.3. Majority Voting Equilibria

In characterizing majority-voting equilibria, the first task is to identify the median voter. Since the wealth effect from social security rises monotonically with age, one might expect the median-aged voter m to be the median voter. It turns out that under borrowing constraints that is not always the case. What we have shown elsewhere is that whenever the median-aged voter prefers at least as high a level of social security as the existing level ($\beta_m^* \geq \beta^0$), person m will be the median voter. This includes the case of the steady state for which $\beta^0 = \beta_m^*$. If $\beta_m^* < \beta^0$, m may or may not be the median voter. However, in this case the median voter will also prefer $\beta^* < \beta^0$.

[7] This can be seen by rewriting (10) as:

$$V_t' = -u'(c_t)\eta \Delta_{t,R} + e^{-\delta(R-t)} u'(c_R) \Delta_{R,T}$$

As β increases, $u''(c_t)$ increases and $u''(c_R)$ falls. Provided $u'(c_s) \to \infty$ as $c_s \to 0$, V_t'' will be negative at sufficiently high but still feasible values of β.

[8] In steady states of this model, households will face binding credit constraints and will exhaust their assets at retirement. This feature of the model probably does not literally describe many retirees. However, at least one important asset for many retirees, housing, is relatively illiquid. Moreover, some liquid assets are probably held on a precautionary basis to deal with adverse random shocks. Although these considerations are not formally represented in our model, the spirit of our analysis suggests that only the stock of liquid "discretionary" assets need be exhausted at retirement for the borrowing constraint to be binding.

As to the properties of the majority voting equilibrium, we know that such an equilibrium exists. We also know that β_∞, the steady state majority voting equilibrium, is unique. This is because the function $\beta^*(\beta^0)$, which is decreasing in β^0, will map into itself at a single value of $\beta < \beta_{max}$. Furthermore, we know that the steady state equilibrium β_∞ will be stable if $|\partial \beta^* / \partial \beta^0| < 1$. Unfortunately, the interpretation of this condition is not intuitive.

We have worked out some dynamics for this model as well. Consider the case in which the steady state is stable and the median-aged voter m is also the median voter. Starting at $\beta^0 = 0$, say, β initially overshoots β_∞, then undershoots, then continues in damped cycles to β_∞. If m is not the median voter, the initial overshooting will still occur, and β will fall as long as $\beta > \beta_\infty$, but it may not fall enough to undershoot and cause cycles. For example, β might gradually approach β_∞ from above [9].

If the steady state is not stable, there will not be a tendency for β to approach β_∞ over time. Thus, if $|\partial \beta^* / \partial \beta^0| > 1$ throughout its range, and if m is always the median voter, starting at $\beta = 0$ will result in a repetition of undamped cycles $(\beta_l, 0)$ where $\beta_1 = \beta^*(0)$. This result will be moderated if m is not the median voter in the downswing.

The preceding discussion has described the nature of a majority voting equilibrium when the interval of time between successive votes is at least T. Although this assumption achieves great analytical simplifications, it is clearly restrictive. In practice, it would appear feasible for votes on social security to occur with greater frequency than this. We therefore consider the nature of voting equilibria when votes occur at regular intervals of length $v < T$. In this situation, when considering the voting behavior of an individual of age $t < T - v$, one must make some hypothesis concerning that voter's expectations about future votes, and the possible linkage between the outcome of the current vote and future votes. Section 4 discusses some of the assumptions that have been used in earlier literature. In our own analysis, we assume *zero conjectural variations* (ZCV), that is, we suppose that all households currently voting believe that the outcomes of future votes are determined independently of the current vote. We also assume that voters have correct expectations or perfect foresight (PF), in that the anticipated and actual outcomes of future elections coincide. Finally, we restrict attention to steady states, so that social security benefits and taxes are constant over time, and so that all households are following the same steady-state life cycle paths of consumption and asset accumulation.

One can show that there is a critical value for the length of the voting cycle, v^*, such that the majority voting equilibrium level of social security benefits β^* in a PF-ZCV steady state is zero for voting cycles of length $v < v^*$. For $v > v^*$, β^* is positive and bounded above zero. Also, β^* is increasing in v. The rationale for these results is that, under the ZCV assumption, voting for higher social security benefits now only guarantees higher taxes and benefits until the next vote. The larger the value of v, the further into the retirement period of the

[9] The pheonomenon of initial overshooting followed by a reduction is a property of this model even when more general borrowing constraints apply.

median age individual will this next vote occur. Thus, the median age household can secure higher benefits further into its own retirement period when the length of the voting cycle is long. Hence, longer voting cycles result in higher preferred levels of social security. Note that $v^* > R - m$. This is because for voting cycles of length $v < R - m$, an incremental increase in β merely results in higher taxes in the pre-retirement period for individuals of age $t \leq m$, with no expected increase in retirement benefits. In fact, v^* is determined so that the wealth effect of social security over the period m to $m + v^*$ is just zero for the household of median age. For $v > v^*$, the median age individual, and all other individuals, are assured of positive wealth effects from any additional benefits for which they vote.

3. Demographic Effects on Majority Voting Equilibrium

In this section, we use the basic model of section 3 to investigate the effect of a change in the population growth rate on the equilibrium social security program. To begin with, we focus on the relatively simple problem of comparing steady states. We then consider some of the short-run effects of changes in population growth.

In general, population growth can affect the majority voting equilibrium level of social security in two ways. First, it can change the desired level of social security for voters of a given age. Second, it can change the age distribution of the population, and thus the identity of the median voter.

Ignoring the second effect for the moment, consider the ideal level of social security, β_t^*, for a voter of age t. For $t < R$, this will be the unique value of β such that

$$V_t'(\beta;\, \beta^0) = -u'(c_t)\eta\Delta_{t,R} + \mathrm{e}^{-\delta(R-t)}u'(c_R)\Delta_{R,T} = 0, \tag{13}$$

which follows from (10) and (3). As we have seen, V_t' is strictly diminishing in β, i.e. $V_t'' < 0$. We can therefore think of (13) as an equation implicitly determining β_t^* as a function of the population growth rate parameter n, such that

$$\frac{\partial \beta_t^*}{\partial n} = -\frac{\partial V_t'/\partial n}{V_t''} \tag{14}$$

To understand how n affects β_t^*, we must analyze $\partial V_t'/\partial n$.

Since we are interested in comparing steady-state majority voting equilibria, and since we know that the benefit constraint is binding in such equilibria, it follows that c_R is independent of n. This is because c_R is the starting point of a retirement consumption path that grows at rate $(r - \delta)/\varepsilon$ and that has a present value equal to $\beta\Delta_{R,T}$; these two conditions completely determine c_R in terms of parameters other than n. Similarly, one can easily check that the other parts of the second term in (13) are independent of n.

In fact, n affects V_t' only through its impact on η. From the government budget constraint (1), it is obvious that an increase in n will increase the proportion of working relative to retired households, and thus reduce η. (See Appendix.) This affects V_t' because η appears directly in the first term in (13). In addition, η enters the household budget constraint (2), and a change in η therefore changes the optimal pre-retirement consumption stream. In particular, for a fully-anticipated change in η, lasting for the entire pre-retirement period, we have [10)]

$$\frac{\partial c_t}{\partial \eta} = -\exp\left(\frac{r-\delta}{\varepsilon}t\right)\beta\Delta_{0,R}\left\{\int_0^R \exp\left[\left(\frac{r-\delta}{\varepsilon} - r\right)s\right]\mathrm{d}s\right\}^{-1} < 0. \tag{15}$$

Using (15), we can differentiate V_t' with respect to n. Recalling that $-cu''/u' = \varepsilon$, we find

$$u'(c_t)^{-1}\frac{\partial V_t'}{\partial n} = \left(\varepsilon\eta\frac{\partial \log c_t}{\partial \eta} - 1\right)\Delta_{t,R}\frac{\partial \eta}{\partial n} > 0, \tag{16}$$

where the inequality follows from (15) and from $\partial\eta/\partial n < 0$. Hence from (14), we have

Proposition 1: *An increase in the population growth rate raises the steady-state desired level of social security for voters of every age $t < R$.*

Intuitively, this result is easily understood. First, an increase in n, by lowering η, lowers the cost per unit of social security benefits. This makes workers demand greater levels of β. Second, by lowering η, pre-retirement consumption is increased. This lowers $u'(c_t)$, the weight on the pre-retirement consumption loss from social security taxes, relative to $u'(c_R)$, the weight on the post-retirement consumption gain from social security benefits. Both effects work in the same direction, resulting in an unambiguous increase in β_t^*. Note that the size of the derivative in (16) depends positively on ε. In particular, one can see from (15) that $\partial c_t/\partial \eta \approx -\beta$ when ε is very large, while

$$c_t \approx c_0 \approx \int_0^R (w_s - \eta)\,\mathrm{e}^{-rs}\,\mathrm{d}s\,\Delta_{0,R}^{-1}.$$

Thus, the derivative in (16) can be made arbitrarily large by increasing ε [11)].

Although Proposition 1 tells us that voters in every age category would prefer

[10)] To see this, note that the optimal pre-retirement consumption grows at rate $(r - \delta)/\varepsilon$, so that $c_t = c_0^{((r-\delta)/\varepsilon)t}$. Substituting this into the pre-retirement budget constraint, we can solve for c_0 in terms of η from

$$c_0\int_0^R \exp\left[\left(\frac{r-\delta}{\varepsilon} - r\right)s\right]\mathrm{d}s = \int_0^T (w_s - \eta)\,\mathrm{e}^{-rs}\,\mathrm{d}s.$$

[11)] With ε very large the pre-retirement consumption stream is almost horizontal, and an increase in η which raises taxes by $\beta\mathrm{d}\eta$, will lower this stream by almost exactly the increase in taxes. The expression for c_0 follows from the preceding footnote.

a larger social security program when n rises, this does not necessarily mean that the majority voting equilibrium level of social security will increase. The reason for this ambiguity is that younger workers prefer smaller social security programs than older workers, and an increase in n increases the relative proportion of young workers. In particular, the median age m is a function of the population growth rate n, such that $\partial m/\partial n < 0$. Hence, although an increase in n raises the desired level of social security for the voter of median age, it also lowers this voter's median age, and the net impact on the majority voting equilibrium depends on which effect dominates.

To investigate this more formally, think of the majority voting equilibrium condition (13) for $t = m$ as depending on the parameter m, i.e., on the age of the median voter. Differentiating with respect to m,

$$\frac{\partial V_m'}{\partial m} = -u''(c_m)\dot{c}_m\eta\Delta_{m,R} + ru'(c_m)\eta\, e^{-r(R-m)} + \delta\, e^{-\delta(R-m)}\, u'(c_R)\Delta_{R,T} \quad (17)$$

Using $\dot{c}_m/c_m = (r-\delta)/\varepsilon$ from the characterization of the optimal consumption path, the first term in (17) becomes $(r-\delta)u'(c_m)\eta\Delta_{m,R}$. Since this expression is to be evaluated at an equilibrium, we can use (13) to obtain a cancellation with the third term, so that

$$u'(c_m)^{-1}\frac{\partial V_m'}{\partial m} = \eta\left[r\Delta_{m,R} + e^{-r(R-m)}\right] = \eta > 0 \quad (18)$$

where the last equality follows from $\Delta_{m,R} = r^{-1}[1 - e^{-r(R-m)}]$. Intuitively, this result states that the marginal benefit of social security is higher, for a slightly older household, by the amount of extra taxes that such a household is able to escape on account of its greater age.

If we now combine the two parts of our analysis, and consider the simultaneous effects of n on both η and m, we find from (16) and (18) that

$$u'(c_m)^{-1}\frac{dV_m'}{dn} = \left(\varepsilon\eta\frac{\partial \log c_m}{\partial\eta} - 1\right)\Delta_{m,R}\frac{\partial\eta}{\partial n} + \eta\frac{\partial m}{\partial n} \quad (19)$$

which is ambiguous in sign, since the first term is positive and the second is negative. However, as already remarked, the first term can be made arbitrarily large by choosing ε sufficiently large. Hence,

Proposition 2: *For sufficiently concave preferences, an increase in the population growth rate will increase the steady-state majority voting equilibrium level of social security.*

On the other hand, it is not necessarily the case that an increase in n increases social security. To see how the opposite case might arise, suppose for simplicity that wages are constant at $w_s = w$ over the pre-retirement life cycle. Then explicit calculations based on (15) show that $\partial \log c_m/\partial\eta = -\beta/(w-\eta)$, which is independent of r. Since $\Delta_{m,R}$ becomes small as r becomes large, the second term in (19) must dominate the first for r sufficiently large. Similarly, one can show that the second term dominates the first for R sufficiently close to m, since again this makes $\Delta_{m,R}$ small [12]. Hence,

Proposition 3: *For r sufficiently large, or for R sufficiently close to m, an increase in the population growth rate will decrease the steady-state majority voting equilibrium level of social security.*

Thus, our analysis shows that there are opposing forces at work on the political equilibrium when the population growth rate changes. If we apply this analysis to a decrease in fertility such as has been observed in western economies in recent years, it suggests, on the one hand, that the aging of the population will favor higher levels of social security, but, on the other hand, that the higher taxes required to support the system will favor cutbacks in the level of benefits. It is unclear, a priori, which of these tendencies should be expected to be stronger.

Of course, our formal analysis here has been restricted to the comparison of steady states. Although the comparative steady state analysis should provide some insight into real problems, if taken literally it only describes very long-run outcomes. It is therefore worthwhile asking what other considerations should be taken into account in thinking about shorter-run responses to demographic changes.

Our steady-state analysis has highlighted the effect of the population growth rate, n, on the tax per unit of social security benefits and on the age of the median voter. In the Appendix it is shown that, during the transition from a period of low to high population growth, both the tax rate and the age of the median voter "overshoot" their new steady-state values, dropping at first and then rising. This would tend to accentuate both of the opposing forces that we have previously identified. If also the change in n is unanticipated, then households will have planned their life cycle consumption and private asset accumulation paths on the basis of incorrect expectations. An unanticipated rise in n results in an unanticipated fall in η, and hence in unexpectedly higher net wages over the remaining pre-retirement life cycle for the working population. Households will have consumed too little, and saved too much, relative to what they would have done had the stream of taxes been correctly anticipated. On this account, an increase in n would tend to ease the benefit constraint, lowering $u'(c_m)$ for the median voter and thus favoring a higher equilibrium value of social security benefits in the short run.

Thus, the short-run implications of changing the population growth rate, as is true in the long run as well, are ambiguous. It appears that some of the same forces are at work in the short run as in the long run. But the analysis in the short run is complicated by the interactions between voting behavior and asset accumulation.

[12] It is shown in the Appendix that the other terms in (19) are well-behaved as R approaches m, so that the behavior of $\Delta_{m,R}$ dominates.

Although the purely theoretical analysis of the effects of the population growth rate on the voting equilibrium in our model leads to ambiguous results, it still serves a very useful purpose. In particular, it has clearly identified critical parameters whose magnitudes must be empirically determined in order to make predictions about majority voting equilibria. The theoretical analysis also shows how this empirical information can be put to use once it is obtained. Thus, for example, although such an exercise is beyond the scope of the present paper, it would be a relatively straightforward matter to calculate the sign of the critical expression in (19) for a range of values of the parameters identified there. A more ambitious but probably still feasible exercise would be to simulate the model outside of steady states. We leave such problems for future investigation.

4. Issues in Public Choice Models of Social Security

The results of the above analysis and of Boadway and Wildasin (1987) depend on a number of underlying assumptions. This section identifies certain key structural features of the model and contrasts them with other approaches found in the literature. This not only provides a basis for comparison with other analyses, but gives insights into the problem of modelling strategy in this area.

There are three structural features of the present model to which particular attention should be drawn. First, the model is set in continuous time. Second, the political process is one in which voting occurs at fixed intervals of time v, where, in most of the analysis, $v \geq T$. Third, there is an explicit capital market which households can use to accumulate and decumulate real assets. By contrast, most previous contributions to the literature use a discrete-time framework in which the number of periods in the life cycle is either 2 (Verbon, 1986, forthcoming) or 3 (Hu, 1982; Sjoblom, 1985), although Browning (1975) considers a multi-period life cycle. Also, with the exception of Browning, this literature assumes that voting occurs every period, i.e., at intervals of $T/2$ or $T/3$. Finally, all of these models implicitly or explicitly abstract from the existence of capital (e.g., Browning, Sjoblom) or, at least, they do not attempt to model the way that the distribution of assets across households might influence voting behavior. Let us consider the implications of these differences.

To begin with, note that analyzing the determination of social security in a life-cycle voting model without intergenerational altruism is likely to lead to one or the other of two problems. The first problem arises when the median voter is a retiree, since the model then yields the rather uninteresting conclusion that the equilibrium level of social security is the maximum feasible level. Empirically, however, the population of retirees constitutes less than half of the voting population, and any model which reflects this fact will yield median voters who are still of working age. A two-period life-cycle model with positive population growth achieves this, because the young (working) generation is larger than the old (retired) generation. A multi-period or continuous-time life-cycle model will also have less than half the population retired, even with zero or negative population growth, if the retirement portion of the life-cycle is small enough.

However, if the median voter is a worker-taxpayer, this raises another problem. If the voting cycle is short enough, an increase in social security will directly affect the median voter only by raising his taxes. The question then arises as to why the voter would favor any positive level of social security at all. Indeed, our model shows that the equilibrium program size is zero, with zero conjectural variations, for sufficiently short voting cycles. This is also an empirically uninteresting conclusion.

How is this conclusion to be avoided? One possibility, pursued by Verbon (1986), is to assume that while voting occurs every period, the young are altruistic toward the old. This approach leads to a model of social security as a kind of Pareto-optimal redistribution. Following this approach obviously requires one to drop the assumption of non-altruistic behavior, and indeed it gives altruism an essential role in the explanation for the existence of social security. Moreover, it requires intergenerational altruism in the direction opposite to that often assumed, i.e., the altruism must run from young to old rather than old to young. It is certainly of interest to see whether there is some other way to develop a model with positive levels of social security.

Another alternative is provided by Hu (1982) and Verbon (forthcoming). These analyses assume that future levels of social security will be positively related to the level of social security currently being determined. For example, Hu assumes that the predicted period $t+1$ level of social security, β_{t+1}, is a function $\Gamma(\beta_t, \varepsilon)$ of the current level of the program β_t and of an unspecified random variable ε, where ε is meant to represent uncertainty about the outcome of future votes. Hu assumes that $\Gamma_1 = \partial\Gamma/\partial\beta_t > 0$. One rationale for this critical assumption, given by Verbon (forthcoming), is that taxpayers in period $t+1$ will be more confident about the continuation of the program, and hence more willing to pay taxes to support it, if it is currently large. Given $\Gamma_1 > 0$, voters at t have nonzero conjectural variations – they believe that an increase in the current level of the program will raise their own retirement benefits. If this effect is sufficiently strong, there will be a majority voting equilibrium with a positive level of benefits. Verbon uses a very similar technique, by having young voters, in a 2-period model, assume that the probability of the existence of a social security program when they are retired is a geometrically-declining function of current and past decisions as to whether or not to have a program.

There are several questions that one can raise about these approaches. First, in Hu's analysis, it is not obvious that $\Gamma_1 > 0$ is an appropriate assumption. Indeed, our model suggests that a higher initial level of social security will *reduce* the subsequently desired level of social security for all voters (since $\partial\beta^*/\partial\beta^0 < 0$). This would correspond to $\Gamma_1 < 0$. A second question is why there should be any uncertainty about future votes in these models. There is no intrinsic uncertainty in them since factor prices, population growth rates, preference structures, etc. are all deterministic. There is therefore no reason, within the context of these models, why each voter should not expect a zero level of social security in the future, since holding this expectation, self-interested voting would lead to a majority-voting equilibrium in which the level of social security is, in fact, zero. In such a world, no one would perceive any uncertainty about future votes, and

Finally, let us mention the role of capital markets and asset accumulation in voting models of social security. As noted, Browning and Sjoblom assume, explicitly or implicitly, that no private capital market exists. In both cases, this is an essential feature of the analysis. An increase in social security in these models implies an increase in old age consumption and a decrease in consumption during the working years because there is no capital market that allows a separation of current consumption from current income. The existence of interior individually-preferred levels of social security, which is crucial for the analysis, then follows from the existence of an optimal life cycle consumption path with positive consumption in all periods.

Assuming away capital markets greatly simplifies the analysis of social security, and Sjoblom, for example, exploits this simplicity in order to obtain quite clear-cut results on the dynamic evolution of the social security system. On the other hand, this abstraction makes it impossible to investigate how social security interacts with savings behavior. Since it is widely argued that savings decisions are in fact heavily influenced by social security, this may be a significant drawback. More particularly, our analysis shows that the dynamic evolution of the system can depend quite importantly on the way that social security affects the level and intergenerational distribution of assets. In models without liquidity constraints, such as Hu's, the distribution of asset holdings need not be of critical importance if one confines attention to steady-state voting equilibria. However, in a model with borrowing constraints, and even in models without such constraints if one is attempting to analyze the dynamics of the system, asset accumulation is quite important because the voting behavior of households of a given age will depend on the amount of wealth they hold. This sets up a complex dynamic linkage running from the wealth distribution to the voting equilibirum and then, via the influence of taxes and benefits on savings and consumption behavior, back to the ensuing wealth distribution. This linkage, although difficult to analyze, must be an important part of any attempt to model the dynamic evolution of the social security system.

5. Appendix

The purpose of this appendix is to develop some basic results on the effects of the population growth rate, n, on crucial variables in our model.

[13] On the face of it, the assumption that the voting cycle is very long does not have much appeal. In most countries, social security programs are potentially subject to modification in virtually every legislative session. There may be implicit constraints, however, that limit the ease with which these programs can be changed. In other legislative areas, so-called "grandfather clauses" are frequently used to exempt, from the appication of new laws, those who have made important committments on the basis of past laws. "Grandfather clause" is a particularly apt expression for capturing the notion that legislatures will not suddenly and unexpectedly make large cuts in the size of social security benefits for existing retirees--an implicit constraint on voting that is arguably present and that might roughly be represented in our model by a lengthening of the voting cycle.

First, consider the effect of n on η_t, the tax per unit of social security benefits at time t. Let the number of individuals born at time $t = 0$ be denoted by N_0, and suppose that the population grows steadily at rate n until some time t_0. Assume that $t_0 > T$, so that a steady-state population distribution has been achieved at t_0. Now suppose the population growth rate rises to $n' > n$, and remains permanently at this new, higher level.

Under these assumptions, the number of people born at date t will be

$$N_t = \begin{cases} N_0\, e^{nt}, & t \le t_0, \\ N_{t_0}\, e^{n'(t-t_0)} = N_0\, e^{nt_0}\, e^{n'(t-t_0)}, & t > t_0 \end{cases}$$

We define

$$\eta_t = \frac{\int_{t-T}^{t-R} Ns\, ds}{\int_{t-R}^{t} Ns\, ds}$$

This is the per-worker head tax required to balance the government's budget at time t. We wish to calculate the time derivative of η_t: this will show the effect of an increase in the population growth rate on the time path of taxes.

First, let us consider the path of η_t in the interval $(t_0, t_0 + R)$, that is, during the time that higher population growth has affected the working population but not the retired population. For t in this interval, we can write

$$\eta_t = \frac{\int_{t-R}^{t-R} e^{ns}\, ds}{\int_{t-R}^{t_0} e^{ns}\, ds + \int_{t_0}^{t} e^{nt_0}\, e^{n'(s-t_0)}\, ds}$$

$$= \frac{e^{nt}(e^{-nR} - e^{-nT})}{e^{nt_0} - e^{n(t-R)} + e^{(n-n')t_0}\left(e^{n't} - e^{n't_0}\right)}.$$

Letting D denote the denominator of this expression, and differentiating with respect to t, one obtains

$$\dot{\eta}_t = \frac{\eta_t}{D}\left[(n - n')\, e^{(n-n')t_0}\, e^{n't}\right] > 0.$$

Thus, as expected, an increase in the population growth rate results in a reduction in taxes at first.

Now take $t \in (t_0 + R, t_0 + T)$. In this interval,

$$\eta_t = \frac{\int_{t-T}^{t_0} e^{ns}\,ds + \int_{t_0}^{t-R} e^{nt_0}\, e^{n'(s-t_0)}\,ds}{\int_{t-R}^{T} e^{nt_0}\, e^{n'(s-t_0)}\,ds}$$

$$= \frac{e^{nt_0} - e^{n(t-T)} + e^{(n-n')t_0}\left[e^{n'(t-R)} - e^{n't_0}\right]}{e^{(n-n')t_0}\left[e^{n't} - e^{n'(t-R)}\right]}$$

Again letting D denote the denominator, differentiation yields

$$\dot{\eta}_t = \frac{1}{D}(n' - n)\, e^{n(t-T)} > 0.$$

Thus, once the relatively more numerous generations begin to retire, the tax rate must begin to rise.

However, the tax rate will remain permanently lower at the higher population growth rate. To see this, define

$$P(x, n) = \frac{\int_0^x e^{-ns}\,ds}{\int_0^T e^{-ns}\,ds} = \frac{1 - e^{-nx}}{1 - e^{-nT}}$$

to be the proportion of the population of age less than or equal to x, under a regime of steady population growth at rate n. Then, in steady states,

$$\eta = \frac{1 - P(R, n)}{P(R, n)} = \frac{1}{P(R, n)} - 1.$$

If an increase in n raises $P(R, n)$, i.e. the working proportion of the population, it will lower η. But

$$\frac{\partial \log P(R, n)}{\partial n} = \phi(R) - \phi(T)$$

where $\varphi(x) = x/(e^{nx} - 1)$. The numerator of this expression is linear in x while the denominator is exponential, hence $\phi' < 0$ and hence $P(R, n)$ is increasing in n. More explicitly,

$$\frac{\partial \eta}{\partial n} = -(1 + \eta)[\phi(R) - \phi(T)].$$

Thus, after an increase in n, η_t falls for the first R years, and then it rises to a new steady-state value that is lower than the initial steady-state value.

So far, we have been looking at the effect of n on the relative sizes of the working and retired populations, i.e. on the relative numbers of those below and above a given age R. For the purposes of the analysis, however, R is simply some arbitrary age between 0 and T. From this observation, it is immediate that what we have said about the time path of η_t is also true of the time path of the median age in the population. We need not go through all the details, therefore. However, it is useful to record the effect of n on the steady-state value of m. This steady-state value is implicitly defined by $P(m, n) = 1/2$. Therefore, with some manipulation, one obtains

$$\frac{\partial m}{\partial n} = -\frac{\partial P/\partial n}{\partial P/\partial m} = \frac{e^{nm} - 1}{n}[\phi(m) - \phi(T)].$$

If we compare this with the expression for $\partial\eta/\partial n$, we see that the bracketed expressions are nearly equal when m is close to R. Also, η approaches 1 as R approaches m. Thus, both steady-state derivatives approach non-zero finite quantities as m and R set close. This result is necessary to complete the proof of Proposition 3.

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References

Aaron, H.J., (1966), "The Social Insurance Paradox", *Canadian Journal of Economics and Political Science*, 32, 371-76.

Barro, R.J., (1974), "Are Government Bonds Net Wealth ?", *Journal of Political Economy*, 82, Nov.-Dec., 1095-1117.

Boadway, R.W. and Wildasin, D.E., (1987), *Median Voter Model of Social Security*, CORE Discussion Paper no. 8714, (Revised version to appear in *International Economic Review*.)

Browning, E.K., (1975), "Why the Social Insurance Budget is Too Large in a Democratic Society", *Economic Inquiry*, 13, Sept., 373-88.

Cass, D., and Yaari, M.E., (1967), "Individual Saving, Aggregate Capital Accumulation, and Efficient Growth", in K. Shell (ed.), *Essays on the Theory of Optimal Economic Growth*, Cambridge, MIT Press, 233-68.

Diamond, P.A., (1965), "National Debt in a Neoclassical Growth Model", *American Economic Review*, 55, Dec., 1125-50.

Feldstein, M.S., (1974), "Social Security, Induced Retirement and Aggregate Capital Accumulation", *Journal of Political Economy*, **82**, Sept-Oct, 905-26.
Feldstein, M.S., (1977a), "The Surprising Incidence of a Tax on Pure Rent : A New Answer to an Old Question", *Journal of Political Economy*, **85**, April, 349-60.
Feldstein, M.S., (1977b), "Does the United States Save Too Little ?", *American Economic Review*, **67**, May, 116-21.
Hu, S.C., 1982), "Social Security, Majority-Voting Equilibrium and Dynamic Efficiency", *International Economic Review*, **23**, June, 269-87.
Kotlikoff, L.J., (1984), "Taxation and Savings : A Neoclassical Perspective", *Journal of Economic Literature*, **12**, Dec., 1566-1629.
Samuelson, P.A., (1958), "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money" *Journal of Political Economy*, **66**, Dec., 467-82.
Sjoblom, K., (1985), "Voting for Social Security", *Public Choice*, **45**, 225-40.
Summers, L.H., (1981), "Capital Taxation and Accumulation in the Life Cycle Model", *American Economic Review*, **71**, Sept., 533-44.
Verbon, H.A.A., (1986), "Altruism, Political Power and Public Pensions", *Kyklos*, **39**, 343-58.
Verbon, H.A.A., forthcoming, "The Rise and Evolution of Public Pension Systems", *Public Choice*.