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OPTIMAL TAX-SUBSIDY POLICIES FOR INDUSTRIAL ADJUSTMENT TO UNCERTAIN SHOCKS*

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1. Introduction

THE notion that a government should intervene in an economy in order to protect its disadvantaged or unlucky members is a familiar one. This general idea is often used to provide a normative justification for a wide range of public policies, including general income redistribution, progressive taxation, social insurance programs of all kinds, and policies targeted at particular industries, occupational categories, or regions. Direct and indirect subsidies to firms, tax preferences for particular industries or regions, protectionist policies such as tariffs or quotas, regulatory policies which may have protective effects, and job training programs for workers in depressed regions or industries all provide examples of the latter, more specific, types of policies.

It is often argued that such policies will promote a general social objective of distributive justice, for example by helping workers who would otherwise have unacceptably low wages. However, it is observed empirically that the beneficiaries of protective policies are often not particularly poor.[1] The normative focus thus tends to shift to remedying "dislocation," which might be defined in terms of "failed expectations." For example, one might try to justify protective tariffs or trade quotas for the auto industry on the grounds that foreign competition causes workers in the industry to experience wages which, even if not low enough to make them truly poor, nonetheless are low relative to expectations held at the time of initial attachment to the industry, or relative to prior investment in skill acquisition. Protective policy might in this case serve a "social insurance" objective in cushioning workers from large negative quasi-rents.

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[1] For instance, Richardson (1982) compares a sample of recipients of aid under US provisions for "trade adjustment assistance" (TAA) with unemployed workers receiving unemployment insurance (UI) benefits. In the year before their job separations, TAA recipients had higher incomes than the UI recipients. Prior to separation, only 3.7% of UI recipients had incomes below the official poverty line; an even smaller percentage (1.9%) of TAA recipients fell below the poverty line in the year prior to separation. (These findings are perhaps not surprising since the programs involved are not means-tested.)

Some economists analyze such policies from a positive rather than from a normative perspective. In the specific case of protectionist trade policies, for example, many economists are skeptical of the "social insurance" rationale for government intervention. As Baldwin (1982) points out, these policies might just as well arise from rent-seeking behavior by political pressure groups, a view that Magee (1982) strongly endorses. Indeed, a large literature has developed on the positive political economy of trade policy, including, e.g., Findlay and Wellisz (1982), Feenstra and Bhagwati (1982), Mayer (1984), Baldwin (1984a, b), Hillman *et al.* (1987), and references cited therein.[2] This literature is itself a part of a more general development in political economy that explains government policy intervention as the manipulation of public sector power by influential groups pursuing their own self-interest.[3]

These observations are indicative of widely varying interpretations of the role and function of many important types of economic policy. In this paper, we investigate the *normative* foundations of government policy intervention in the face of industry-specific risk. We examine the problem of determining optimal policy in a social welfare maximization framework, rather than taking the more positive approach of trying to explain how policy emerges as the equilibrium of some political process. As will be seen, our analysis indicates conditions under which certain types of policy interventions may or may not have a normative rationale. In this sense, a normative analysis may complement positive political economy by identifying circumstances in which, for example, the "social insurance" justification for policy is weak and when it is strong.

Our approach to this problem begins with the observation that individuals expose themselves to risk when they become attached to particular industries, occupations, or regions.[4] In choosing types and locations of jobs, individuals can never be certain ex ante whether they have chosen well or poorly because future employment conditions will depend on all sorts of uncertain events such as technological changes and world price fluctuations. In a world with complete markets, these risks would be more or less inconsequential, since they could be avoided through the purchase of

[2] It should be noted that some writers who are generally unsympathetic to protectionist trade policies (e.g., Lawrence and Litan [1986]) are opposed not to the general notion of government intervention for social insurance purposes, but to departures from free trade as a mechanism for achieving this objective; that is, they are unhappy with the choice of trade policy as an instrument, not with social insurance or redistribution as a target.

[3] Note, however, that the conflict between the "social insurance" and "self-interest" views of policy intervention can be exaggerated. The two are revealingly intertwined in the comments of Fisher and Fisher (1942, p. 96) on the use of taxation to further self-interest and justice: "For the original purpose of society was and is mutual protection. Call it mercy and sometimes justice, but always it is the duty of somebody—and of society if there is nobody else—to throw a protecting arm around the unfortunate."

[4] To illustrate, Flaim and Sehgal (1985) find earnings losses for "displaced workers" on the order of 20–40% of prior wages.

appropriate insurance contracts. Empirically, however, such markets are seen to be incomplete.[5] As a result, these random occurrences impose substantial risks on workers, as exemplified by many examples of "boom" and "bust" industries, regions and occupations. In such an economy, government policy *may* be able to perform a valuable social insurance function.

In discussions of practical policy problems, actions to aid distressed industries, occupational groups, etc., often look appealing from a "short run" perspective, taking as given the historical circumstances that have resulted in the present economic situation of the industry or group. However, concerns are sometimes expressed about the "long run" consequences of such policies. In particular, a policy response to some *particular* case of economic distress may be taken as a signal of a *policy regime* in which all "disadvantaged" groups will be protected from the rigors of the market place by the government. Note that the distinction here is not just one of "long run" vs. "short run" in the traditional sense of fixity of capital or other resources. Rather, it is a question of whether the full response of economic agents to public policy is taken into account in the evaluation of the policy—this is, whether one takes the structure of the economy as historically given and independent of policy decisions, or whether one takes into account the fact that the choice of policy will affect the structure of the economy into which random shocks will eventually be introduced. We distinguish the two viewpoints with the terms "ex ante" and "ex post."[6]

These issues are discussed more concretely within the context of the model that we analyze in this paper. Throughout most of the paper we study an economy in which a random shock causes one industry to be advantaged

[5] We do not seek here to model *why* markets are incomplete, but simply take this as a datum. One can provide plausible informal explanations for incomplete markets. For example, suppose that young workers were to attempt to buy insurance against income fluctuations later in life. Since a worker's realized income depends on individual effort and skill, such insurance would dull the incentive to work; moreover, it would be especially attractive to workers who know themselves to have low skill levels. These moral hazard and adverse selection problems militate against private insurance for uncertain incomes. On the other hand, government policies operating at the industry level (e.g., subsidies triggered by an industry-wide shock, as opposed to low earnings for an individual worker) might be less susceptible to such problems. Comparable private insurance at the industry level might be difficult to organize because of large transactions costs. Incomplete markets are often assumed in general equilibrium models with technological uncertainty. For just two examples of such models, see Kihlstrom and Laffont (1979) and Kanbur (1979).

[6] As will become clear, the ex post–ex ante distinction is related to the problem of dynamic consistency discussed by Kydland and Prescott (1977) and subsequent authors such as Hillier and Malcomson (1984) and Staiger and Tabellini (1987). Hillman *et al.* (1987) look at a related but different question. They suppose that government behavior gives firms reason to believe that their prospects of obtaining some sort of protection in the event of adverse future contingencies will be higher, the greater the amount of labor they employ. (The intuition is that higher employment implies that future adverse events will hurt labor more severely, thereby increasing the political influence of the adversely-affected firm.) We do not consider the possibility of such manipulation of the political process here.

or disadvantaged relative to another. By "ex post" optimal policies we mean policies that are undertaken to optimize some government objective function after the event has occurred and the industry has been found to be a disadvantaged or declining one. By "ex ante" ones we mean contingent policies formulated before it is known which particular industry will be a disadvantaged one. One might expect that policies undertaken ex post and anticipated by agents in the economy will induce an ex ante misallocation of workers. For example, if it is known that the government will bail out declining industries, this may induce an excess of labor into potentially declining industries ex ante. Ex ante optimal policies, unlike ex post optimal ones, will anticipate this response. We therefore investigate the differences between the two.

Several prior studies in the literature provide a background to ours. First, the literature on optimal income taxation can be given a social insurance interpretation, and Varian (1980), for example, explicitly discusses the role of government redistribution, including income taxation, as a form of insurance. Research in this tradition, however, focuses on innate, unalterable, personalized risks (owing to differences in ability, for instance), rather than on sectoral risks that can be partly mitigated by intersectoral labor flows. Second, some recent work on protectionism in international trade is quite closely related to ours, although our paper is not specifically concerned with trade issues. Grossman and Eaton (1985) study the role of tariff policy as a form of insurance against terms of trade uncertainty, but do not allow ex post intersectoral mobility of disadvantaged factors (in their case, capital rather than labor), effectively assuming that relocation costs are infinite. Staiger and Tabellini (1987) do consider the possibility that workers can migrate, at a cost, once terms of trade uncertainty is resolved. Like other writers in the trade literature, they focus on trade restrictions in the form of tariffs. Even though our model can be interpreted as an open-economy with terms of trade uncertainty, we analyze primarily the role of instruments such as industry subsidies, migration subsidies, and other non-tariff policies. Diamond (1982) has analyzed the role of such instruments in a model with costly migration, and, like us, considers the implications of restrictions on the instruments available to the government. His analysis focuses only on ex post policy evaluation, however.[7]

Our discussion begins, in Section 2, by laying out the basic structure of the model. Section 3 introduces the normative criteria that we will study, and clarifies the distinction between ex ante and ex post policy environments. Section 4 contains most of the main results, including a comparison of optimal policy under different assumptions about the types of policy instruments that the government can use. In Section 5, we note that these results can be given rather varied interpretations. Furthermore, this section

[7] As discussed further below, there are also several specific aspects of the structure of Diamond's model that differ from ours, such as the specification of technology, which make direct comparisons of results somewhat difficult.

shows that it is not difficult to extend many of them to contexts that relax many of the simplifying assumptions that are imposed in the basic model. A concluding section discusses some applications of the analysis.

2. The model and notation

The economy consists of two competitive industries, 1 and 2, operating under conditions of technological uncertainty. Each employs labor as the sole variable input. When uncertainty is resolved, one of two states of the world occurs: a "good" state, which occurs with probability π, and which results in a favorable technological shock in industry 2; and a "bad" state, which occurs with probability $(1-\pi)$ and which results in an unfavorable shock to industry 2. For concreteness, we suppose that the uncertainty takes the form of a multiplicative shift parameter in the production function of industry 2. Variables and functions in the good and bad states will be denoted, respectively, by corresponding roman and greek letters.

The aggregate supply of labor in the economy is n. Ex ante, workers allocate themselves to each of the two industries in the amounts n_1 and $n_2(=n-n_1)$. Ex post, when conditions in one industry have been revealed favorable relative to those in the other, some workers may have an incentive to change occupations, or migrate. This migration will be from industry 1 to 2 in the good state, and conversely in the bad state, with the amount of migrant workers denoted m or μ. Thus, the ex post allocations of labor are (n_1-m, n_2+m) and $(n_1+\mu, n_2-\mu)$ in each of the two states. The outputs in industry 1 are $[f(n_1-m), \phi(n_1+\mu)]$ in each of the two states, and $[g(n_2+m), \gamma(n_2-\mu)]$ in 2, where the production functions (f, ϕ, g, γ) are twice differentiable, strictly increasing and strictly concave, and satisfy $f'(0)=\phi'(0)=g'(0)=\gamma'(0)=\infty$. Our assumption of multiplicative uncertainty in industry 2 means that $\gamma(\cdot)=\theta g(\cdot)$, where $0<\theta<1$, and that $f(\cdot)=\phi(\cdot)$.[8] Output prices are exogenously fixed at unity.[9] Workers in each industry and state are paid their marginal products.

Since relative prices are fixed, output can be treated as a single homogenous good. Let (c_i, ψ_i) denote the income or consumption of this good in each state by non-migrant workers in industry i. Preferences are represented by a common utility function $u(c_i)$ with $u'>0>u''$. Consumption for non-migrants in industry i can differ from their marginal products because they may receive (positive or negative) subsidies (s_i, σ_i) from the government. (Examples of policies corresponding to such subsidies will be discussed below.) Thus:

$$c_1=f'(n_1-m)+s_1 \qquad \psi_1=\phi'(n_1+\mu)+\sigma_1$$
$$c_2=g'(n_2+m)+s_2 \qquad \psi_2=\gamma'(n_2-\mu)+\sigma_2.$$

[8] We distinguish f and ϕ because, as discussed below, the results actually generalize to the case where there is uncertainty in both industries.

[9] One can think of this as a "corn economy" if desired, in which the two different industries correspond to regions with different rainfall realizations. Somewhat more generally, we need only require that the products of the two industries be perfect substitutes in consumption.

The consumption of migrants in each of the two states is denoted by (z, ζ). The consumption of migrants differs from their marginal productivity not only because of subsidies (denoted (s_m, σ_m)) but also because of a fixed migration cost x which we assume to be the same for both states.[10] Migrant consumption in the two states is therefore given by

$$z = g'(n_2 + m) + s_m - x \qquad \zeta = \phi'(n_1 + \mu) + \sigma_m - x.$$

where migrants move from 1 to 2 in the good state and from 2 to 1 in the bad state. Note that $x \geqslant 0$, with $x = 0$ corresponding to the special case of costless migration. Migrant utility in each state is given by $u(z)$ and $u(\zeta)$, where $u(\cdot)$ is the same utility function as for non-migrants.

Ex post migration is undertaken when it is advantageous for some workers to do so, and will continue until workers are indifferent between locations. Thus, the level of migration in each state, m and μ, must satisfy the equilibrium conditions

$$u(c_1) - u(z) = 0 \qquad u(\psi_2) - u(\zeta) = 0 \tag{1}$$

when $m > 0$ and $\mu > 0$. (These equalities are replaced by $\geqslant$ at corner solutions where $m = 0$ or $\mu = 0$.) Ex ante, all workers are identical and can choose to work in either industry at zero (or equal) cost. In equilibrium, they must be allocated so expected utilities are equalized across industries, i.e.

$$\pi u(c_1) + (1 - \pi)u(\psi_1) = \pi u(c_2) + (1 - \pi)u(\psi_2). \tag{2}$$

Since the production functions are strictly concave, rents are generated in each industry. These rents accrue to entrepreneurs who are assumed to be risk-neutral.[11] The government may impose a (positive or negative) tax of t or τ on entrepreneurs in each state, so that aggregate net rents, denoted r and ρ, are:

$$r = f(n_1 - m) - (n_1 - m)f'(n_1 - m) + g(n_2 + m) - (n_2 + m)g'(n_2 + m) - t$$
$$\rho = \phi(n_1 + \mu) - (n_1 + \mu)\phi'(n_1 + \mu) + \gamma(n_2 - \mu) - (n_2 - \mu)\gamma'(n_2 - \mu) - \tau$$

in each of the two states.

[10] The implication of state-dependent migration costs are discussed later. Treating migration costs as resource costs simplifies the analysis; in Boadway and Wildasin (1987) we have treated the more general case in which migration costs include non-resource (psychic) costs.

[11] It is commonplace to assume risk-neutrality on the part of entrepreneurs, or at least that entrepreneurs are less risk-averse than workers. See, for example, Kihlstrom and Laffont (1979). There are, however, various modelling strategies that one could consider here. In some earlier work, we investigated the case in which all rents were collected by the public sector on behalf of the workers (Boadway and Wildasin (1987)). The qualitative results on subsidies were basically the same. Alternatively, entrepreneurs could be made risk averse. Doing so adds little insight to the analysis while complicating matters considerably. Other authors use different assumptions: Staiger and Tabellini (1987) assume that all rents are collected by the government and paid out as a proportional wage subsidy to workers, while Diamond (1982) assumes that labor's marginal product is constant in each industry, so that there are no rents generated in the model.

Finally, government policies must be feasible, satisfying the government budget constraints in each state:

$$(n_1 - m)s_1 + n_2 s_2 + m s_m = t \qquad n_1\sigma_1 + (n_2 - \mu)\sigma_2 + \mu\sigma_m = \tau. \tag{3}$$

This essentially completes the description of the economy. From an ex ante perspective, given the values of $(s_1, \sigma_1, s_2, \sigma_2, s_m, \sigma_m, t, \tau)$, an equilibrium is a set of values for (n_1, n_2, m, μ) satisfying (1), (2), and (3). Once these values are fixed, equilibrium production, wages, rents, consumption, and utilities are all determined. From an ex post perspective, given (n_1, n_2) and the state-dependent policy instruments (e.g., (s_1, s_2, s_m, t) for the good state), the ex post migration equilibrium condition (1) determines the amount of migration (e.g., the left-hand expression in (1) determines m), and thus the equilibrium allocation of labor, wages, rents, etc.[12]

It is important to notice that workers incomes come entirely from wages determined on spot labor markets. That is, workers are assumed not to be able to insure themselves against technological shocks. In principle, there are two ways that this might be done. First, if capital markets were perfect, workers could buy shares ex ante in each industry.[13] Note that profit sharing within each industry would *not* accomplish the same outcome. Second, if entrepreneurs (or insurance companies) could offer full state-contingent wage contracts to workers, including those who migrate, the same Pareto-optimal allocation would be achieved. As noted in the introduction, we assume that workers do not have access to such private insurance arrangements. Instead, government policy will fulfill this risk-sharing role. To the extent that workers can insure, government's role will be diminished.

As mentioned, the model can be given an open-economy interpretation. This is of interest because it facilitates a comparison of our results with some of the literature on trade policy. Suppose that the output of industry 2 is sold on world markets where its price (relative to the output of industry 1, which is taken as numéraire) is subject to random fluctuations. This replaces technological uncertainty as the source of risk in the model. If we assume that the output of industry 2 is not consumed domestically, then we need not be concerned with consumption price risk, and the model as written applies directly.[14]

[12] For expository reasons, the discussion here and in the next section makes only passing reference to corner solutions. These are discussed in somewhat more detail later. Note here, however, that $f'(0) = \infty$ and related conditions imply that $n_1 - m > 0$, $n_2 + m > 0$, $n_1 + \mu > 0$ and $n_2 - \mu > 0$. Along with positive migration costs, these conditions imply $n_1 > 0$ and $n_2 > 0$.

[13] The model would then be like that of Diamond (1967) with migration costs imposed. The same concept of constrained Pareto efficiency would apply if there were more states of the world than types of shares.

[14] Consumption price risk is explicitly incorporated in Grossman and Eaton (1985) and Staiger and Tabellini (1987). However, as Diamond (1982) notes, this is a relatively minor concern if the industry in question accounts for a small share of the consumers' budgets. In any event, we wish to focus attention on income risk.

3. Ex post and ex ante policy equilibria

We distinguish between two settings for government policymaking. The first, the ex post optimization framework, arises when the government maximizes the sum of utilities after the state of the world is known, taking as exogenously given the initial allocation of labor across industries. This represents a short-run approach to policy, or, alternatively, it represents how policies might be chosen when the random shock is totally unforeseen. In the second, ex ante optimization, the government chooses its policy instruments taking full account of the effect of its policies on the initial allocation of labor.

Ex post optimization

Let $a=(s_1, s_2, s_m, t, m)$ denote the set of policy instruments used by the government in the good state, and let $d(a, n_1)=n_1u(c_1)+n_2u(c_2)+r$ denote total utility plus net rents in the good state. Since entrepreneurs are risk-neutral, their utility is linear in income. Thus, the function $d(a, n_1)$ is simply a classical utilitarian measure of aggregate utility, with individual utilities normalized such that for entrepreneurs, utility equals income. Then, in the good state, the government's ex post policy problem is to

$$\max_{\langle a\rangle} d(a, n_1) \tag{4}$$

subject to the good-state versions (left-hand side) of the migration equilibrium and government budget constraints (1) and (3), and taking n_1 (and $n_2=n-n_1$) as given.[15] In the event that the government is restricted in the number of instruments available to it, only a subset of the vector a would act as controls. The solution to this problem yields a as a function of n_1. Let $L(n_1)$ denote the maximum value function for problem (4), that is the value of $d(a, n_1)$ at the optimum. Similarly, the ex post problem for the bad state is

$$\max_{\langle a\rangle} \delta(\alpha, n_1) \tag{5}$$

subject to (1) and (3) (right-hand side), given n_1, where $\alpha=(\sigma_1, \sigma_2, \sigma_m, \tau, \mu)$ is the instrument vector and $\delta(\alpha, n_1)=n_1u(\psi_1)+n_2u(\psi_2)+\rho$ is the objective. The maximum value function is denoted $\Lambda(n_1)$.

An *ex post policy equilibrium* is a vector (a^*, α^*, n_1^*) such that for $n_1=n_1^*$, a^* and α^* solve (4) and (5) and the ex ante migration equilibrium condition (2) is satisfied.

[15] For the sake of technical convenience only, m is treated as a policy instrument to be chosen subject to (1). This is equivalent to using (1) to solve for m and dropping m as an instrument.

Ex ante optimization

In the ex ante problem, the government maximizes aggregate expected utility plus expected net rents, taking ex ante migration equilibrium into account. Let $W(a, \alpha, n_1) = n[\pi u(c_1) + (1-\pi)u(\psi_1)] + \pi r + (1-\pi)\rho$. Then the ex ante problem is

$$\max_{\langle a, \alpha, n_1 \rangle} W(a, \alpha, n_1) \tag{6}$$

subject to (1), (2), (3), and possibly subject to additional institutional constraints on the instruments.[16] A solution $(\bar{a}, \bar{\alpha}, \bar{n})$ to (6) is called an *ex ante policy equilibrium.*

To compare the ex ante and ex post policy equilibria, we could, in principle, use constraints (1) and (3) to eliminate the instruments (m, μ, t, τ). Let $s = (s_1, s_2, s_m)$ and $\sigma = (\sigma_1, \sigma_2, \sigma_m)$ denote the truncated instrument vectors for the ex post problems (4) and (5), and note that the ex ante migration equilibrium condition (2) could now be used to solve for $n_1(s, \sigma)$. It follows that an ex post policy equilibrium is a vector (s^*, σ^*) that solves[17]

$$\frac{\partial d(s^*, n_1[s^*, \sigma^*])}{\partial s} = \frac{\partial \sigma(\sigma^*, n_1[s^*, \sigma^*])}{\partial \sigma} = 0. \tag{7}$$

Note here that it is the vector of *partial* derivatives of d and δ with respect to their *first* arguments alone that are equated to zero. To constrast this with an ex ante policy equilibrium, note that $W(s, \sigma, n_1[s, \sigma]) = \pi d(a, n_1[s, \sigma]) + (1-\pi)\delta(\sigma, n_1[s, \sigma])$. In an ex ante policy equilibrium $(\bar{s}, \bar{\sigma})$,

$$\begin{aligned}&\frac{\partial W(\bar{s}, \bar{\sigma}, n_1[\bar{s}, \bar{\sigma}])}{\partial s} + \frac{\partial W(\bar{s}, \bar{\sigma}, n_1(\bar{s}, \bar{\sigma}])}{\partial n_1}\frac{\partial n_1(\bar{s}, \bar{\sigma})}{\partial s}\\ &\qquad = \frac{\partial W(\bar{s}, \bar{\sigma}, n_1[\bar{s}, \bar{\sigma}])}{\partial \sigma} + \frac{\partial W(\bar{s}, \bar{\sigma}, n_1[\bar{s}, \bar{\sigma}])}{\partial n_1}\frac{\partial n_1(\bar{s}, \bar{\sigma})}{\partial \sigma} = 0,\end{aligned}$$

i.e.,

$$\begin{aligned}&\pi\frac{\partial d(\bar{s}, n_1[\bar{s}, \bar{\sigma}])}{\partial s} + \left[\pi\frac{\partial d(\bar{s}, n_1[\bar{s}, \bar{\sigma}])}{\partial n_1} + (1-\pi)\frac{\partial \delta(\bar{\sigma}, n_1[\bar{s}, \bar{\sigma}])}{\partial n_1}\right]\frac{\partial n_1}{\partial s}\\ &= (1-\pi)\frac{\partial \delta(\bar{s}, n_1[\bar{s}, \bar{\sigma}])}{\delta \sigma} + \left[\pi\frac{\partial d(\bar{s}, n_1[\bar{s}, \bar{\sigma}])}{\partial n_1} + (1-\pi)\frac{\partial \delta(\bar{\sigma}, n_1[\bar{s}, \bar{\sigma}])}{\partial n_1}\right]\frac{\partial n_1}{\partial \sigma}\\ &= 0.\end{aligned} \tag{8}$$

Comparing (7) and (8), it is clear that an ex post policy equilibrium

[16] n_1 is treated as a policy instrument, subject to (2), for technical convenience. See fn. 15.

[17] In writing d as a function of s rather than a, the functional dependence of (m, t) on s is of course implicitly taken into account. Similarly for δ and W.

(s^*, σ^*) will also be an ex ante policy equilibrium if (and only if)[18]

$$[\pi L'(n_1[s^*, \sigma^*]) + (1-\pi)\Lambda'(n_1[s^*, \sigma^*])]\frac{\partial n_1}{\partial s}$$
$$= [\pi L'(n_1[s^*, \sigma^*]) + (1-\pi)\Lambda'(n_1[s^*, \sigma^*])]\frac{\partial n_1}{\partial \sigma} = 0. \quad (9)$$

where $L(\cdot)$ and $\Lambda(\cdot)$ are the maximum value functions for the ex post problem as defined earlier.

Thus in a formal sense, the distinction between ex post and ex ante optimal policies is like the distinction between partial and total derivatives. In investigating policy equilibria in the next section, we will be interested in evaluating whether (9) is satisfied in various cases. Since $\partial n_1/\partial s$ and $\partial n_1/\partial \sigma$ are generally non-zero for the problems we consider, this amounts to evaluating $[\pi L'(\cdot) + (1-\pi)\Lambda'(\cdot)]$, a term which has an intuitive interpretation. It is the expected change in social welfare from an incremental increase in n_1, starting at the ex post policy equilibrium. If it is zero, labor is allocated optimally ex ante. If it is positive, expected social welfare will increase if labor is reallocated to industry 1, and vice versa.

One can also contrast the ex post and ex ante policy evaluations environments in terms of the nature of the equity/efficiency tradeoffs that they involve. The analysis of ex post optimal policy reflects government equity objectives as expressed in a utilitarian social welfare function. In particular, workers in different sectors will have unequal incomes in the absence of offsetting public policies, and this may provide some role for policies with redistributive impacts (an equity objective). In the ex ante case, we may also think of government policy as being set in accordance with social welfare objectives. However, when workers are able to choose the industry to which they are initially assigned at no cost (costless ex ante migration), any equilibrium will be characterized by equal ex ante utility for all workers. Thus, aggregate expected utility for workers will coincide with expected per capita utility and the objective function of the government, in its treatment of workers, is then better interpreted as reflecting a valuation for insurance for ex ante identical individuals (an efficiency objective) as opposed to an aversion to inequality between fortunate and unfortunate workers.

4. Characterization of optimal policies

In this section, we derive the rules that the government should follow in selecting the set of taxes and subsidies for the ex post and ex ante problems. The rules turn out to depend both upon migration costs and upon whether or not the set of subsidy instruments that the government uses is restricted.

[18] Ignoring complications concerning corners, which are discussed further below, our problem is sufficiently well-behaved that necessary conditions for interior solutions are also sufficient.

In fact, polar extreme sorts of results can be obtained by imposing restrictions either on the available instruments or on the size of migration costs. Since these extreme results are both interesting and intuitive, we organize the presentation of our results so as to highlight them. We begin with the case in which the government is restricted in the number of instruments it uses, and then treat the unrestricted case. Our restrictions concern only the manner in which the government is able to discriminate among workers. The government is assumed always to be able to redistribute between entrepreneurs and workers.[19]

There are various interpretations that can be put on the restricted use of instruments. One is that the government itself may choose to restrict itself to particular instruments to address the problem of worker dislocation. Another is that certain instruments may be difficult to apply for informational or incentive reasons, such as those which require identification of workers according to the industry in which they originate. Finally, the analysis of restricted instruments is a useful pedagogical device for learning about the nature of the policy problem and equivalences among instruments, as well as their limitations.

Case (i). Restricted Instruments: Industry Subsidies Only

Suppose the government subsidizes workers according to where they work, and does not or cannot distinguish between migrants and non-migrants. The subsidies may, of course, be state-contingent. Institutionally, this could correspond to two alternative interpretations. First, it could represent the case in which the government subsidizes firms via output or employment subsidies or taxes. Equivalently, the government could directly subsidize and tax workers in each of the two industries on the basis of the industry in which they are actually employed (not on the basis of their ex ante occupational choice). As an example, a subsidy to the dairy industry or to agriculture in general would benefit those who work in the industry. Workers who leave the industry are no longer beneficiaries of these programs, but instead join the population of taxpayers who pay for them. This would be true, as well, of industry-specific protectionist policies, or of

[19] We note here some of the specific similarities between our analysis and that of Diamond (1982). First, we note that Diamond's model is similar to ours in that it exhibits costly migration from one industry to another. However, Diamond does not consider the distinction between ex ante and ex post optimal policies, restricting attention instead to the problem of ex post policy analysis. Within the ex post context, the approaches are again similar, since Diamond uses a utilitarian social welfare function. As noted earlier, Diamond assumes a linear production technology so that the wage of labor in each industry is independent of the level of employment and so that there are no rents. Intersectoral migration is equilibrated through rising migration costs (i.e., some workers have low migration costs and are the first to move, followed by others with higher migration costs, etc.), rather than through endogenous wage adjustments, as in our model. These structural differences make comparisons of our results with Diamond's somewhat difficult, for example because Diamond's assumption of person-specific variations in moving costs introduces an element of worker heterogeneity that has no counterpart in our model. However, where they might be expected to overlap—i.e., in the characterization of ex post optimal policies—our results do seem broadly consistent with Diamond's.

regionally-targeted policies (e.g., aid to the "rust belt" or to businesses in inner cities). Second, a subsidy that is conditioned only on the industry in which a worker is employed could represent a redistributive progressive wage income tax system in which the income tax base does not allow a deduction for migration costs. This is, the tax base in our model would correspond with c_1, c_2, $z + x$, etc. Since these policies treat all workers in an industry identically regardless of whether or not they are migrants, they are equivalent to imposing the restrictions $s_m = s_2$ and $\sigma_m = \sigma_1$ in the two states.

The ex post problems with these instruments can be written formally as in (4) and (5), where the constraints are (1), (3), and $s_m = s_2$, $\sigma_m = \sigma_1$. One can substitute from the government budget constraint (3) into the objective function (4), eliminating (t, τ) as instruments.[20] This leaves only constraints (1). If one associates Lagrange multipliers (b, β) to these constraints, then one can characterize the ex post optimum policies by deriving necessary conditions in the usual way. For the good state, the Lagrangian is

$$n_1 u(c_1) + n_2 u(c_2) + r + t - s_1(n_1 - m) - s_2(n_2 + m) + b[u(c_1) - u(z)],$$

which is optimized with respect to (s_1, s_2, m). Assuming $b > 0$, $m > 0$, the first-order conditions reduce to:

$$s_1 = s_2 \tag{10}$$

$$\frac{n_1}{n} u'(c_1) + \frac{n_2}{n} u'(c_2) = 1. \tag{11}$$

From (10), no differential treatment of the favored and disfavored industries is called for. That is, *there should be no industrial policy.*[21] There will, of course, be redistribution between the workers as a whole and the entrepreneurs. It will be governed by (11) which states that the average marginal utilities of the workers equals that of the entrepreneurs (which is unity here). Given that instruments are restricted, the redistributive potential of the tax-subsidy system is limited and the utilities of the workers remain unequal.

Industry subsidies are a completely ineffective instrument for redistribution, given the migration equilibrium condition, since they cannot distinguish between non-migrants originally in the favored industry and migrants coming to the favored industry (who must have the same utility as non-migrants in the declining industry). Differential industry subsidies would simply distort the migration decision, reducing total utility for all

[20] This substitution is legitimate if there are no binding non-negativity constraints on the choice of (t, τ) at an optimum. (The most natural constraints to impose are perhaps that $t \geq 0$, $\tau \geq 0$, $r \geq 0$, $\rho \geq 0$). Constraints of this nature will be non-binding under weak restrictions on the curvature of u. Essentially, the choice of (t, τ) is a decision on how much to redistribute from entrepreneurs to workers. Our analysis requires that these transfers be operative (e.g., $t > 0$, $\tau > 0$) but not indefinitely large (e.g., $r > 0$, $\rho > 0$). It would distract attention from the main purposes of our analysis to pay formal attention to cases where these weak conditions do not obtain, and they are assumed to hold henceforth.

[21] This result also applies more generally to the case of non-pecuniary adjustment costs. See Boadway and Wildasin (1987).

persons without reducing utility differentials (which are determined here by migration costs). To see this, notice that $z + x = c_2$. By (1), it then follows that $c_1 < c_2$ whenever $x > 0$, but that $c_1 \to c_2$ as $x \to 0$. The optimal policy is thus one which allows market-generated interworker inequalities to persist. These inequalities are arbitraged away to some extent by migration, the inequalities becoming arbitrarily small as migration costs become small.

Now suppose that x is sufficiently large that $m = 0$ and $b = 0$ – i.e., $x > g'(n_2) - f'(n_1)$.[22] In this case, (10) no longer holds. With no migration, subsidies should be differentiated across industries in order to achieve distributive objectives. In particular, it is not difficult to show that one would optimally set $s_1 - s_2 = g'(n_2) - f'(n_1) > 0$, so that $c_1 = c_2$, $u'(c_1) = u'(c_2) = 1$, and full equalization of incomes is achieved. Notice that when $x > g'(n_2) - f'(n_1)$, there would be no migration in the absence of any redistributive policy (i.e., if the government followed a *laissez-faire* strategy); conversely, there would be some migration under a *laissez-faire* regime if $x < g'(n_2) - f'(n_1)$, and, as we have seen, there will also be the same amount of migration under the optimal policy. Thus, in either case, optimal government policy does not change the amount of migration that takes place, as compared with the amount under *laissez-faire,* and, in particular, it never changes what would otherwise be a no-migration outcome to one with migration, or *vice versa.*[23]

It is clear that identical results characterize the ex post optimal policies in the bad state. Moreover, when $m > 0$ and $\mu > 0$, differentiation of the Lagrangians for the good and bad states and application of the envelope theorem imply that $L' = u(c_1) - u(c_2)$ and $\Lambda' = u(\psi_1) - u(\psi_2)$. Hence, in an ex post policy equilibrium with positive migration, since (2) is satisified, we must have $\pi u(c_1) + (1-\pi)u(\psi_1) - \pi u(c_2) + (1-\pi)u(\psi_2) = \pi L' + (1-\pi)\Lambda' = 0$, and thus, by (9), an ex post policy equilibrium must also be an ex ante policy equilibrium. This can be verified by working out the ex ante problem and generating the same first order conditions as for the two ex post problems. To summarize:

Proposition 1. Suppose that the government pays subsidies to workers according to the industry in which they are employed in each state. Then:
(a) ex post and ex ante optimal policies coincide if migration occurs in both states,

[22] One can show that the optimal $m > 0$ if and only if $f'(n_1) < g'(n_2) - x$. This is an intuitively natural condition which states that (starting from 0 migration) the marginal product of a migrant worker in industry 2, net of the cost of migration, is higher than in industry 1. In this case, at least some migration will be optimal, and will proceed to the point where $f'(\cdot) = g'(\cdot) - x$.

[23] The first-order conditions in the general case where we allow for $m \geq 0$ can be summarized as (11) plus

$$s_1 \geq s_2 \qquad g' - x \leq f'$$

$$m(s_1 - s_2) = 0 \qquad m(g' - x - f') = 0$$

$$(s_1 - s_2)(c_1 - c_2) = 0.$$

(b) these policies involve no interworker redistribution at all in states where the (optimal) level of migration is positive,
(c) these policies involve complete equalization of incomes across workers in states where there is no migration.

Case (ii). Restricted Instruments: Migration Subsidies

Consider now a second type of policy in which workers that migrate from one industry to another are singled out for special treatment relative to non-migrants. This means imposing the constraints $s_1 = s_2$ and $\sigma_1 = \sigma_2$ in the two states, but allowing $s_m \neq s_1$ and $\sigma_m \neq \sigma_1$. The differences $(s_m - s_1)$ and $(\sigma_m - \sigma_2)$ could be thought of as a subsidy to migration per se in addition to a general subsidy to all workers in each state.

Job retraining programs for workers provide one example of a policy that is targeted at workers who are going to change industries or occupations. In the US, such programs are sometimes instituted for industries that are determined to have been harmed by import competition. As another example, think of migrants as "voluntarily unemployed" workers from an industry in which they have been laid off, so that migration costs would include forgone earnings and job search costs during an unemployment spell. Then a migration subsidy could be interpreted as a subsidy to job search or as unemployment insurance (of a lump-sum type however, such as severance pay). Speaking very broadly, even policies such as publicly-supported general education can be seen as lowering the costs of occupational change, and hence as a form of migration subsidy.

The ex post problems under this specification of instruments are as in (4) and (5), where the constraints are (1), (3), and $s_1 = s_2$, $\sigma_1 = \sigma_2$. As before, we use the government budget constraints to eliminate (t, τ) as instruments, and we use the institutional constraints to eliminate (s_2, σ_2). With (b, β) as Lagrange multipliers for the constraints (1), the good state Langrangian becomes

$$n_1 u(c_1) + n_2 u(c_2) + r + t - (n - m)s_1 - ms_m + b[u(c_1 - u(z)].$$

The first-order conditions for this problem, for the case $m > 0$, reduce to:

$$s_m - s_1 = n_2[u'(c_2) - 1][f''(n_1 - m) + g''(n_2 + m)] \tag{12}$$

$$\frac{n_1}{n} u'(c_1) + \frac{n_2}{n} u'(c_2) = 1. \tag{13}$$

Again, (13), which governs the extent of redistribution between entrepreneurs and workers, states that average marginal utility from consumption of the workers equals that of the entrepreneurs (i.e., unity). By (12), we see that $s_m > s_1$ if $u'(c_2) < 1$. That is, subsidies to migrants would be desirable if workers in the favored industry are relatively well off. It is straightforward to show that $c_2 > c_1$ at the restricted optimum, so $u'(c_2) >$

$1 > u'(c_1)$ by (13) and hence $s_m > s_1$ by (12).[24] Of course, perfectly symmetric results hold for the bad state.

We note that the case where migration costs are sufficiently high that m or $\mu = 0$ is not very interesting when the analysis of optimal policy focuses on the differential treatment of migrants relative to non-migrants, as is true under the current institutional restrictions. If migration costs are zero, $s_m = s_1$. Market arbitrage fully equalizes net income in this case. Summarizing the above results:

Proposition 2. If migration costs are positive but sufficiently small that migration occurs, the optimal ex post migration subsidy is greater than the subsidy paid to non-migrants but not great enough to equalize net incomes across industries. If migration costs are zero, migrants do not receive differentially higher subsidies than other workers.

Finally, note that using the envelope theorem we can derive:

$$L' = u(c_1) - u(c_2) + (s_1 - s_m) \qquad \Lambda' = u(\psi_1) - u(\psi_2) + (\sigma_1 - \sigma_m).$$

Therefore, at an ex post policy equilibrium,

$$\pi L' + (1-\pi)\Lambda' = \pi(s_1 - s_m) + (1-\pi)(\sigma_m - \sigma_1) \gtrless 0.$$

This implies that, in general, the ex post policy equilibrium will not be ex ante optimal. However, we cannot say a priori whether there are too many workers in 1 or too few.

Next consider the ex ante optimization problem. Beginning with the problem stated as (6), use the government budget constraints (3) to eliminate (t, τ), let (b, β) be Lagrange multipliers for (1), and let λ be a Lagrange multiplier for the ex ante migration constraint, so that the Lagrangian for this problem is

$$\begin{aligned}&\pi[nu(c_1) + r + t - (n-m)s_1 - ms_m]\\ &\qquad + (1-\pi)[nu(\psi_1) + \rho + \tau - (n-\mu)\sigma_1 - \mu\sigma_m]\\ &\qquad + b[u(c_1 - u(z)] + \beta[u(\psi_2) - u(\zeta)]\\ &\qquad + \lambda[\pi u(c_1) + (1-\pi)u(\psi_1) - \pi u(c_2) - (1-\pi)u(\psi_2)],\end{aligned}$$

which is to be optimized with respect to s_1, s_m, σ_1, σ_m, μ, and n_1. The Appendix develops the first-order conditions for this case in some detail. For an optimum in which m, $\mu > 0$, the condition on n_1 reduces to:

$$\pi(s_1 - s_m) - (1-\pi)(\sigma_1 - \sigma_m) = 0.$$

This indicates that if migrants are subsidized relative to other workers in one state the same must be true in the other state as well. Furthermore, the following result is proven in the Appendix:

Proposition 3. Suppose migration costs are non-zero but sufficiently small

[24] Proof: Suppose the contrary. If $c_1 \geq c_2$, $z > c_2$ by (1). Thus, $g' + s_m - x \geq g' + s_1$ and hence $s_m > s_1$. Thus, $u'(c_2) < 1$ by (12). But $c_1 \geq c_2$ also implies $u'(c_1) \leq u'(c_2)$, so $u'(c_2) \geq 1$ by (13). This contradiction implies $c_1 < c_2$. Q.E.D.

that positive migration occurs in both states. Then ex ante optimal migration subsidies are positive but not large enough to equalize net incomes across industries. If migration costs are zero, then migrants do not receive differentially higher subsidies.

Thus, Propositions 2 and 3 show that there are qualitative similarities between ex post and ex ante optimal policies. However, the two will not coincide in general. Furthermore, we cannot say a priori in which direction n_1 differs in the ex post policy equilibrium as compared with the ex ante one.

Case (iii). Restricted Policy Instruments: State-Independent Subsidies

Suppose now that the subsidies offered to workers are not state-contingent. However, we imagine that they may vary according to the industry in which workers are located ex ante, and so may affect the allocation of workers across industries. Taxes collected from entrepreneurs will therefore also be state-independent. These restrictions are equivalent to requiring that $s_1 = \sigma_1 = s_m$, $s_2 = \sigma_2 = \sigma_m$, and $t = \tau$.

In this context, the distinction between ex ante and ex post policies is meaningless. The instruments of necessity are imposed ex ante. Working through the ex ante policy problem with the above constraints on instruments, we obtain the result that $s_1 = s_2$. That is, there is no differentiation among workers—all receive the same subsidy regardless of the industry to which they are initially assigned. This means that differential treatment of workers in different industries can only be justified, in the present framework, when state-contingent instruments are available.

Case (iv). Unrestricted Policy Instruments

Suppose that we allow the government freedom to choose all subsidy rates independently (subject, of course, to budget balance). An unrestricted choice of subsidies should allow the government to attain a constrained Pareto-optimal allocation of resources, where the constraints are the ex ante and ex post migration equilibrium conditions.[25] As we show later, one interpretation of this is that differential subsidy rates (s_1, s_2, s_m, σ_1, σ_2, σ_m) could be attained by an appropriate combination of the previous two policies—industry subsidies and migration subsidies.

The first-order conditions from the ex post optimization problem in the good state (4), when $m > 0$, reduce to:

$$u'(c_1) = u'(c_2) = 1 \tag{14}$$

$$s_1 = s_m. \tag{15}$$

Condition (15) indicates that workers initially assigned to 1 should be

[25] The allocation is constrained-optimal in the sense that migration equilibrium constraints, which are binding here, prevent the government from obtaining social welfare optima with different utility levels of migrants from non-migrants. This phenomenon is familiar from the literature of urban economics and local public goods. See Wildasin [1986a,b] for discussion and references.

subsidized identically, whether or not they migrate.[26] Thus, the migration decision should not be distorted. The reason for this is that migrants and those workers remaining in industry 1 will end up with equal utilities in any equilibrium. Therefore, no distributional objective is furthered by subsidizing them differentially.[27] A symmetric analysis for the bad state shows that $\sigma_2 = \sigma_m$ (provided $\mu > 0$). These results can be used from now on to drop s_m and σ_m from explicit consideration in states with positive migration.

Condition (14) determines the extent of redistribution of income between workers and entrepreneurs. They imply that the net incomes of workers are equalized at an optimum. Moreover, $c_1 = z - x = c_2$ implies $f' + s_1 = g' + s_m - x = g' + s_1 - x = g' + s_2$, and hence $g' - f' = s_1 - s_2 = x$. This equality uniquely determines the final allocation of labor $(n_1 - m, n_2 + m)$ independently of the ex ante allocation of labor (n_1, n_2).[28]

Symmetric results of course apply in the bad state. In particular, $u'(\psi_1) = u'(\psi_2) = 1$. Thus, not only are net incomes equalized across workers in different industries in a given state, they are equalized across states as well, i.e. $c_1 = c_2 = \psi_1 = \psi_2$. Since the government budget is balanced in each state, it follows that net entrepreneurial rents are equal to total output less nc_1, the total consumption of all workers. All risk is therefore born by entrepreneurs. It is somewhat intuitive that this is optimal, since they are risk neutral while workers are risk averse. Hence, the structure of taxes and subsidies is set so as to exactly offset gross wage differentials across industries, which in turn will exactly equal migration costs.

Summarizing the results so far, including symmetric results for the bad state, we have

Proposition 4. a. With unrestricted policy instruments, the optimal ex post policy (with positive migration) requires setting $s_1 = s_m$ and $\sigma_1 = \sigma_m$, i.e., workers are subsidized according to the industry to which they are *initially* assigned.

b. Net incomes and utility levels are equalized across all workers and states, so that $c_1 = z - x = c_2 = \psi_1 = \zeta - x = \psi_2$. The optimal subsidy to the workers in the disadvantaged industry exceeds that to those in the advantaged industry by the amount of migration costs: $s_1 - s_2 = \sigma_2 - \sigma_1 = x$. The ex post allocation of labor is independent of the ex ante allocation of labor, i.e. $n_1 - m$ and $n_2 + m$ uniquely satisfy $f' + x = g'$ and $n_1 + \mu$

[26] This condition relies on $m > 0$. If $x > g'(n_2) - f'(n_1)$, then it will be optimal for there to be no migration, i.e., $m = 0$. In this case, the only meaningful requirement that s_m must satisfy is that it is sufficiently small that migration does not occur. In the absence of migration, the distinction between industry subsidies and worker-specific subsidies vanishes. The characterization of the optimal industry subsidies with zero migration—i.e., full equalization of consumption across workers in the two industries—must therefore apply directly here as well.

[27] This result also holds when migration costs take a non-pecuniary form.

[28] When migration costs take a non-pecuniary form, (14) no longer applies. Instead, the average marginal utility of workers originating in 1 is equated to unity. Migration equilibrium prevents (14) from being achieved for migrants and non-migrants simultaneously.

and $n_2 - \mu$ uniquely satisfy $\phi' = \gamma' + x$, *independently* of the value of n_1 (and $n_2 = n - n_1$). All production uncertainty is borne by entrepreneurs.

To see whether the ex post policy equilibrium is also ex ante optimal, we compute $L'(n_1) = u(c_1) - u(c_2) - (s_1 - s_2)$ and $\Lambda'(n_1) = u(\psi_1) - u(\psi_2) - (\sigma_1 - \sigma_2)$. Therefore, $\pi L' + (1-\pi)\Lambda' = -[\pi(s_1 - s_2) - (1-\pi)(\sigma_2 - \sigma_1)]$. In general, this expression might be of either sign or zero, so that an ex post policy equilibrium will generally differ from an ex ante policy equilibrium. We return to a more detailed analysis of this difference below.

Consider now the ex ante policy problem. Associating multipliers (b, β) with constraints (1) and λ with constraint (2) and eliminating (t, τ) via the government budget constraints (3), one obtains first-order conditions for the instruments (s, σ, m, n_1). Manipulation of these conditions yields

$$\frac{n_1}{n}u'(c_1) + \frac{n_2}{n}u'(c_2) = 1 \tag{16}$$

$$\frac{n_1}{n}u'(\psi_1) + \frac{n_2}{n}u'(\psi_2) = 1 \tag{17}$$

$$\frac{n_1}{n}u'(\psi_1) + \frac{n_2}{n}u'(c_2) = 1 \tag{18}$$

$$s_1 - s_m = \sigma_2 - \sigma_m = 0 \tag{19}$$

$$\pi(s_1 - s_2) - (1-\pi)(\sigma_2 - \sigma_1) = 0. \tag{20}$$

Equations (18)–(20) rely on the assumption that $m > 0$ and $\mu > 0$.

Condition (19) is equivalent to (15) from the ex post problem. It indicates that subsidies to workers should be based entirely on industry of origin and should not be conditioned on the migration decision.[29] This corresponds with the result obtained for the ex post policy case, although the levels of subsidies will differ. Condition (20), which states that the expected subsidy payments to workers are equated across industries, implies that $sgn(s_1 - s_2) = sgn(\sigma_2 - \sigma_1)$. That is, workers with differentially higher subsidies in one state must receive differentially lower subsidies in the other state.

Conditions (16) and (17) govern the extent of redistribution between workers and entrepreneurs in each state. They state that the marginal utilities of workers in each state average to unity (the marginal utility of entrepreneurs). These conditions are implied by, but do not necessarily imply, the ex post optimality condition (14). Condition (18) has a similar interpretation, but it involves the marginal utilities in different industries and in different states. Conditions (16), (17) and (18) together imply that $u'(c_1) = u'(c_2) = u'(\psi_1) = u'(\psi_2) = 1$. To show this, suppose for example that $u'(\psi_2) < 1$. Then (17) implies $u'(\psi_1) > 1$ and (18) implies $u'(c_2) < 1$. In turn, (16) implies $u'(c_1) > 1$. But then $c_1 < c_2$ and $\psi_2 < \psi_2$, which is

[29] The same proviso as in footnote 26 applies here. Condition (19) applies as well when migration costs are non-pecuniary.

inconsistent with the ex ante migration equilibrium condition (2). Thus, $c_1 = c_2 = \psi_1 = \psi_2$ at an optimum with pecuniary migration costs, as in the ex post case.

There are thus certain similarities between the ex ante and ex post optimal policies. More precisely, in an ex post policy equilibrium, the ex ante optimality conditions (16)–(19) are all satisfied, but (20) is generally not. We know that $u'(c_1) = u'(c_2) = u'(\psi_1) = u'(\psi_2) = 1$ in both ex post and ex ante policy equilibria. Thus, the consumption and welfare of workers is identical in each case. Moreover, the ex post allocation of labor satisfies the conditions $f' = g' - x$ and $\phi' - x = \gamma'$ in both equilibria, and hence the ex post allocation of labor, and thus total production and gross rents, are identical in each case. It follows that the non-migrant subsidies s_1, s_2, σ_1 and σ_2 must also be identical in the ex post and ex ante policy equilibria. (For example, $s_1 = c_1 - f'$.) Despite all these similarities, however, an ex post policy equilibrium may not be ex ante optimal. To see how they may differ, it is useful to note the following:

Proposition 5. Let $\underline{n}_1$ satisfy $\phi'(\underline{n}_1) - x = \gamma'(\underline{n}_1)$, and let $\bar{n}_1$ satisfy $f'(\bar{n}_1) = g'(\bar{n}_1) - x$. Then for each value of $n_1 \in [\underline{n}_1, \bar{n}_1]$, there is an ex post policy equilibrium. The vector of optimal subsidy rates $(s_1^*, s_2^*, s_m^*, \sigma_1^*, \sigma_2^*, \sigma_m^*)$ is the same for each of these ex post policy equilibria.

To establish this proposition, start with some arbitrary $n_1 \in [\underline{n}_1, \bar{n}_1]$. Choose (m, μ) such that $f'(n_1 - m) = g'(n_1 + m) - x$ and $\phi'(n_1 + \mu) - x = \gamma'(n_1 - \mu)$. Noting that f', γ' thus determined is independent of n_1, determine (s_1^*, σ_2^*), independently of n_1, as solutions to $u'(f' + s_1^*) = u'(\gamma' + \sigma_2^*) = 1$. Setting $s_m^* = s_1^*$, $\sigma_m^* = \sigma_2^*$, $s_2^* = s_1^* - m$, and $\sigma_1^* = \sigma_2^* - m$, it is straightforward to check that $(s_1^*, s_2^*, s_m^*, \sigma_1^*, \sigma_2^*, \sigma_m^*)$ support an ex post policy equilibrium with the given value of n_1. Thus, there are many ex post policy equilibria, all with identical policies but with differing initial allocations of labor and ex post levels of migration. This establishes Proposition 5.

Consider now the welfare implications of different ex post policy equilibria. Let n_1', n_1'' both lie in the interval $[\underline{n}_1, \bar{n}_1]$, and let $W(n_1)$ denote the value of $\pi[u(c_1) + r + t - s_1 n_1 - s_2 n_2] + (1 - \pi)[u(\psi_1) + \rho + \tau - \sigma_1 n_1 - \sigma_2 n_2]$ at the associated ex post policy equilibria. Since worker consumption, gross rents, and non-migrant subsidies are constant across such equilibria, and since $s_1 - s_2 = \sigma_2 - \sigma_1 = x$, it follows that[30]

$$\begin{aligned} W(n_1'') - W(n_1') &= \pi[s_1(n_1' - n_1'') + s_2(n_2' - n_2'')] \\ &\quad + (1 - \pi)[\sigma_1(n_1' - n_1'') + \sigma_2(n_2' - n_2'')] \\ &= \pi(s_1 - s_2)(n_1' - n_1'') + (1 - \pi)(\sigma_1 - \sigma_2)(n_1' - n_1'') \\ &= x[\pi - (1 - \pi)](n_1' - n_1'') \\ &= x(1 - 2\pi)(n_1'' - n_1'). \end{aligned}$$

[30] Explicit differentiation of the value functions L and Λ from the ex post problems shows that $\pi L' + (1 - \pi)\Lambda' = c(1 - 2\pi)$, the differential version of the result.

The difference in welfare across ex post policy equilibria therefore appears in the form of a difference in expected net rents arising from a difference in expected worker subsidies. This in turn is equal to the difference in expected migration costs. Thus, if $n_1'' > n_1'$, $W(n_1'') < (>) W(n_1')$ as $\pi > (<) \frac{1}{2}$. Since $\pi < \frac{1}{2}$ means that migration is most likely to be in the direction of industry 1, expected migration costs are lowered by moving workers toward industry 1 ex ante.

Different ex ante policy equilibria therefore differ in terms of their initial labor allocations, and have different welfare implications. If $\pi < \frac{1}{2}$, ex ante welfare is increasing in n_1 until $n_1 = \bar{n}_1$, whereas if $\pi > \frac{1}{2}$, welfare decreases in n_1 until $n_1 = \underline{n}_1$. In fact, the ex ante policy equilibrium occurs precisely at one of the extremes of the interval $[\underline{n}_1, \bar{n}_1]$, depending on the value of π. (Of course, if $\pi = \frac{1}{2}$, the value of n_1 is irrelevant for ex ante welfare, and the continua of ex post and ex ante policy equilibria coincide.)

An interesting question is how the optimal ex ante allocation of labor can be achieved. Suppose that $\pi < \frac{1}{2}$, for example, so that it is optimal to set $n_1 = \bar{n}_1$. In this case, it is optimal to have $\mu = 0$ if the bad state should occur. In order to insure that $n_1 = \bar{n}_1$, the government can set $\sigma_m < \sigma_2$. This insures that if $n_1 < \bar{n}_1$, the welfare of workers in industry 2 in the bad state would be less than that of workers in industry 1. Thus, expected utility for workers in industry 2 would be less than that of workers in industry 1 whenever $n_1 < \bar{n}_1$, driving the system to an ex ante equilibrium with $n_1 = \bar{n}_1$.

Summarizing these results, we have shown

Proposition 6. Any ex ante policy equilibrium is also an ex post policy equilibrium. If $\pi > \frac{1}{2}$, ex ante policy equilibrium is characterized by $n_1 = \underline{n}_1$, $m = 0$, and $s_m < s_1$. If $\pi < \frac{1}{2}$, ex ante policy equilibrium is characterized by $n_1 = \bar{n}_1$, $\mu = 0$, and $\sigma_m < \sigma_2$. If $\pi = \frac{1}{2}$, the set of ex ante equilibria coincides with the set of ex post policy equilibria.

5. Interpretation and Extension of the Results

The first part of this section discusses the results of Section 4 from the perspective of the general question of how ex ante and ex post optimal policies differ from each other. In the second part, we compare the results of Cases (i), (ii) and (iv) of Section 4, and establish some instrument equivalence results. The last part provides several extensions of the results.

Ex Ante Versus Ex Post Policies

In general, one would expect that ex ante and ex post policy should differ. When policymakers treat random shocks effectively as one-time-only events, neglecting the impact of their policies on the incentives of agents to choose different actions ex ante, policies would be presumed to differ from the case where these incentive effects are not neglected. Although our analysis generally supports this conclusion, it shows that it is not always correct and can be rather misleading.

For example, in Case (i) (industry subsidies), the optimal policy is not to subsidize or tax workers differentially, regardless of whether an ex ante or ex post policy view is taken. One might have thought that some equalization might be desirable from an ex post perspective, since it ignores the effect of policy on the initial assignment of workers to the riskless and risky industries, and that the ex ante optimal policy would involve less equalization because it would take this effect into account. Our analysis shows that this intuition is definitely incorrect, however. Instead, no attempt should be made to even out the gains and losses experienced by advantaged and disadvantaged workers in either case.

By contrast, in Case (iv) (unrestricted subsidies), all such gains and losses will be completely evened out, whether policymakers take the incentive effects on ex ante labor allocations into account or not. Somewhat unexpectedly, however, "complete equalization of incomes" turns out not to be a uniquely specified policy. There are many ex post policy equilibria which have this property, only one of which (ignoring the case $\pi = \frac{1}{2}$) is ex ante optimal. These ex post policy equilibria are characterized by having too few workers initially assigned to the safe industry if $\pi < \frac{1}{2}$, and too many if $\pi > \frac{1}{2}$. Thus an ex post policy perspective may result in deficient or excessive entry into the risky industry, and does not systematically bias the ex ante allocation of labor toward or away from safe or risky industries, per se.

Finally, although ex ante and ex post policy equilibria may coincide in Cases (i) and (iv), they do not coincide in general, even though the form of the policy rules (i.e., their implicit characterization in terms of necessary conditions) always share certain similarities.

Instrument Equivalence Results and Time Consistency

Cases (i), (ii) and (iv) differ because of the instruments that are assumed to be at the government's disposal. In Case (i), workers are subsidized or taxes according to the industry in which they are employed, i.e., subsidies effectively go to industries rather than workers. In Case (ii), migrants are treated differently from all other workers. In Case (iv), at an optimum, workers are taxed or subsidized only according to their industry of origin. These cases are related as follows:

Proposition 7. Any (ex ante or ex post) optimal policy with unrestricted instruments can be replicated by an appropriate combination of industry and migration subsidies combined with general redistributive transfers between entrepreneurs and workers.

To prove this result, let $(s_1^*, s_2^*, s_m^*, \sigma_1^*, \sigma_2^*, \sigma_m^*)$ be a given optimal policy with unrestricted instruments. Let $(\bar{s}_1, \bar{s}_2, \bar{\sigma}_1, \bar{\sigma}_2)$ denote some policy of industry subsidies, and let $(s_1', s_m', \sigma_1', \sigma_m')$ denote a policy of migration subsidies. Then set

$$\bar{s}_i = s_i^*, \qquad \bar{\sigma}_i = \sigma_i^*, \qquad s_1' = \sigma_i' = 0 \qquad i = 1,2$$

$$s_m' = s_m^* - s_2^*, \qquad \sigma_m' = \sigma_m^* - \sigma_1^*.$$

It is clear, then, that the combined effect of the policies $(\bar{s}_1, \bar{s}_2, \bar{\sigma}_1, \bar{\sigma}_2)$ and $(s_1', s_m', \sigma_1', \sigma_m')$ so defined is that non-migrant workers in industry i receive $\bar{s}_1 = s_1^*$, $\bar{\sigma}_1 = \sigma_1^*$, migrants from industry 1 to 2 (good state) receive $\bar{s}_2 + s_m' = s_m'$, and migrants from 2 to 1 (bad state) receive $\bar{\sigma}_1 + \sigma_m' = \sigma_m^*$. This exactly replicates the optimal policy with unrestricted instruments, proving the proposition.

This is an interesting result because, in some applications, one might not be able to set ex ante worker subsidies directly. What Proposition 7 shows, however, is that any desired result can be achieved by industry and migration subsidies together—and both of these instruments might be available. A surprising corollary of Proposition 7 follows from a comparison of Propositions 2, 4 and 7 (for the ex post case) or of Propositions 3, 6 and 7 (for the ex ante case). Recall that when only industry subsidies are available, the government does not equalize incomes across industries at all. When only migration subsidies are available, the government equalizes incomes across industries, but only partially. It is remarkable, then, that when both instruments are available, the degree of income equalization, instead of being greater than zero but less than in the migration subsidy case, becomes *complete*.

There are other types of instruments that are equivalent to the set of unrestricted instruments considered in Case (iv). For example, suppose the government has the following two types of instruments at its disposal: first, a *state-independent* subsidy I_i paid to workers initially located in industry i, and, second, state contingent subsidies $(S_1, S_m, \Sigma_2, \Sigma_m)$ which are paid in the good and bad states, respectively, to workers initially assigned to industries 1 and 2, the disadvantaged industry in each state. Then we have

Proposition 8. Let $(s_1, s_2, s_m, \sigma_1, \sigma_2, \sigma_m)$ be the subsidy rates in an ex post or ex ante policy equilibrium with unrestricted instruments. Then there exists a vector $(I_1, S_1, S_m, I_2, \Sigma_2, \Sigma_m)$ of ex ante migration subsidies and subsidies to workers in disadvantaged industries that results in an identical equilibrium.

To prove this proposition, simply set

$$\begin{aligned} I_1 + S_1 &= s_1 \qquad & I_2 &= s_2 \\ I_1 + S_m &= s_m \qquad & I_2 + \Sigma_2 &= \sigma_2 \\ I_1 &= \sigma_1 \qquad & I_2 + \Sigma_m &= \sigma_m. \end{aligned}$$

(It is straightforward to verify the feasibility of these policies.)

This result is of some interest because it shows that the number of state-contingent policies need not be very great: it is only necessary to apply appropriate subsidies to the workers in the industry that happens to be harmed. Of course, if one imagines that these policies are actually imposed sequentially, a time-consistency problem can arise. Suppose that an ex ante optimal set of policies $(I_1, S_1, S_m, I_2, \Sigma_2, \Sigma_m)$ is designed, and that the ex ante subsidies (I_1, I_2) are "implemented," i.e., binding commitments to pay these subsidies are made. Once uncertainty is resolved, the question is whether the government in this ex post situation would choose to implement the remaining part of the policy, that is, (S_1, S_m, Σ_2, Σ_m). The answer is yes, if the ex ante policy equilibrium is also an ex post policy equilibrium, but no if these differ. We have shown in Proposition 6 that the two coincide when migration costs are pecuniary, but in general they do not. Thus, the policy (I_1, S_1, S_m, I_2, Σ_2, Σ_m) is not generally time-consistent.

Similar results obtain when the government is allowed to pay ex ante (state independent) subsidies (I_1, I_2) to workers in each industry, to use unrestricted instruments ($\Sigma_1, \Sigma_2, \Sigma_m$) in one state of nature, e.g., the bad state, and to pay a migration subsidy S_m in the other state. Then we have

Proposition 9. Let (s_1, s_2, s_m, σ_1, σ_2, σ_m) be an ex ante or ex post policy equilibrium with unrestricted instruments. Then there exists a set (I_1, I_2, Σ_1, Σ_2, Σ_m, S_m) of ex ante migration subsidies, ex post industry subsidies for one state only (e.g. The bad state), and a migration subsidy for the other state, that results in the same equilibrium.

To establish this result, set

$$\begin{aligned} I_1 &= s_1 & I_1 + \Sigma_1 &= \sigma_1 \\ I_2 &= s_2 & I_1 + \Sigma_2 &= \sigma_2 \\ S_m &= s_m & I_1 + \Sigma_m &= \sigma_m. \end{aligned}$$

To see why this result might be of interest, suppose that one state—e.g. the bad state—is very unlikely to occur. Then, provided that the good state occurs, only one state-contingent instrument, the migration subsidy, would be required. The probability of having to use a full set of instruments ex post is just the probability that the bad state would occur. Probabilistically speaking, this result shows that the government can economize on the number of instruments that it uses.

The same time-inconsistency problem arises here as in the preceding case: once uncertainty is resolved the government will not pursue the ex ante optimal policy unless it coincides with the optimal ex post policy. Indeed, there is a general problem that arises even with the instruments considered previously in Cases (i)–(iv). To achieve ex ante policy equilibria requires that government policy be known to households when the initial allocation of workers across industries is made. If this policy is subject to revision ex post, then ex ante policy equilibria will not in general be attainable.

The general problem of time inconsistency is familiar (Kydland and Prescott [1977]) in models with a temporal structure such as ours. Hillier and Malcomson (1984) show, in the context of the Kydland-Prescott model, that time inconsistency will occur only if announced (ex ante) policies are not first-best optimal. In our model, unrestricted worker subsidies are first-best optimal when migration costs are pecuniary, so ex ante optimal policies are time-consistent. However, if instruments are restricted, time-inconsistency may arise, although it need not. An example of this is in the industry-subsidy case which, although not first-best optimal, is nevertheless a case in which optimal policy is time-consistent. Staiger and Tabellini (1987) examine the issue of time-consistency in a model that is not unlike ours in that it incorporates costly migration by workers following a trade shock. However, Staiger and Tabellini take the ex ante allocation of labor as exogenously fixed, and thus restrict attention to what we term the ex post policy problem. In contrast to our model, they allow for the possibility that the government may be able to change its policy after migration has occurred. They find that free trade may be the optimal policy when it is possible for the government to precommit itself not to change this policy after migration has occurred, but that this is not a time-consistent strategy. This problem does not arise in our model because we assume that migration can always occur *after* the announcement of policy.

Some Extensions

It is useful to note that some of the simplifying assumptions underlying our analysis can easily be dispensed with. This expands the range of potential applications of the model.

First, having written the production functions for industry 1 as $f(\cdot)$ and $\phi(\cdot)$ for each of two states of the world, and similarly $g(\cdot)$ and $\gamma(\cdot)$ for industry 2, simple reinterpretation of the existing model allows us to apply the analysis to the case of uncertainty, not necessarily multiplicative, in both industries simultaneously. All that is essential is that migration should go from industry 1 to industry 2 in one state, and the reverse in the other. Thus, our model can accommodate a very wide range of technological uncertainty. The simple good state–bad state dichotomy used so far has been useful because it allows one to make statements about a "safe" and "risky" industry, but the results are robust to relaxation of this dichotomization.

Second, the essential results extend directly to the multi-state case. This is obvious for all of the ex post results, since they refer essentially to only one state of the world. Only a slight amount of additional analysis, involving no new theoretical principles, confirms this assertion for the ex ante results as well.

Third, we can relax the assumption that migration costs are identical across states of the world. The basic analysis does not change at all if we assume that migration costs take on some state-contingent values x_j in state of the world j. For example, we might have $x_j = 0$ for some state j. As noted previously, this case is of interest because zero migration costs in state j means that the realized utility of all workers will be identical in this state in the absence of any public policy. Essentially, zero migration costs results in full equalization of income across workers. One can easily show, then, that no redistributive transfers are called for in this state of the world. More generally, if migration costs are very small, the optimal level of taxes and subsidies will also be very small. The analysis thus implies that the level of redistribution among workers or industries is inversely linked to the level of inter-industry mobility.

6. Conclusions: Applications of the Analysis

Throughout our discussion, we have focused on the case where uncertainty consists of an adverse or favorable shock to a particular industry. This industry has been described as a "declining industry" when the shock is unfavorable, whereas when the shock is favorable it is the other industry that declines. We now want to remark on a variety of possible applications of our results, and on the institutional interpretation of the policy instruments that we have analyzed.

First, as noted in the preceding subsection, our assumption of uncertainty in only one industry is inessential. Moreover, our identification of productive activities 1 and 2 as "industries" is also inessential. In the broadest terms, our model describes how agents must choose between two risky activities, where the initial choice is costless (or equally costly) and where ex post moves from one activity to the other are possible but costly. The decisions by workers as to the industry in which they will be employed is clearly one example of such a choice. Another example might be locational choice. This could be illustrated by imagining a group of young people just completing school and trying to decide in which region of a country to find jobs. If their initial moving costs are relatively small when young, unsettled and unburdened by established family ties, prior home ownership, and the like, these costs might be treated as negligible and hence we have an approximation to ex ante costless migration. A third example is the choice of training or investment in human capital. Thus, activity 1 might refer to unskilled labor (or training in the humanities, or training as a chemist) and activity 2 might refer to skilled labor (or training in the sciences, or training as a physicist, respectively), and migration costs would then refer to retraining costs. Of course, our model does not perfectly describe any of these situations, but it captures an important common element of each. In each of these applications, migration costs have a different specific interpretation. They might refer to search costs, actual pecuniary costs of moving from one location to another, the disruption of personal ties that accompany a change of occupation or location, retraining costs, etc.

Our analysis has shown that optimal policy is highly dependent on the instruments that are assumed to be available for the government to use. The distinction between "industry subsidies" and "worker subsidies," for instance, is crucial: it is optimal to have *no* differential subsidies, and thus no equalization of income among workers, when industry subsidies are the only available instrument, whereas *full* equalization is desired when worker subsidies are possible. Furthermore, the analysis of Section 3, and the instrument equivalence result of Proposition 7, shows that migration subsidies are a very important weapon in the arsenal of public policy. Although optimal industry subsidies are zero when migration subsidies are infeasible, the two in combination can be used to achieve any desired set of effective worker subsidies—even if worker subsidies per se are not institutionally feasible. And, as we have just seen, this changes the optimal policy dramatically, to one of full equalization of incomes.

In terms of particular applications, this might mean, for example, that there is a very strong case for the retraining of agricultural workers, or subsidies to facilitate their movement to urban areas, if agricultural subsidies are to be justified on the basis of adverse shocks. Conversely, the case for agricultural subsidies could be very much weakened if they are not accompanied by policies that encourage exit from the industry. It is interesting to recall, in this context, some of the proposals in Lawrence and Litan (1986). In particular, they advocate (pp. 112–113) offering more generous assistance to workers from industries adversely affected by international trade if those workers obtain employment at reduced wages in other industries. The wage reduction that workers experience could represent lost human capital associated with occupational change, which, in the context of our model, would be another form of migration cost. Our analysis suggests that such assistance for relocated workers can be a very important component in the overall policy toward workers and industries that may be harmed by trade fluctuations.

Finally, it is interesting to observe that wage income taxation can be used to deal with inequalities arising from random shocks to industries, regions, or occupations. A wage income tax with no deduction for migration costs would tax workers on the basis of their gross income, i.e., their wage income in the industry in which they are employed ex post. Such a tax would reduce incomes in the favored industry relative to the unfavored industry, and would thus be equivalent to a form of industry subsidies. If this is the only available policy instrument, then the optimal tax in our model would be a poll tax, i.e., a tax with a zero marginal rate, since this would correspond to zero industry subsidies. On the other hand, suppose that migration cost deductions are used so that workers are taxed only on their income net of migration costs. In this case, ex post migration equilibrium ensures that all workers originating in the disadvantaged industry have identical taxable incomes, which are lower than those of the workers in the fortunate industry. A wage income tax in this world is equivalent to a system of worker subsidies, and the optimal income tax would be a fully progressive tax, with 100% marginal rates that would equalize incomes across industries. Thus, the deductibility of migration costs is a crucial matter in determining the optimal rate structure for the income tax.[31]

[31] With non-pecuniary migration costs, a wage income tax cannot achieve ex ante or ex post optimal outcomes because it is not possible to differentiate across worker categories in the appropriate way.

In concluding, let us compare our results with those of Varian [1980], who examines the role of redistributive policies as a form of insurance against adverse random events of the general type we have analyzed here. In Varian's model, individuals are uncertain as to their eventual earning power, and they choose to set government policies to protect them if they do badly, even at the expense of being made worse off by those policies if they do well. An important difference between Varian's model and ours is that in the former, risks are person-specific whereas risks in our model attach to industries, and individual agents can mitigate these risks to some degree by moving to other industries. This in turn spreads the risk to other industries through the response of equilibrium factor prices. In the extreme case where migration costs are so large as to preclude any migration in response to unfavorable shocks, our model becomes essentially equivalent to Varian's. In this sense, our analysis can be seen as a generalization of Varian's to allow for the realistic possibility that people are not irrevocably tied to a particular occupation, industry, or region. Our analysis shows that when one relaxes this strong assumption, optimal government policy can be quite different. Indeed, if migration costs are zero, as noted above, there is no scope for any government policy. Moreover, as we have been discussing, the scope for policy will depend crucially on what instruments are open to the government and how they are coordinated. This involves particularly the availability and optimal use of instruments that do not inhibit exit from the unfavorably-affected industry. Such possibilities simply cannot arise within the context of models with innate and unalterable personal risks, and the results of our analysis show that they carry significant implications for policy.

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APPENDIX

Proof of Proposition 3

First let us write down the first-order conditions for the migration subsidy problem. In each case, the variable being optimized is noted at the left. The arguments of the production functions can be suppressed without confusion.

$$s_1\colon\ \pi[nu'(c_1) - (n-m)] + bu'(c_1) + \lambda\pi[u'(c_1) - u'(c_2)] = 0 \tag{A.1.1}$$

$$s_m\colon\ -\pi m - bu'(z) = 0 \tag{A.1.2}$$

$$\sigma_1\colon\ (1-\pi)[nu'(\psi_1) - (n-\mu)] + \beta u'(\psi_2) + \lambda(1-\pi)[u'(\psi_1) - u'(\psi_2)] = 0 \tag{A.1.3}$$

$$\sigma_m\colon\ -(1-\pi)\mu - \beta u'(\zeta) = 0 \tag{A.1.4}$$

$$m:\ -\pi[nu'(c_1)f''-(n_1-m)f''+(n_2+m)g'']+\pi(s_1-s_m)$$
$$-b[u'(c_1)f''+u'(z)g'']-\lambda\pi[u'(c_1)f''+u'(c_2)g'']=0 \qquad (A.1.5)$$

$$\mu:\ (1-\pi)[nu'(\psi_1)\phi''-(n_1+\mu)\phi''+(n_2-\mu)\gamma'']+(1-\pi)(\sigma_1-\sigma_m)$$
$$-\beta[u'(\psi_2)\gamma''+u'(\zeta)\phi'']+\lambda(1-\pi)[u'(\psi_1)\phi''+u'(\psi_2)\gamma'']=0 \qquad (A.1.6)$$

$$n_1:\ \pi[nu'(c_1)f''-(n_1-m)f''+(n_2+m)g'']$$
$$+(1-\pi)[nu'(\psi_1)\phi''-(n_1+\mu)\phi''-(n_2-\mu)\gamma'']$$
$$+b[u'(c_1)f''+u'(z)g'']-\beta[u'(\psi_2)\gamma''+u'(\zeta)\gamma'']$$
$$+\lambda\pi[u'(c_1)f''+u'(c_2)g'']+\lambda(1-\pi)[u'(\psi_1)\phi''+u'(\psi_2)\gamma'']=0 \qquad (A.1.7)$$

Substitution of (A.1.5) and (A.1.6) into (A.1.7) yields:

$$\pi(s_1-s_m)-(1-\pi)(\sigma_1-\sigma_m)=0, \qquad (A.2)$$

a result discussed in the text and used further below.

The ex post migration equilibrium conditions (1) imply that $c_1=z$ and $\psi_2=\zeta$, so that $u'(z)=u'(c_1)$ and $u'(\zeta)=u'(\psi_2)$. Making use of this fact and using (A.1.2) and (A.1.4) to eliminate $(b,\ \beta)$, the remaining conditions become

$$n[u'(c_1)-1]=-\lambda[u'(c_1)-u'(c_2)] \qquad (A.3.1)$$

$$n[u'(\psi_1)-1]=-\lambda[u'(\psi_1)-u'(\psi_2)] \qquad (A.3.2)$$

$$s_1-s_m=(n[u'(c_1)-1]+\lambda u'(c_1)+n_2)f''+[n_2+\lambda u'(c_2)]g'' \qquad (A.3.3)$$

$$\sigma_m-\sigma_1=(n[u'(\psi_1)-1]+\lambda u'(\psi_1)+n_2)\phi''+[n_2+\lambda u'(\psi_2)]\gamma'' \qquad (A.3.4)$$

Although the expected utility condition (2) has been formulated as an equality constraint, it could just as well be expressed in inequality form for the purposes of the present problem. The essential role of this constraint is to insure that the ex ante expected utility of the workers whose expected utility is maximized in the government's optimization problems is no higher than that of the other workers. Thus, we can assume $\lambda\leq 0$.

Now suppose $\lambda=0$. Then (A.3.1) implies $u'(c_1)=1$. Using this and $\lambda=0$ in (A.3.3) implies $s_1<s_m$. Similarly, (A.3.2) and (A.3.4) imply that $\sigma_m<\sigma_1$ when $\lambda=0$. However, $s_1<s_m$ and $\sigma_m<\sigma_1$ contradict (A.2). This contradiction and $\lambda\leq 0$ imply $\lambda<0$.

Let us now assume that $x>0$, i.e., migration costs are strictly positive. Later, we consider what happens when $x=0$.

The first task is to show that $s_1\neq s_m$ and $\sigma_1\neq\sigma_m$. To see this, note to begin with that substitution of (A.3.1) into (A.3.3) and of (A.3.2) into (A.3.4) yield

$$s_1-s_m=(n[u'(c_1)-1]+\lambda u'(c_1)+n_2)(f''+g'') \qquad (A.4.1)$$

$$\sigma_m-\sigma_1=(n[u'(\psi_1)-1]+\lambda u'(\psi_1)+n_2)(\phi''+\gamma''). \qquad (A.4.2)$$

Now suppose that $s_1=s_m$. By (A.2), this implies $\sigma_1=\sigma_m$. Hence (A.4) implies that $n[u'(c_1)-1]+\lambda u'(c_1)+n_2=n[u'(\psi_1)-1]+\lambda u'(\psi_1)+n_2$, i.e.,

$$(n+\lambda)u'(c_1)=(n+\lambda)u'(\psi_1). \qquad (A.4.5)$$

There are now two cases.

Case (i): If $n+\lambda\neq 0$, (A.5) implies $c_1=\psi_1$. But $s_1=s_m$ implies $c_1=f'+s_1=g'+s_m-x=g'+s_1-x<g'+s_1=c_2$, and similarly $\sigma_1=\sigma_m$ implies $\psi_1>\psi_2$. Hence, $u'(c_2)<u'(c_1)$ and $u'(\psi_2)>u'(\psi_1)$. However, $c_1=\psi_1$ implies, by (A.3.1) and (A.3.2), that $sgn[u'(c_1)-u'(c_2)]=sgn[u'(\psi_1)-u'(\psi_2)]$, a contradiction.

Case (ii): If $n+\lambda=0$, then (A.3.1) implies that $u'(c_2)=1$. (A.3.3) then reduces to $s_1 - s_m = -n_1(f''+g'')>0$, another contradiction.

Now we show that $s_m < s_1$ is impossible. Suppose the contrary. Then, by (A.2), $\sigma_m < \sigma_1$ also. By (A.4), it follows that $(n+\lambda)u'(c_1) < n_1 < (n+\lambda)u'(\psi_1)$. Hence $n+\lambda>0$ and $u'(c_1) < u'(\psi_1)$. However, $c_2 = g' + s_1 = c_1 + s_1 - s_m + x$ by the ex post migration equilibrium condition, and $s_1 > s_m$ thus implies $c_2 > c_1$. By (A.3.1), this implies $u'(c_1)>1$. Similarly, $\sigma_1 > \sigma_m$ implies $\psi_1 > \psi_2$ and, by (A.3.2), $u'(\psi_1)<1$. Thus $u'(\psi_1) < u'(c_1)$, a contradiction.

So far, then, we have shown $s_1 < s_m$ and $\sigma_1 < \sigma_m$, i.e. there are positive subsidies to migrants. Next we show that these are not large enough to equalize utility across industries.

First, suppose $c_1 = c_2$ and $\psi_1 = \psi_2$. Then, by (A.3.1) and (A.3.2), $u'(c_1) = u'(\psi_1) = 1$. Then (A.4.1) implies $sgn(s_1 - s_m) = -sgn(\sigma_1 - \sigma_m)$, which contradicts (A.2). Next, suppose $c_1 > c_2$. Then $m=0$. But $m=0$ implies $f' < g'$, so that $c_1 = f' + s_1 < g' + s_1 = c_2$, a contradiction. Similarly, $\psi_2 > \psi_1$ is impossible. Thus, $c_1 \leqslant c_2$, $\psi_1 \geqslant \psi_2$, with at least one strict equality. Subsidies to migrants are therefore not fully equalizing in at least one state.

Finally, consider the situation where migration costs are zero. Free migration implies $c_1 = c_2$ and $\psi_1 = \psi_2$, and, by (A.3.1) and (A.3.2), this implies $u'(c_1) = u'(\psi_1) = 1$. Substituting into (A.4), it must be the case that $sgn(s_1 - s_m) = sgn(\sigma_m - \sigma_1)$. Together with (A.2), this implies that $s_1 = s_m$ and $\sigma_1 = \sigma_m$. This completes the proof of Proposition 3.

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