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A MEDIAN VOTER MODEL OF SOCIAL SECURITY*

BY ROBIN W. BOADWAY AND DAVID E. WILDASIN[1]

This paper presents a theoretical median voter analysis of the determination of the level of social security. The framework for the analysis is a continuous-time overlapping-generations model with non-altruistic households facing borrowing constraints in the capital market. A majority voting equilibrium is shown to exist, in which the median voter is liquidity constrained. The desired level of social security for each voter is a declining function of the pre-existing level of social security. As a consequence, in a sequence of votes on social security beginning with a zero level, the program initially overshoots its steady state value.

1. INTRODUCTION

Many government policies involve transfers of purchasing power among generations. The most widely recognized of these are increases in unfunded social security (public pension) plans and debt-financed tax reductions both of which involve an ongoing transfer from younger to older generations as long as they are in force. However, the phenomenon is not restricted to these particular types of policies. Other examples abound, as has been made clear in the recent survey by Kotlikoff (1984). For example, tax policy changes which substitute a tax collected early in the life cycle of taxpayers for one collected later have the same intergenerational redistributive impact as social security (though they may well impose different sorts of efficiency losses). A change from a consumption to a wage tax, such as that analyzed in Summers (1981), would be of this sort.[2] Similarly, many public expenditure schemes have intergenerational impacts, including the provision of health and welfare services.

It is well-known that the long-run consequences of such programs may well be to reduce the average lifetime utility levels of future generations (see Diamond 1965). Yet, they are typically associated with transitional gains to some gener-

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[2] Summers compares three tax regimes—a consumption tax, a wage tax and income tax. In his model with fixed labor supply and government budget balance, a switch from a consumption tax to a wage tax is equivalent to a lump-sum tax-transfer across generations. This effect is analyzed explicitly by Auerbach, Kotlikoff and Skinner (1983), who, unlike Summers, analyze the transition between steady states when such tax changes are made.

ations alive when the policy is instituted (with corresponding losses to other generations if the policy is removed). Given this potential for current generations to redistribute in their favor at the expense of future generations, a natural question to ask is to what extent would they have an incentive to exploit it. The purpose of this paper is to analyze formally, in the context of a fully specified model and using simple majority voting as a decision procedure, the collective choice of net intergenerational transfers of a particular sort. We will refer to these transfers as social security, but any policy which involves the same flow of transfers across generations in the accounting sense will be understood to be included in our notion of social security. The key feature of social security for our purposes is that it involves a continuous transfer from workers to retirees such that changes in the level of transfers entails windfall gains or losses for retirees. In addition to public pensions, our notion of social security could include tax-financed expenditures on health or other services to the retired, or a substitution of wage taxation for consumption taxation.

Much of the literature on social security studies the positive and normative effects of such intergenerational transfers.[3] By contrast, we are concerned here with the determinants of the existence and level of the program. The literature here is much more limited. An early paper by Aaron (1966) shows that the median-aged voter might have an incentive to perpetuate an unfunded social security system even if everyone's lifetime utility is reduced by it.[4] Browning (1975) presents the first detailed analysis of the median voter's choice of social security. He considers an overlapping-generations model and shows that there is "too much" social security in a steady-state equilibrium. Social security contributions are viewed as a form of forced saving. The level of social security (and thus saving) chosen by the median voter exceeds the amount that maximizes lifetime utility (which is the amount that would be preferred by the youngest voter). However, this analysis assumes that there are no capital markets so that social security is the only form of savings. Also, voters expect the system they vote for to remain unchanged for the rest of their lives. Finally, there is no attempt to analyze the dynamic process by which a long-run equilibrium is reached.

A recent paper by Hu (1982) remedies some of these limitations. His model consists of overlapping generations of households who live for three periods, earning given amounts of wages in the first two and saving for consumption in the retirement period. When a vote is taken, voters of the younger two ages are assumed to face uncertainty about the level of social security in the future. The uncertainty is meant to capture uncertainty over future demographic conditions, and uncertainty over the behavior of future voters. Given certain assumptions about the probability distribution of future outcomes, he shows that social secur-

[3] See the recent surveys by Kotlikoff (1984) and Atkinson (1987). Some seminal contributions include Feldstein's (1974) empirical investigation of the effects of social security on aggregate capital accumulation; Summer's (1981) simulation study of the long-run effects of tax reforms; Diamond's (1965) analysis of the effect of national debt on economic welfare; and Samuelson's (1958) characterization of inefficiency in infinite-horizon growth models with overlapping generations.

[4] See also Townley (1981) who derives the circumstances in which this will be the case.

ity levels will be lower in the presence of uncertainty, and savings higher. Uncertainty plays a crucial role in this model by generating an internal solution to the level of social security preferred by the median voter. In the absence of such uncertainty, the median voter would either prefer zero or an indefinitely large amount of social security depending on whether the implicit rate of return on social security over his remaining life was negative or positive.

Hu's analysis has the merit of incorporating social security decision-making into a model in which capital markets and saving play an explicit role. However, its formulation of uncertainty about future voting is rather ad hoc since it is simply postulated that the future level of social security is a random variable whose expected value increases with the level of social security currently chosen. This formulation links present decisions to future outcomes and is critical in generating the results.[5] Other assumptions which play a key role in the analysis include the specification of a three-period lifetime, the supposition that votes are taken each period (and thus three times per lifetime) and the implicit assumption that the previously existing level of social security has no impact on levels desired by current voters. Finally, the qualitative nature of Hu's results do not allow him to analyze the path that social security levels follow over time.

Part of the purpose of our paper is to analyze the dynamic evolution of the social security system over time in a stylized economy. We use a model of overlapping generations in which a large number of generations are alive at any one time, and in which internal majority voting equilibria are obtained by imposing realistic capital market constraints on voters. In a sense, our model is a compromise between Browning's assumption of no capital markets and the assumption that capital markets are perfect. A full specification of voter saving behavior under these constraints allows us to show how preferred social security levels depend upon previously existing ones.

It quickly becomes apparent when undertaking median voter analyses of social security levels in a fully specified model that the problem is inherently complicated, both conceptually and analytically, by its dynamic nature and by the manner in which different generations of households interact with one another. The median voter's choice of social security will depend both upon past social security levels, which determine the circumstances under which past savings decisions have been made, and upon the future evolution of the system, which may depend upon how future median voters behave. A median voter's decision should take account of the fact that his choice of social security will have an influence on the level chosen by subsequent median voters during his lifetime. Since these next median voters will affect subsequent ones and so on into the future, the forward-looking median voter would have to solve the future evolution of the system taking into account the mutual consistency of his decision with those of the sequence of future median voters. Such a dynamic system is immensely difficult to analyze.

[5] A recent analysis by Verbon (1987) has developed Hu's notion of uncertain outcomes further by suggesting that voters make predictions of future social security levels by extrapolating past trends according to a sort of adaptive expectations process. Current decisions then effect future decisions through this process.

Different authors have made different assumptions to resolve this problem. Browning (1975) assumes that current voters behave as if the scheme they choose will be in effect for the remainder of their lives. This would be the case if votes were taken infrequently or if one were only considering long-run equilibria. Hu (1982) and Verbon (1987) assume that the outcomes of future median votes are uncertain, but positively linked to the current outcome. Sjoblom (1985) treats the choice of social security levels of successive generations as a dynamic game in which failure by a generation to maintain the existing level of social security is punished by the next generation.

In common with previous authors, we adopt a set of institutional and behavioral assumptions which allow us to abstract from some of these complexities. Depending on the context, our analysis can be interpreted either as applying for steady states as in Browning (1975) or for economies in which votes are taken infrequently over social security levels. In either case, the model is not fully descriptive of the dynamic evolution of a general economy with frequent voting.[6] Nonetheless, our results are suggestive of the sorts of forces at work, which one would expect to reappear, though possibly with added complexitites, in more detailed analyses.

The paper is organized as follows. Section 2 outlines the institutional and behavioral assumptions of the model. This is followed by an analysis of the life-cycle savings-consumption pattern of a typical household of arbitrary age under a social security system of given size. Section 4 investigates the consequences of changing the size of the social security program on household welfare and, from this, deduces the preferences for social security levels. Two key features of this analysis are the demonstration that preferences for social security levels are single-peaked and the finding of an inverse relationship between the most-preferred social security level and the previously-existing level. Section 5 analyses how the preferred level of social security varies with age, allowing us to identify the median voter and to show certain properties of the median voter equilibrium. Section 6 traces out the dynamic evolution of median voter equilibria in an economy in which votes are taken infrequently. It is shown that starting from zero the chosen level of social security typically overshoots the steady state level and then falls. In certain circumstances, it gradually converges to the steady state level; in others it does not. A concluding section reiterates some key implications of our analysis and considers some directions for future research.

2. THE INSTITUTIONAL AND BEHAVIORAL ASSUMPTIONS OF THE MODEL

The model we use abstracts from all aspects of policy except for intergenerational transfers. In particular, all households are assumed identical to avoid

[6] Among other things, our assumptions permit us to ignore the anticipated effect of current votes on future votes. Other sorts of "conjectural variations" of the effect of current choices on subsequent choices are clearly possible, as discussed in Section 7 of the earlier version of this paper (Boadway and Wildasin, 1987) and in a companion paper, Boadway and Wildasin (forthcoming). The analysis in those papers indicates the importance of the frequency with which votes are taken as well as the perceptions held by current voters about the behavior of future voters.

intragenerational redistribution effects. The model we use is essentially the continuous-time overlapping-generations model with perfect certainty originally analyzed by Cass and Yaari (1967). Each household begins economic life at age 0, works until age R and is retired until life's end T. Labor supply is fixed during the working period and a given stream of labor income is received, denoted w_s ($0 \leq s \leq R$). During retirement, no labor income is earned; consumption is financed from earlier savings or from social security benefits. Households leave no bequests, although that does not seem to affect the qualitative nature of the analysis unless bequests are operative in the sense of Barro (1974). Households are born (and die) continuously at a rate which increases exponentially at the rate n. This fixed exponential growth rate ensures a constant age-distribution of the population. Of particular importance for us is the person of median age, denoted m. Since the age-distribution of the population is fixed, so will be the median age. The production side of the model is kept very simple. It is assumed that producer prices are exogenously fixed. Thus, the interest rate τ is fixed and so is the wage rate in terms of the single consumption good, though wage payments w_s can vary over the life cycle due to variable amounts of effective labor supplied. This assumption of fixed interest and wage rates can be interpreted as assuming either a linear production technology or a small open economy.[7]

Intergenerational transfer policy is modelled as a tax-transfer scheme according to which all working persons pay a constant stream of taxes of τ per unit of time and all retirees receive a constant stream of benefits per unit of time β. The government budget is assumed to be balanced instantaneously, so

$$\tau \int_0^R e^{-ns}\, ds = \beta \int_R^T e^{-ns}\, ds \quad \text{or} \quad \tau = \eta\beta \tag{1}$$

where $\eta = \eta(R, T, n) = (e^{-nR} - e^{-nT})/(1 - e^{nR})$. Thus, for given demographic variables (R, T, n), the tax rate τ is a constant proportion of β and we can treat the level of β as being the only policy choice variable. The choice of β is assumed to be made by simple majority voting. Provided preferences are single-peaked, the social choice of β will be the most-preferred level of the median voter. The level of β (and τ) chosen by a median voter remains in force until it is changed by some subsequent median vote. We let β_∞ denote the value of β in a steady state.

The median voter is assumed to choose the value of β which maximizes his remaining lifetime utility. It is assumed that the median voter believes that, once chosen, the value of β will remain unchanged for the remainder of his lifetime. This is a strong assumption which will be satisfied under one of the two circumstances discussed above—infrequent voting or steady state. Note that in a steady state, not only will future values of β be unchanged at β_∞ but also the value of β which existed for the prior portion of the median voter's life will be β_∞ as well.

[7] Variability of factor prices would complicate the dynamic analysis of policy changes enormously. The median voter would have to predict the future path of factor prices resulting from the change in capital stock due to policy changes. At the same time, as discussed below, the assumption of fixed factor prices eliminates one avenue by which the welfare of the median voter would eventually stop rising as social security levels increase.

Under the infrequent voting assumption, when $\beta \neq \beta_\infty$ at the time of voting, the median voter may choose to change β. Which of these two assumptions is being made will be specified in the analysis.

For a voter of a given age, the net present value of a dollar's worth of social security benefits can be calculated. Following Feldstein (1974), we refer to this as the net social security wealth per dollar of social security benefits. If capital markets are perfect, the utility of the median voter will be monotonically increasing in net social security wealth, and hence monotonic in β. If net social security wealth is negative, the median voter chooses $\beta = 0$. If it is positive, he chooses the maximum possible level of social security. This level, denoted $\beta_{\max}$, is the level which would tax away all the wealth of the working generations.[8] These extreme outcomes were those obtained by Hu (1982) for the case of infrequent voting.

There are various ways to amend the model so as to avoid such extreme outcomes. One might make factor prices, pre-retirement labor supply or retirement age endogenous. Or, one could introduce uncertainty about the future as in Hu (1982). We assume instead that capital markets are imperfect in one or both of two ways. First, households may not be able to borrow against future after-tax wage income. The empirical validity and some of the long-run consequences of this assumption have been discussed in detail by Hubbard and Judd (1986). Second, households may be unable to borrow against future social security benefits. This restriction might be viewed as appropriate in view of the absence of explicit contractual guarantees on future social security levels. In addition, the fact that many benefits in retirement may be in-kind transfers or may be thought of as annuities whose asset value becomes zero on death makes the inability to borrow against them reasonable.[9] As analyzed below, these constraints imply that a positive wealth effect due to social security increases is eventually overcome by a loss of welfare due to the inability to borrow.

3. THE LIFE-CYCLE BEHAVIOR OF LIQUIDITY-CONSTRAINED HOUSEHOLDS

Consider a person of age t who has accumulated a certain amount of wealth according to his utility-maximizing asset profile under the social security scheme previously in existence, denoted β^o. Let us first characterize the allocation problem for the remainder of his life under some new scheme β which begins at t and continues for the remainder of his life cycle $(T - t)$. Adopting an additively-separable lifetime utility function with strictly concave instantaneous utility u, the consumer problem is:

$$\max \int_t^T e^{-\delta(s-t)} u(c_s)\, ds \tag{2}$$

subject to

$$\dot{A}_s = w_s + \beta_s - \tau_s + rA_s - c_s \qquad t \leq s \leq T$$

$$A_s \geq 0 \qquad \forall\, s \in S$$

[8] If net social security wealth were just zero, the median voter's choice of β would be indeterminate.

[9] Browning's (1975) assumption that social security taxes are the only form of saving for retirement can be viewed as being an extreme case of this assumption.

with the initial condition $A_t = \bar{A}_t(\beta^o)$.[10] Here, c_s is consumption, A_s is assets, w_s is the exogenous wage ($=0$ for $s > R$), β_s is social security benefits and τ_s is taxes. Note that $\beta_s = 0$ for $t \le s \le R$ and β for $s > R$, and $\tau_s = \tau$ for $t \le s \le r$ and 0 for $s > R$. Recall from (1) that $\tau = \eta\beta$. The non-negativity constraint on A_s will be used to characterize various liquidity constraints facing households. Depending on the interval S, the constraint will correspond to an inability to borrow against future social security benefits, future wages, or both. As discussed below, each of these cases will involve imposing certain regularity conditions on the paths of wages, consumption, and social security benefits, for simplicity.

The constant value Hamiltonian function for this problem may be written:

$$H = e^{-\delta(s-t)}u(c_s) + \lambda_s(w_s + \beta_s - \tau_s + rA_s - c_s) + \mu_s A_s.$$

The first-order conditions are:

$$\frac{\partial H}{\partial c_s} = e^{-\delta(s-t)}u'(c_s) - \lambda_s = 0 \tag{3}$$

$$\frac{-\partial H}{\partial A_s} = -\lambda_s r - \mu_s = \dot{\lambda}_s \tag{4}$$

$$\mu_s A_s = 0. \tag{5}$$

At some finite number of points, λ_s may jump discontinuously so (4) will not apply. The points at which this occurs will be discussed below. Taking the time derivative of (3), substituting into (4) and simplifying we obtain:

$$\varepsilon \frac{\dot{c}_s}{c_s} = r - \delta + \frac{\mu_s}{\lambda_s} \tag{6}$$

where $\varepsilon = -u''c_s/u'$ (the elasticity of the marginal utility of consumption), which we take to be constant for simplicity. This implies the utility function takes the form $(1-\varepsilon)^{-1}c_s^{(1-\varepsilon)}$, $\varepsilon > 0$. Note also that $\mu_s = 0$ if $A_s > 0$; conversely, if $\mu_s > 0$ then $A_s = 0$. In general, the path of c_s will depend upon the exogenous parameters, including β. For future use, note that c_s will be twice differentiable in β except at those points where λ_s jumps discontinuously.

From (6), the path of consumption depends on whether and when a liquidity constraint is binding. We can distinguish four cases as follows.

Case 1. Neither liquidity constraint binding. This is the conventional unconstrained life-cycle model in which, from (6), c_s changes exponentially at the rate $(r-\delta)/\varepsilon$. For expositional purposes, we shall assume in what follows that $r > \delta$. A typical individual's consumption, income and asset paths might be as depicted in Figure 1.

Case 2. Benefit constraint binding. We refer to the inability to borrow against future social security benefits as the benefit constraint. This constraint will bind if

[10] The dependence of $\bar{A}_t$ on the existing level of social security β^o is of critical importance for the subsequent analysis. As will become apparent, $\partial\bar{A}_t/\partial\beta^o \le 0$, with the inequality applying when $\bar{A}_t > 0$.

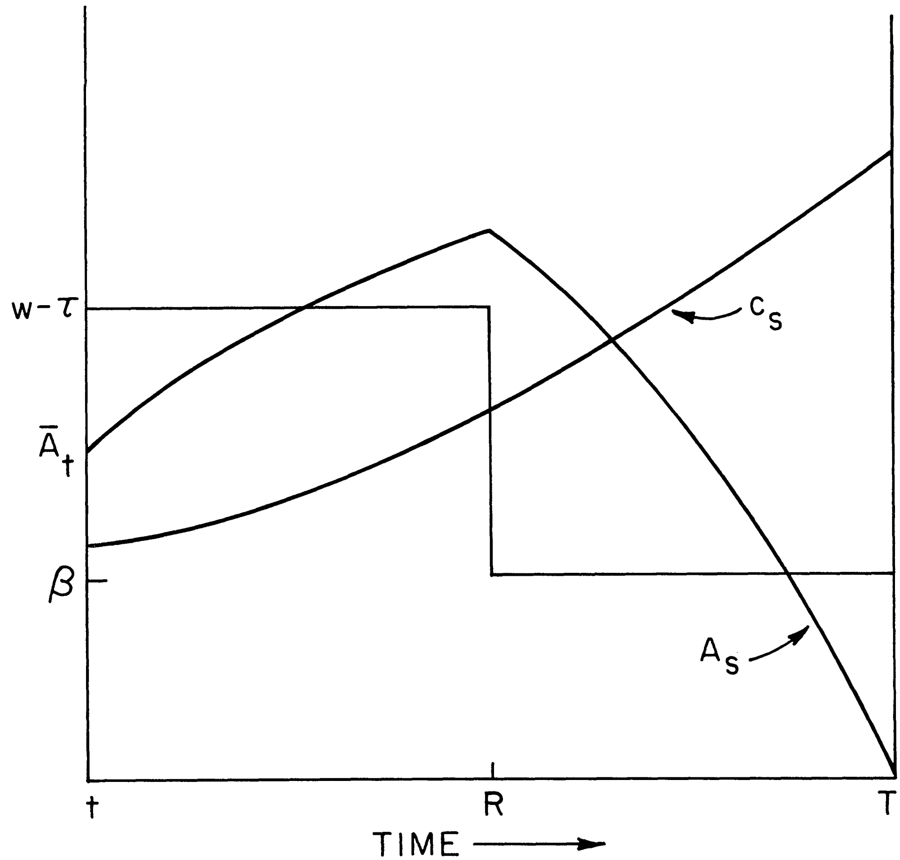

FIGURE 1
LIQUIDITY CONSTRAINTS NOT BINDING

the household would like to consume in his working years an amount which is, in present value terms, greater than $\bar{A}_t$ plus the present value of his net wages. To simplify the analysis, we assume that the consumption path c_s is steeper than the wage profile w_s in this case. Then the non-negativity constraint in (2) can be characterized as applying over the interval $S = [R, T]$. If the consumption stream in retirement is rising, as we assume, then the constraint will only be binding at $s = R$. The case in which this constraint alone operates is depicted in Figure 2 where we have assumed w_s to be constant for simplicity. At retirement, $A_R = 0$, $\mu_R > 0$ and the values of λ_s and c_s take a discontinuous jump, c_s rising and λ_s falling. Notice that for this constraint to bind, it is necessary that $\beta > (w_R - \tau)$. Although this may at first sight seem implausible, it should be remembered that β could include in-kind transfers which benefit the elderly, such as health services and leisure facilities.

Case 3. Earnings constraint binding. The inability to borrow against future wages is referred to as the earnings constraint. For this constraint to arise, we

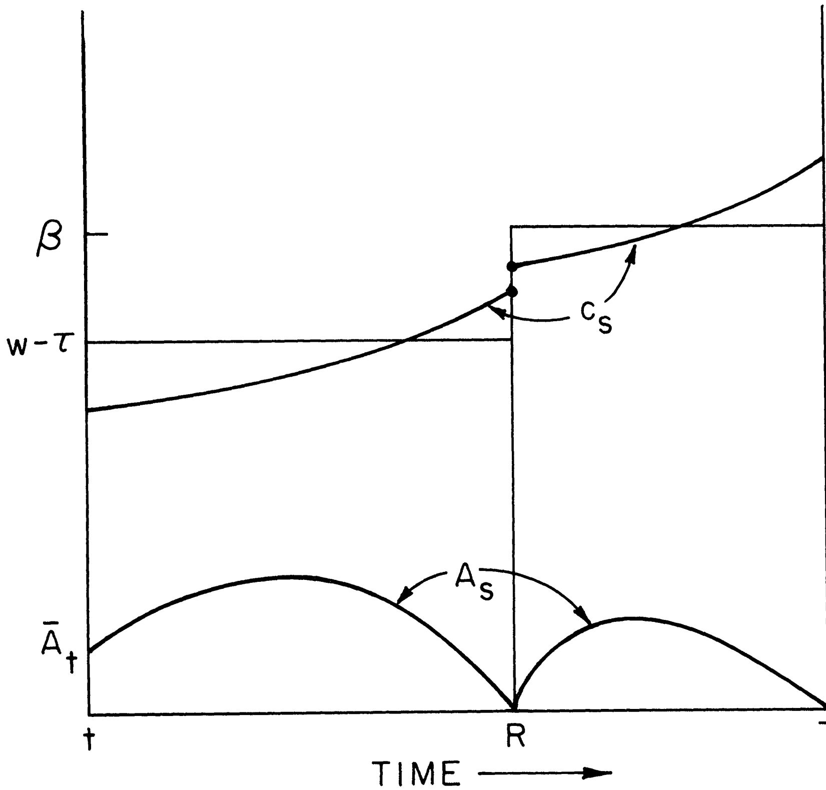

FIGURE 2
BENEFIT CONSTRAINT BINDING

require that the wage stream rise more rapidly than the consumption stream. We will also assume that the consumer's optimal consumption stream is such that $A_R > 0$, to rule out the possibility of borrowing against future social security benefits. In these circumstances, the non-negativity constaint in (2) applies only over the interval $S = [0, R]$. Then, when the constraint is binding, $A_s = 0$ so $c_s = w_s - \tau$ from the consumer's budget constraint. A typical lifetime profile might appear as in Figure 3. In this Figure, the household at age t has positive wealth $\bar{A}_t$. Initially, he would like to consume in excess of his after-tax wage and can do so until wealth is run down to zero at age b. He is then constrained until age d at which point his net wages begin to rise above the consumption stream. From then on positive wealth is held until death.[11] For $b < s < d$, $\mu_s > 0$, and elsewhere $\mu_s = 0$. Conversely, λ_s, the shadow value of wealth, falls exponentially in the intervals $t \leq s \leq b$ and $d \leq s \leq T$. In the constrained period λ_s is determined by (3) with $c_s = w_s - \tau$.

[11] Notice that the point b will coincide with t if the household is earnings-constrained to begin with. Also, for some earning streams, it would be possible for there to be more than one segment of the working period when the earnings constraint binds. For simplicity, we ignore that possibility in this paper. Note that both b and d depend upon β.

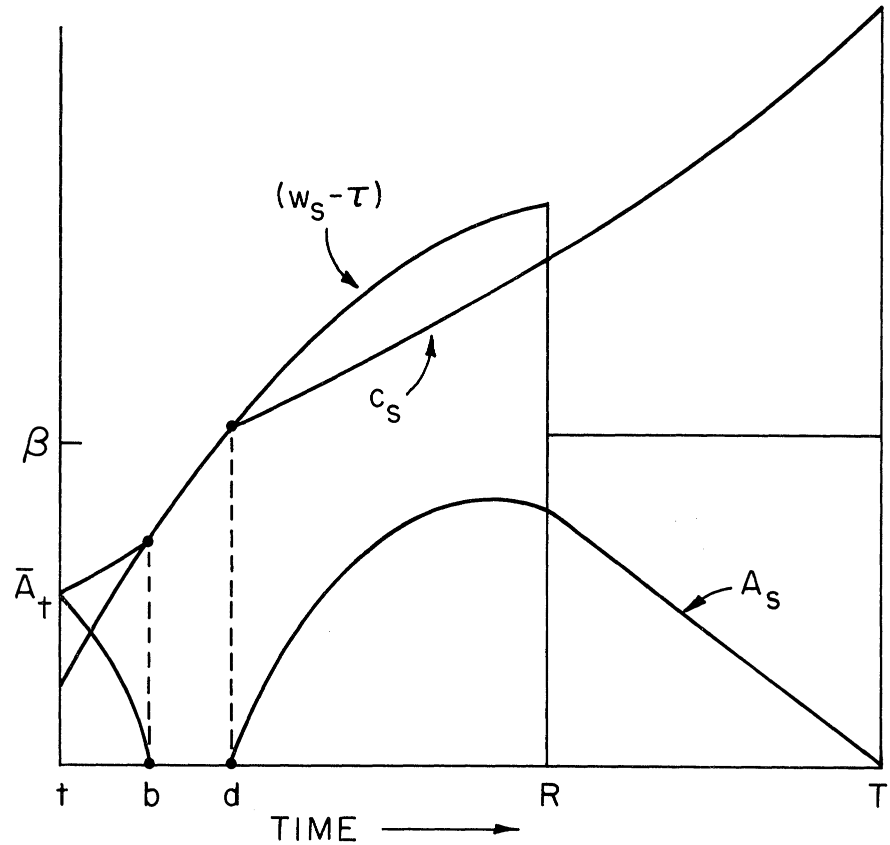

FIGURE 3
EARNINGS CONSTRAINT BINDING

Case 4. Both constraints binding. It is also possible that borrowing against future wages and future social security both be ruled out. For this general case, the non-negativity constraint is applicable over the entire interval $S = [0, T]$. This case could readily be depicted as a combination of Figures 2 and 3. Asset wealth would be zero for some age interval $[b, d]$ in the pre-retirement period and consumption would equal net wages. Also, at R, $A_R = 0$ and consumption jumps discontinuously to a higher level.

4. VOTER PREFERENCES OVER SOCIAL SECURITY LEVELS

Voter preferences for β depend upon how the welfare of the voter varies with β. In this section, we present a welfare analysis of the effects of changes in β on household utility over the remaining lifetime as a basis for deducing preferences defined over β.

For given values of factor prices (w_s, r) and demographic variables (n, R, T), the time path of consumption c_s can be viewed as depending upon the level of social security in existence up to age t, β^o, and the level of social security to prevail in the future, β. Given the values of w_s, r, n, R and T, we write the level of lifetime utility remaining to a household aged t as the indirect utility functional:

$$V_t(\beta;\, \beta^o) = \int_t^T e^{-\delta(s-t)} u[c_s(\beta;\, \beta^o)]\, ds \tag{7}$$

where c_s has been chosen optimally. Our objective is to determine the properties of V_t and to demonstrate its single-peakness in β. To evaluate the effect of changes in social security on lifetime utility, we differentiate (7) with respect to β to yield:

$$V_t'(\beta;\, \beta^o) = \int_t^T e^{-\delta(s-t)} u' \frac{\partial c_s}{\partial \beta}\, ds = \int_t^T \lambda_s \frac{\partial c_s}{\partial \beta}\, ds \tag{8}$$

using the first-order condition (3) where V_t' denotes the derivative with respect to β.[12]

Before investigating how (8) applies in various cases, we show that for all of the four cases discussed earlier, $V_t''(\beta;\, \beta^o) < 0$. Differentiating (8) by β yields:

$$\begin{aligned} V_t''(\beta;\, \beta^o) &= \int_t^T e^{-\delta(s-t)} \left\{ u' \frac{\partial^2 c_s}{\partial \beta^2} + u'' \left(\frac{\partial c_s}{\partial \beta} \right)^2 \right\} ds \\ &= \int_t^T \lambda_s \frac{\partial^2 c_s}{\partial \beta^2}\, ds + \int_t^T e^{-\delta(s-t)} u'' \left(\frac{\partial c_s}{\partial \beta} \right)^2 ds. \end{aligned} \tag{9}$$

The second term is always negative provided $u'' < 0$. The first term can be shown to equal zero.[13] Hence $V_t''(\beta;\, \beta^o) < 0$ establishing the following proposition:

[12] When carrying out the differentiation in (8) for the liquidity-constrained case, the boundaries b and d may change. However, this need not be explicitly accounted for in the differentiation since c_s and thus $u(c_s)$ are continuous at both b and d. They are discontinuous at R, but that is of no concern since R is given.

[13] To see this, we break it into intervals for the general case:

$$\begin{aligned} \int_t^T \lambda_s \frac{\partial^2 c_s}{\partial \beta^2}\, ds &= \lambda_t \int_t^b e^{-r(s-t)} \frac{\partial^2 c_s}{\partial \beta^2}\, ds + \int_b^d \lambda_s \frac{\partial^2 c_s}{\partial \beta^2}\, ds \\ &\quad + \lambda_d \int_d^R e^{-r(s-t)} \frac{\partial^2 c_s}{\partial \beta^2}\, ds + \lambda_R \int_R^T e^{-r(s-t)} \frac{\partial^2 c_s}{\partial \beta^2}\, ds \\ &= \lambda_t \partial^2 \left[\int_t^b e^{-r(s-t)} (w_s - \tau)\, ds + \bar{A}_t \right] \Big/ \partial \beta^2 + \int_b^d \lambda_s \frac{\partial(-\eta)}{\partial \beta}\, ds \\ &\quad + \lambda_d \partial^2 \left[\int_d^R e^{-r(s-t)} (w_s - \tau)\, ds \right] \Big/ \partial \beta^2 + \lambda_R \partial^2 \left[\int_R^T e^{-r(s-R)} \beta\, ds \right] \Big/ \partial \beta^2 = 0 \end{aligned}$$

where we have used the budget constraint for each subperiod as well as the fact that $\partial c_s / \partial \beta = -\eta$ for the constrained interval (b, d). Here and subsequently, λ_R is understood to be the right-hand limit of λ evaluated at R.

PROPOSITION 1. *Voter preferences are single-peaked over β. Therefore, a majority voting equilibrium will exist and will yield a level of social security corresponding to that preferred by the median voter.*

The evaluation of $V_t'(\beta; \beta^o)$, including its sign, will depend upon the borrowing constraint in existence, the size of β^o and the age of the voter, t. Consider again the most general case where both constraints may bind. For this case, (8) can be expanded as:

$$V_t'(\beta; \beta^o) = \lambda_t \int_t^b e^{-r(s-t)} \frac{\partial c_s}{\partial \beta} ds - \eta \int_b^d \lambda_s \, ds + \lambda_d \int_d^R e^{-r(s-d)} \frac{\partial c_s}{\partial \beta} ds \tag{10}$$

$$+ \lambda_R \int_R^T e^{-r(s-R)} \frac{\partial c_s}{\partial \beta} ds \qquad \text{(using (1), (2) and (4))}$$

$$= -\lambda_t \eta \Delta_{t,b} - \eta \int_b^d \lambda_s \, ds - \lambda_d \eta \Delta_{d,R} + \lambda_R \Delta_{R,T}$$

where $\Delta_{x,y} \equiv \int_x^y e^{-r(s-x)} ds$. The last step involves using the budget constraint applicable over the intervals (t, b), (d, R), and (R, T).[14] The properties of $V_t'(\beta)$ for each of the four cases can be determined by considering special cases of (10).

Neither liquidity constraint binding. For this case, (10) becomes:

$$V_t'(\beta; \beta^o) = -\lambda_t \eta \Delta_{t,R} + \lambda_R \Delta_{R,T}. \tag{11}$$

Furthermore, since $\mu_s = 0$ for all $t \leq s \leq T$ and $\lambda_R = \lambda_t e^{-r(R-t)}$,

$$V_t'(\beta; \beta^o) = \lambda_t (e^{-r(R-t)} \Delta_{R,T} - \eta \Delta_{t,R}). \tag{12}$$

The expression in brackets is the present value at age t of future benefits less taxes from an increment of social security. It is independent of the level of β. Also, since $\lambda_t = u'(c_t) > 0$, V_t is monotonic in β. In particular, $V_t'(\beta)$ is proportional to what we will refer to as the "wealth effect" of a dollar of social security from the point of view of a person of age t. This wealth effect is the term in brackets in (12).

Two properties of this wealth effect are relevant. First, for $t = 0$, the wealth effect will be positive or negative according to whether $n > (<) r$.[15] Second, the

[14] Thus, for example, in the interval (R, T),

$$\int_R^T e^{-r(s-R)} c_s \, ds = \int_R^T e^{-r(s-R)} \beta \, ds.$$

So,

$$\int_R^T e^{-r(s-R)} \frac{\partial c_s}{\partial \beta} ds = \int_R^T e^{-r(s-R)} ds = \Delta_{R,T}.$$

[15] PROOF: From (1),

$$\eta = \left(\int_R^T e^{-ns} ds \right) \Big/ \left(\int_0^T e^{-ns} ds \right).$$

Substitution into (11) yields the result.

wealth effect increases as t rises (or becomes less negative). For $t \geq R$, the wealth effect is necessarily positive. Hence, if $n > r$, preferences will be monotonically increasing in β for voters of all ages. If $r > n$, there will be a voter of a particular age for whom the wealth effect is zero. All younger voters will have negative wealth effects and monotonically decreasing preferences for β; all older voters will have positive wealth effects and monotonically increasing preferences for β.

Benefit constraint binding. Suppose either that workers can borrow against future wages or that $w_s = w$ $(0 \leq s \leq R)$ so that the liquidity constraint will only arise as a result of an inability to borrow against future social security benefits (Figure 2). In this case, $V_t'(\beta;\ \beta^o)$ is given by (11). Define $\delta_R = e^{-r(R-t)}\lambda_t - \lambda_R$, where $\delta_R > 0$, due to the liquidity constraint which requires $A_R = 0$. Equation (11) can therefore be rewritten:

$$(13) \qquad V_t'(\beta;\ \beta^o) = \lambda_t(e^{-r(R-t)}\Delta_{R,T} - \eta\Delta_{t,R}) - \delta_R \Delta_{R,T}.$$

The term with the coefficient λ_t is the wealth effect (as in (12)) and the last term is the reduction in welfare due to the presence of the distortion. Thus, an increase in β will have a wealth effect as before (which may be positive or negative) and a negative distorting effect. Note that the distorting effect will only apply for persons aged $t < R$. V_t' will be negative for large enough β. To see this, use (3) to rewrite (11) as:

$$(14) \qquad V_t'(\beta;\ \beta^o) = -u'(c_t)\eta\Delta_{t,R} - e^{-\delta(R-t)}u'(c_R)\Delta_{R,T}$$

and note that c_t falls and c_R rises as β rises. Since, for the constant elasticity utility function, $u'(c_s) \to \infty$ as $c_s \to 0$, V_t' will be negative for sufficiently large β. Since $V_t'' < 0$, preferences of pre-retirement voters will be single-peaked either at $\beta = 0$ (if the wealth effect is negative) or at some positive value of β below the maximum feasible one, $\beta_{\max}$. For retired voters, preferences are monotonically increasing in β.

Earnings constraint binding. The effect of being constrained by an inability to borrow against future wages yields qualitatively similar results. Consider an individual for whom this constraint alone is binding (Figure 3). V_t' will be given by (10) with $\lambda_R = \lambda_d e^{-r(R-d)}$ and $\lambda_b = \lambda_t e^{-r(b-t)}$. Notice also, using (4), that

$$(15) \qquad \lambda_s = \lambda_b e^{-r(s-b)} - \int_b^s e^{-r(s-x)}\mu_x\, dx \quad \text{for} \quad b \leq s \leq d.$$

Using (15) and the expressions for λ_R and λ_b, (10) can be rewritten as:

$$(16) \qquad V_t'(\beta;\ \beta^o) = \lambda_t[e^{-r(R-t)}\Delta_{R,T} - \eta\Delta_{t,R}]$$
$$- \left[\Delta\mu_{b,d}(e^{-r(R-d)}\Delta_{R,T} - \eta\Delta_{d,R}\right) - \eta \int_b^d \Delta\mu_{b,s}\, ds\right]$$

where $\Delta\mu_{x,y} = \int_x^y e^{-r(y-s)}\mu_s\, ds$. The interpretation of (16) is similar to before. The first term in square brackets is the wealth effect. The second term in square brackets is the marginal increment in the cost of the distortion from an increase in β where the distortion occurs over the period (b, d).[16] Increasing the size of β reduces the stream of after-tax wages in the pre-retirement period and increases the magnitude of this distortion. The value of μ_s in the liquidity-constrained interval will eventually rise by enough to cause the distortion effect to outweigh the wealth effect. Then, with a positive wealth effect, preferences of earnings-constrained pre-retirement voters will peak at some $\beta < \beta_{\max}$. With a negative wealth effect, the peak is at $\beta = 0$. For the retired, preferences increase monotonically with β.

Both constraints binding. This case simply combines the above two cases and it is unnecessary to repeat the analysis. The patterns of preferences over β are the same.

So far in discussing voter preferences, we have taken as given the parameters exogenous to the problem. In analyzing possible patterns for the evolution of social security through time, it will be useful to investigate how consumer preferences, particularly the location of the most-preferred level of β, depend upon the pre-existing level β^o. To do so involves undertaking a comparative static analysis on the implicit demand for social security.

Consider a voter aged $t < R$ with a positive wealth effect. The level of social security which he prefers, denoted β_t^*, will be that satisfying:

$$V_t'(\beta_t^* ; \beta^o) = 0. \tag{17}$$

Differentiating $V_t'(\cdot)$ with respect to β^o yields at the most preferred level of β;

$$\frac{\partial \beta_t^*}{\partial \beta^o} = -\frac{\partial V_t'/\partial \beta^o}{\partial V_t'/\partial \beta_t^*} = -\frac{\partial V_t'/\partial \beta^o}{V_t''}. \tag{18}$$

Since $V_t''(\beta_t^*) < 0$ by (9),

$$\operatorname{sgn}\, [\partial \beta_t^*/\partial \beta^o] = \operatorname{sgn}\, [\partial V_t'(\beta_t^*;\, \beta^o)/\partial \beta^o]. \tag{19}$$

This result applies whichever of the liquidity constraints is binding. Consider the constraints in turn.

Benefit constraint binding. Consider a pre-retirement-aged person for whom the benefit constraint binds. Differentiating (11) evalauted at β_t^* with respect to β^o we obtain:

$$\frac{\partial V_t'(\beta_t^* ; \beta^o)}{\partial \beta^o} = -\eta \Delta_{t,R} \frac{\partial \lambda_t}{\partial \beta^o} \tag{20}$$

since c_R and hence the second term of (11) is invariant with respect to changes in β^o. The term $\partial \lambda_t/\partial \beta^o$ can be decomposed as $(\partial \lambda_t/\partial A_t) \cdot (\partial A_t/\partial \beta^o)$. The first of these terms is negative because of the strict concavity of the utility function. The

[16] Intuition may be assisted by noting that $\lambda_d = \lambda_b e^{-r(s-b)} - \Delta\mu_{b,d}$. Thus, $\Delta\mu_{b,d}$ is a measure of the amount by which λ_d diverges from a path that would be followed if μ_s had been zero.

second is negative because a larger β^o implies a higher prior tax rate τ and therefore lower disposable income out of which to save. Therefore, $\partial V_t'/\partial\beta^o < 0$ and, by (19), $\partial\beta_t^*/\partial\beta^o < 0$. When referring to this decreasing functional relationship we will use the notation $\beta_t^*(\beta^o)$.

Earnings constraint binding. Again we seek to determine the sign of $\partial V_t'/\partial\beta^o$ where now V_t' is given by (16). Differentiating (10), we see that

$$\frac{\partial V_t'(\beta_t^*\,;\,\beta^o)}{\partial\beta^o} = -\eta\Delta_{t,b}\frac{\partial\lambda_t}{\partial\beta^o} \tag{21}$$

since a reduction in initial wealth will only affect the consumption path, and hence the λ_s's, in the pre-constrained period of the life cycle. Notice that (21) is identically zero if the voter is already constrained when the change occurs (i.e., if $\bar{A}_t = 0$). Therefore, we conclude that for the earnings constrained voter, $\partial\beta_t^*/\partial\beta^o \leq 0$ as $\bar{A}_t \geq 0$.

Again the more general case can be constructed from a combination of the above two. We can summarize these results in the following proposition:

PROPOSITION 2. *For a liquidity-constrained voter of age t, the preferred level of social security β_t^* is a decreasing function of the pre-existing level β^o whenever $\bar{A}_t > 0$. If $\bar{A}_t = 0$ (when the voter is already earnings-constrained), β_t^* is invariant with respect to β^o.*

5. MAJORITY VOTING EQUILIBRIA

Because preferences are single-peaked, as shown in Proposition 1, majority voting equilibria always exist. However, not all majority voting equilibria are very interesting. For example, if there are no capital market constraints, individual voters prefer either zero social security ($\beta_t^* = 0$), maximal social security ($\beta_t^* = \beta_{\max}$), or an indeterminate level, depending on whether the wealth effect of the program is negative, positive, or zero. Since the wealth effect of social security is always greater for older individuals, the median voter will be the voter of median age. The sign of his wealth effect will determine which of these extreme outcomes is obtained.

When liquidity constraints are present, extreme types of equilibria need not occur. We shall show this first for the case where only the benefit constraint can arise and later discuss the earnings-constrained case.

Benefit constraint binding. To analyze majority voting equilibria in this case requires first that we investigate the way that the benefit constraint will affect households of different ages. To begin with, note that increases in the level of social security for a person of age t will eventually cause the benefits constraint to become binding. One useful result that can be obtained is that the level of β at which the benefit constraint just becomes binding increases with the age in the life cycle at which the β changes are introduced.

To show this, consider a person of age t who has faced a social security level of β^o up to age t. That person will have planned a stream of consumption c_s^o which changes exponentially at the rate $(r - \delta)/\varepsilon$. Suppose this person is not benefit-constrained given β^o and suppose β is increased until the benefit constraint is just binding. From the consumer's budget constraint, the change in consumption Δc_s must satisfy:

$$\int_t^T e^{r(s-t)} \Delta c_s \, ds = \Delta\beta\left(\int_R^T e^{-r(s-t)} \, ds - \eta \int_t^R e^{-r(s-t)} \, ds\right)$$

where the right-hand side is the change in social security wealth at age t. Also, from the fact that the constraint just binds, retirement consumption is financed from social security benefits. Therefore,

$$\int_R^T e^{-r(s-R)}(c_s^o + \Delta c_s) \, ds = \int_R^T e^{-r(s-R)}\beta \, ds.$$

Next, consider a person aged k, where $t < k < R$. The same change in social security $\Delta\beta$ will cause a change in consumption $\Delta c_s'$ satisfying:

$$\int_k^T e^{-r(s-k)} \Delta c_s' \, ds = \Delta\beta\left(\int_R^T e^{-r(s-k)} \, ds - \eta \int_k^R e^{-r(s-k)} \, ds\right).$$

Since the right-hand side, the change in social security wealth for a person aged k, is greater than the corresponding change for the person aged $t < k$, the change in consumption after age k everywhere dominates that for person t. In particular, after age R, the present value of consumption for the person initially of age k is greater than that for the person of age t. Since they receive the same social security benefits,

$$\int_R^T e^{-r(s-k)}(c_s^o + \Delta c_s') \, ds > \int_R^T e^{-r(s-k)}\beta \, ds.$$

Therefore, $A_R > 0$ for the person aged k, who is therefore not benefit-constrained. By an identical argument, all persons younger than t when the change is introduced will be benefit-constrained if person t is. A similar argument obtains when the person aged t is initially constrained. If so, one reduces β to the level at which t is just constrained. The remainder of the argument goes through with obvious changes. Thus, we have established that if the only borrowing constraint is the inability to borrow against future social security benefits, the level of social security which, when introduced at age t, just causes the benefit constraint to bind will be increasing in t. Equivalently, if a household of age t is benefit-constrained at a given level of social security, all households of age $s < t$ will also be benefit-constrained.

With this preliminary result, we can now discuss majority voting equilibria. First, let m represent median age, and consider β_m^*, the ideal level of social security benefits for voter m, given β^o. Assume that $m < R$, and that the social security wealth effect for this individual is positive. We know that $0 < \beta_m^* < \beta_{\max}$, and thus β_m^* is the implicit solution to

$$V_m'(\beta_m^*; \beta^o) = -u'(c_m)\eta\Delta_{m,R} + e^{\delta(R-m)}u'(c_R)\Delta_{R,T} = 0, \tag{22}$$

where c_R is the right-hand limit of c_t at $t = R$. Voter m must be benefit-constrained at β_m^*.

Now consider the value of β_t^*, the ideal point for a voter aged t, given β^o. If $t > m$, and if the benefit constraint is not binding when $\beta = \beta_m^*$, we must have $\beta_t^* > \beta_m^*$. This follows because the wealth effect for older individuals must be positive, given that it is positive for m, and because the sign of $V'(\beta; \beta^o)$ depends only on the wealth effect when no liquidity constraints are binding. On the other hand, suppose that $t > m$ and that the benefit constraint is binding. Then

$$V_t'(\beta_m^*; \beta^o) = -u'(c_t)\eta\Delta_{t,R} + e^{\delta(R-t)}u'(c_R)\Delta_{R,T} \tag{23}$$

where c_t is the optimal consumption for a voter of age t (given that β has changed from β^o to β_m^*) and where c_R is the same as in (22). (Because both households are benefit-constrained, and hence begin the retirement period with zero wealth, they follow identical retirement-period consumption streams.) Subtracting (22) from (23) and rearranging yields:

$$\begin{aligned} V_t'(\beta_m^*; \beta^o) = \eta(u'(c_m) - u'(c_t))\Delta_{m,R} + \eta u'(c_t)(\Delta_{m,R} - \Delta_{t,R}) \\ + u'(c_R)\Delta_{R,T}(e^{\delta(t-m)} - 1)e^{\delta(R-m)}. \end{aligned} \tag{24}$$

The last two terms in this expression are positive while the sign of the first term depends on the relationship between c_m and c_t. One can show that $c_t \geq c_m$ when $\beta_m^* \geq \beta^o$, so that the first term is non-negative in this case.[17] Hence, $V_t'(\beta_m^*; \beta^o) > 0$ for $t > m$, provided that $\beta_m^* \geq \beta^o$, whether or not the voter of age t is benefit-constrained. If m is benefit-constrained, we have shown above that so must be all households $t < m$. Applying (24) in this case, and noting by symmetry that $c_t \leq c_m$ when $\beta_m^* \geq \beta^o$, it follows that $V_t'(\beta_m^*; \beta^o) < 0$ for $t < m$. Hence,

[17] PROOF: From the budget constraint for the person aged m,

$$\int_m^R e^{-r(s-m)}\Delta c_s \, ds = -\int_m^R e^{-r(s-m)}\Delta\tau \, ds$$

when τ changes by $\Delta\tau$, where Δc_s is the change in consumption for person m. Since $\Delta c_s = \Delta c_m e^{(r-\delta)(s-m)/\varepsilon}$, we have that

$$\left|\int_t^R e^{-r(s-t)}\Delta c_s \, ds\right| > \left|\int_t^R e^{-r(s-t)}\Delta\tau \, ds\right|$$

for $t > m$ for person m. However, for person t,

$$\int_t^R e^{-r(s-t)}\Delta c_s' \, ds = -\int_t^R e^{-r(s-t)}\Delta\tau \, ds$$

where $\Delta c_s'$ is the change in consumption for person t. Therefore, $|\Delta c_s| > |\Delta c_s'|$. This implies that for $\Delta\tau > 0$, $c_t > c_m$.

PROPOSITION 3. *Suppose that only the benefit constraint applies and that $\beta_m^* \geq \beta^o$. Then all voters of age $t > m$ prefer higher social security benefits than m, all voters of age $t < m$ prefer lower social security benefits, and hence β_m^* is the majority voting equilibrium level of β. In particular, m is the median voter for any steady-state majority voting equilibrium ($\beta_m^* = \beta^o$).*

When $\beta_m^* < \beta^o$, we can no longer argue that $V_t'(\beta_m^*; \beta^o)$ is of unambiguous sign for $t > m$ or $t < m$ and hence we cannot be certain that m is the median voter. However, we can show:

PROPOSITION 4. *Given the same institutional environment as in Proposition 3, suppose that $\beta_m^* < \beta^o$. Then the median voter, say an individual of age q, has a preferred level of benefits β_q^* such that $\beta_q^* < \beta^o$.*

In other words, when the voter of median age prefers a reduction in the size of the social security program, so will a majority of voters. The majority voting equilibrium level of β, however, need not coincide with β_m^*.

To prove this proposition, we argue by contradiction. Let q be the median voter,[18] and suppose $\beta_q^* \geq \beta^o$. By argument identical to that given in the proof of Proposition 3, we can show that all voters of age $t > q$ prefer a higher level of β than β_q^*, and all younger voters prefer less. Hence $q = m$ and $\beta_q^* = \beta_m^* \geq \beta^o$. This contradicts the hypothesis that $\beta_m^* < \beta^o$, proving the desired result.

Earnings constraint binding. Essentially identical results obtain when households are earnings-constrained (or when both types of constraints can occur). The one added qualification here is that the path of wages must be sufficiently regular so that, when a vote is taken, each household will choose a consumption path for the rest of the life cycle along which the earnings constraint will be binding for at most one interval of time. A sufficient regularity condition is that the net earnings stream should cut the optimal consumption path only once, lying below it up to a certain critical time and above it thereafter. Given this condition, all of the previous results for the benefit-constrained case apply directly to the general case. That is, if a voter of age t is constrained by either type of constraint (or both), so will be all voters of age $s < t$. If $\beta_m^* \geq \beta^o$, the median-aged voter is the median voter (Proposition 3), and if $\beta_m^* < \beta^o$, the median voter prefers a social security level less that β^o (Proposition 4). Henceforth, Proposition 3 and 4 should be understood as holding in this general case.

The proof of these propositions are omitted here for the sake of brevity. The general line of argument is identical to that given for the benefit-constrained case.

[18] It should be noted that q may not be unique. That is, there may be more than one voter who has the median demand for social security β_q^*. However, the median demand β_q^* will itself be unique by single-peakedness of preferences.

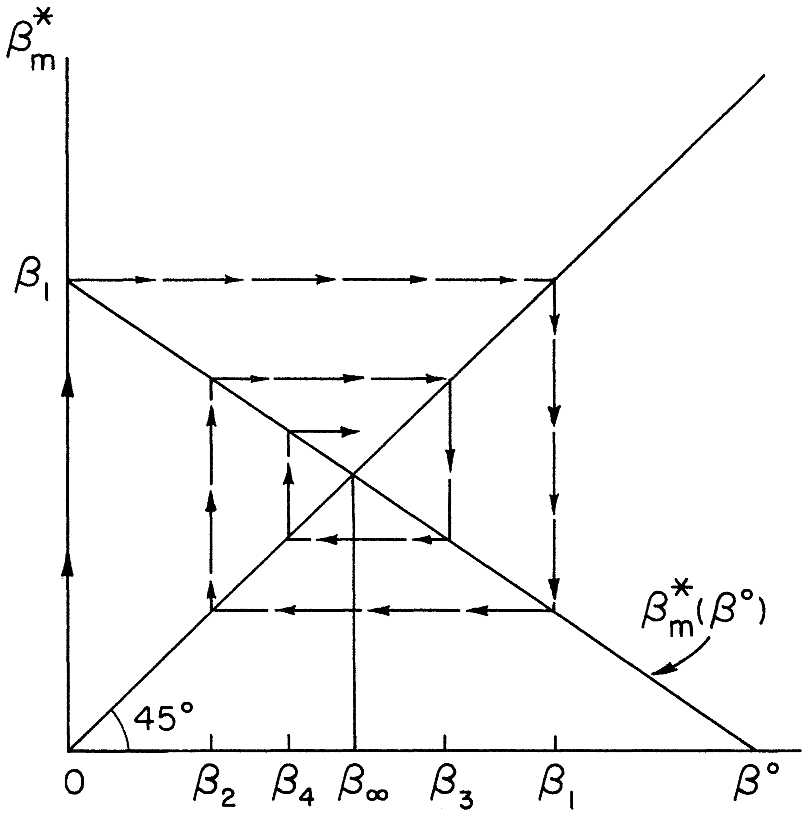

FIGURE 4

6. DYNAMIC EVOLUTION OF MEDIAN VOTER EQUILIBRIA

Tracing out the dynamics of economies responding to policy changes is inherently complicated even in a model as simple as ours in which only one type of economic decision is taken (saving/consumption) and only one policy variable is used (social security β). Some simplifying assumptions must necessarily be made.

We begin by assuming that the median-aged person will be the median voter, and that only the benefit constraint applies. Figure 4 illustrates the dynamics that might occur in this case. The curve labelled $\beta^*_m(\beta^o)$ represents the level of β that the median voter would choose given an existing level β^o. Recall from Proposition 2 that this curve will have a negative slope. Note that the curve $\beta^*_m(\beta^o)$ will cross the 45° line at the steady-state value, denoted β_∞. It is the value of β^*_m which maps into itself in the function $\beta^*_m(\beta^o)$ and, once achieved, would not be changed by subsequent voters. The arrow depicts one possible sequence of adjustments, starting from an initial level of $\beta = \beta_0 = 0$. The first median voter would choose the relatively high level β_1, the next would reduce it to β_2, following by β_3, β_4, etc. until the chosen level approached the steady-state level β_∞. This pattern, characterized by alternative over- and under-shooting cycles which gradually damp down as time goes by, would be the case when the slope of $\beta_m^*(\beta^o)$ is less than one in absolute value throughout its length.

The alternative case in which the slope is everywhere greater than unity in absolute value would result in an endless sequence of identical pairs $(\beta_1, 0)$, never approaching a steady state. If the initial value β_0 were positive, the sequence would be an ever-expanding sequence of cycles gradually approaching $(\beta_1, 0)$. In all cases, the system would be characterized by initial overshooting followed by undershooting. The pattern might then continue cyclically either approaching β_∞ by damped cycles or $(\beta_1, 0)$ by undamped cycles. Other patterns are, however, possible if the slope of $\beta_m^*(\beta^o)$ changes enough over its length (e.g., if it is sometimes >1 and sometimes <1).

We might summarize these results as follows:

PROPOSITION 5. *If the median voter is median-aged and benefit-constrained, and starting at* $\beta^o < \beta_\infty$:

a) *if the absolute value of the slope of* $\beta_m^*(\beta^o)$ *is uniformly* <1, β_m^* *will initially overshoot, then undershoot, then cycle continuously around* β_∞, *gradually approaching it by damped cycles*;
b) *if the absolute value of the slope of* $\beta_m^*(\beta^o)$ *is uniformly* >1, β_m^* *will initially overshoot, then undershoot, then cycle continuously around* β_∞ *by undamped cycles* $(\beta_1, 0)$.

This dynamic analysis presumes that the median voter is the median-aged voter. Proposition 3 shows that this will be the case when β is increased from β^o. However, for decreases in β, the median voter may not be median-aged. Proposition 4 shows that if the median voter prefers β to fall, so will the majority of voters. The implication of this for the above analysis is that the increasing segments of the cycle will be determined as described by the curve $\beta_m^*(\beta^o)$. However, the decreases may differ from that. As a consequence, the overshooting will still occur and the steady state level β_∞ will be unchanged. However, convergence to β_∞ is not guaranteed.

PROPOSITION 6. *If the conditions of Proposition 5 apply, but the median voter is not median-aged for reductions in* β, *overshooting above* β_∞ *will occur whenever* $\beta^o < \beta_\infty$, *and* β *will fall whenever* $\beta^o > \beta_\infty$.

Consider next the case in which the median voter is earnings-constrained. We retain the assumption that the median voter is median-aged. The dynamics of this case admit of a somewhat wider range of possibilities compared with that obtained earlier. Take two examples. If the wage profile of households is such that in the steady state $\bar{A}_m > 0$, then Proposition 5 and 6 continue to apply.

On the other hand, suppose that in the steady-state, the wealth held at m is zero. That is, $b \leq m$ in the steady state so the household is already constrained when the choice is made. Now imagine starting this economy with $\beta^o = 0$. The first voter, if he has some wealth on hand to start with at m, will opt for a level of β above the steady state level, thereby constraining himself (and the subsequent median voter). The next median voter, who has $\bar{A}_m = 0$, will then reduce β to the steady state level where it will stay thereafter. However, suppose at $\beta^o = 0$ the median voter is already liquidity-constrained at age m, so $\bar{A}_m = 0$. Here, the median voter will immediately choose the steady state β_∞. Thus, overshooting depends on the existence of $\bar{A}_m > 0$ at $\beta^o = 0$.

We can summarize these results in the following propositions:

PROPOSITION 7. *If the median voter is earnings-constrained and if $\bar{A}_m > 0$ in the steady state, the sequence of choices of β is as in Propositions 5 and 6.*

PROPOSITION 8. *If the median voter is earnings-constrained and if $\bar{A}_m = 0$ in the steady state:*

a) *if, at $\beta^o = 0$, $\bar{A}_m = 0$, β will go to the steady state immediately;*
b) *if, at $\beta^o = 0$, $\bar{A}_m > 0$, β will go to the steady state after one period of overshooting.*

In the more general case in which both constraints bind, Propositions 7 and 8 continue to apply. As long as $\bar{A}_m > 0$ for the median voter, overshooting can be expected to occur. If, however, $\bar{A}_m = 0$ in the steady state, the system will go to the steady state either immediately or after one period of overshooting.

7. CONCLUDING REMARKS

The intention of this paper has been to analyze, in the context of a fully-specified dynamic model, the sequence of social security levels that would be chosen by median voters. In order to make the analysis manageable we have had to impose strong assumptions. Doing so has allowed us to concentrate our attention on the precise interactions between social security levels and asset accumulation, and on the implications of this interaction for voter preferences over social security levels. Although our analysis cannot be taken literally as a description of history even in the counterfactual sense, it is suggestive of some of the sorts of influences that must be taken into account in further work in this area. We have particularly tried to indicate some of the main issues involved in modelling policies in a dynamic economy.

One of the main messages is that dynamic policy modelling is inherently complicated both analytically and conceptually. When one tries to incorporate overlapping generations and capital market equilibria in a realistic manner, the full evaluation of alternative policies by a current generation of voters will typically require information both on the history of past policies (which can be taken as given) and on the sequence of future policies (which themselves will be partly determined by current decisions). This presents a formidable analytical task. Attempts to avoid or limit these interdependencies by, for example, restricting analysis to two- or three- period life-cycles may unwittingly assume away some important interactions. These models by their very nature restrict both the frequency with which votes are taken over the life cycle and the number of generations which can be alive at a given time. Similarly, analyzing overlapping-generations models without including full capital markets can be misleading.

Despite the restrictiveness of our assumptions, the qualitative nature of the results are in accordance with some stylized facts. The general result that newly-introduced social security systems initially overshoot the long-run equilibrium and then fall seems to accord with a pattern noted by Verbon (forthcoming) for many public pension systems. Also, the result we obtain of positive social security levels above the long-run welfare-maximizing level but less than the maximum-possible level corresponds both with that obtained by previous authors (Browning 1975; Hu 1982; Sjoblom 1985) and with that alleged to exist in practice (Feldstein 1974).

Queen's University, Canada
Indiana University, U.S.A.

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