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TAX-TRANSFER POLICIES AND THE VOLUNTARY PROVISION OF PUBLIC GOODS

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The purpose of this paper is twofold. First, it extends previous models of non-cooperative private funding of pure public goods by allowing both for distortionary taxation of private goods and for subsidies based on contributions to the public goods. Second, it clarifies the type of behavioral and informational assumptions which are needed to insure neutrality of both lump-sum and distortionary policies. The analysis is developed in the context of fiscal federalism.

1. Introduction

It is widely recognized that the effects of redistributive transfers among agents may be nullified by offsetting involuntary transfers in an economy with altruistic utility functions. This was formally demonstrated for the case of intergenerational transfers by Barro (1974) in his resurrection of the Ricardian Equivalence Theorem. According to that theorem, any attempt by the government to redistribute income across generations by, say, unfunded

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pensions or public debt would give rise to equal and opposite changes in bequests when generations were linked by altruism.[1]

Recently, Warr (1982) has shown that altruism is not the only form of interdependence among agents that can render redistributive transfers neutral. He showed that when agents all voluntarily contribute to some pure public good, lump-sum transfers among them will have no effect on resource allocation or welfare. Agents will change their contributions to the public good by amounts equal to the transfers they receive, provided it remains feasible for them to do so.[2] In Warr's model, agents care only about the aggregate supply of the privately-provided public good and take as given both the policy parameters facing them and the behavior of other agents.

In this context, as Warr and subsequent authors show, lump-sum redistributive transfers are neutral. Essentially this result emerges from the fact that inter-agent redistributions will be accompanied, in equilibrium, by offsetting changes in the levels of individual contributions to the public good. This is a striking result, because lump-sum interpersonal transfers are traditionally viewed as an ideal (even if impractical) mechanism for changing the distribution of welfare in the economy. In the voluntary contribution setting, the lump-sum redistribution instrument can no longer operate in this way. Indeed, we show that this result applies in more general settings than those analyzed previously, in that lump-sum redistributions are neutral even when they are carried out in an economy with pre-existing tax and subsidy distortions.

This raises the question of what policy instruments, if any, would have real distributional effects in such an economy. Here our analysis yields some quite striking and surprising results. In addition to lump-sum transfers, we consider transfers which affect the relative price to agents of contributing to the public good, such as per unit subsidies to contributors. Warr has already shown that a uniform subsidy increases total contributions and is therefore non-neutral. However, we show that differential subsidies across agents not only affect total contributions, but affect the welfare of different agents in an unexpected way. For example, we show that an increase in the subsidy on one agent's contributions, financed by a lump-sum tax on another agent, will *lower* the welfare of the agent receiving the subsidy, and *raise* the welfare of the agent paying the tax. Using this analysis, we also show how points on

[1]This was generalized by Bernheim and Bagwell (1988) to show that the linkages could be indirect (i.e. via connecting agents) so that the neutrality phenomenon could be very widespread indeed.

[2]Bergstrom et al. (1986) extend Warr's analysis to examine the case in which some agents are making no contribution. Note also that, as Bernheim (1986) claims, the Warr result applies in a world of many public goods even if some persons make zero contributions to some public goods provided only that all persons are linked indirectly by overlapping contributions with all other persons. Andreoni (1988) argues that few individuals will make positive contributions in equilibrium as the number of agents becomes large.

the utility-possibility frontier can be attained through suitably-determined subsidy rates on the contributions of different agents. Movements along this frontier involve increasing one agent's subsidy rate, and lowering that of another, such that the agent whose subsidy rate is increased experiences a reduction in welfare, and vice versa.[3]

These results appear to conflict with an even more remarkable neutrality result, reported in Bernheim (1986). He has argued that, in a model in which voluntary contributions to a public good are strictly positive for all agents, not only will lump-sum transfers be neutral, so will both government contributions to the public good and the distortionary taxes that may be used to finance them. This result is derived in a model in which allocations are determined by a Nash equilibrium of a two-stage game in which the first stage involves a labor supply decision and the second, contributions to the public good. The strategy space for agents in the second stage involves the decision rules for contributing to the public good. To help clarify the reason for the differences between Bernheim's results and those found in the rest of the literature, we examine the types of informational and behavioral assumptions which are required for the neutrality of various policies in these models. To obtain very strong neutrality results like Bernheim's requires adopting very strong informational and behavioral assumptions. For example, each agent must both 'see through' the government budget and also make correct conjectures about the behavior of other agents. It is on these latter assumptions that Bernheim's neutrality results are based.

In judging the appropriateness of various behavioral assumptions, the institutional setting is relevant. Previous authors have presented their analysis in the context of individual contributions to a common public good, such as charity. In economies with large numbers of individuals, the informational requirements imposed on each individual before rationally assessing their own contributions may be thought to be overly-demanding. We have therefore chosen to cast our analysis in the setting of a federal state in which the contributing agents are local governments. This has a number of advantages. First, the number of agents is small and likely to be reasonably well informed about the consequences of their actions. Second, given that there are national public goods to which the localities might voluntarily contribute (e.g. quality of environment and welfare), the likelihood that all would make positive contributions seems more defensible than in the individual case. Third, the use of the federalism example enables us to examine the effect of various sorts of grants which exist in practice, and

[3]A related result appears in Roberts (1987), who shows that replacement of tax credits for charitable contributions with tax deductions, engineered in such a way that total charitable contributions are held fixed, lowers the welfare of high-income donors if they have elastic demands for the level of charitable activity.

thereby make a contribution to the existing theory of grants.[4] Although we find the federalism application particularly appealing, we emphasize that the model, like previous models in the literature, can also be applied to the case of individual contributions to public goods, such as charities.

The paper is organized along the following lines. First, we present a simple model consisting of a three-good–two-locality economy and describe Nash equilibria in which both localities contribute voluntarily to a national public good. This model is used in section 3 to analyze the effects of inter-community redistribution by the central government. Lump-sum redistributions are shown to be fully neutral, while matching grants based on localities' contributions stimulate public good provision and thus have real effects. This section also derives some strong results on efficient matching grants, and shows that such grants can have surprising distributional implications. Section 4 shows that were localities aware of the central government budget constraint from the start, both lump-sum redistribution and distortionary subsidies would be neutral, thus duplicating Bernheim's results.

2. The basic model

Assume that there are two localities in the economy, 1 and 2, and that each is inhabited by a group of identical immobile households. We shall assume without further loss of generality that the number of individuals in each locality is set equal to one.[5] Although we do not pursue the extension to many jurisdictions in a formal way, note that one could think of locality 1 as a single small locality and locality 2 as the aggregate of all other localities.

2.1. Preferences

In the most general version of the model, we assume that the household in each locality has preferences defined over three commodities. The first of these is a private good, consumption of which is denoted by x_i, $i=1,2$. We may think of this as the basic numeraire good in the model, and the good in which voluntary contributions are made and taxes and subsidies are paid.

[4]The public good to which contributions are made in our model differs from that sometimes used in the theory of fiscal federalism. The latter [see, for example, Williams (1966)] often involves local public goods with externalities where some proportion of a locality's spending spills out to other localities. Thus, one locality's spending is not a perfect substitute for another, unlike in this paper. Our analysis could be extended to these 'impure' public goods but the results are not clearcut. In particular, the neutrality results no longer apply. Nonetheless, the qualitative results of this paper are at least suggestive for these other cases. Furthermore, all the neutrality results require is one public good to which all contribute, or several public goods in which contributions are overlapping.

[5]Models which treat localities as utility-maximizing individuals are common in the literature of local public finance. See Wildasin (1986) for a discussion of the uses and limitations of these models, and for references to the literature.

The second commodity, consumption of which in i is denoted by y_i, is either another private good (or factor, if $y_i<0$) or a strictly local public good (or factor, if $y_i<0$), benefiting only the household in the locality in which it is provided. By interpreting y_i as a vector, we could actually accommodate all of these cases simultaneously. The third commodity is a 'national' public good, the level of provision of which is denoted by g. When this public good is provided, it is consumed jointly by the households in both localities. A single public good is assumed only for convenience as our results can be readily extended to the case in which g is a vector. The utility function for locality i is $u^i(x_i, y_i, g)$, assumed strictly increasing in each argument, strictly quasi-concave, and twice continuously differentiable. Note that utility functions can vary across localities.

2.2. *Technology and endowments*

In order to avoid unnecessary complications, assume that relative prices do not change, and that units of all commodities are chosen so that their prices are equal to 1. (A linear production technology assures this.) Locality i has an exogenously-given income or wealth of w_i. The national public good g is produced from the contributions of the localities.[6] Let g_i denote the contribution of locality i. Then

$$g=\sum_i g_i. \tag{1}$$

2.3. *Central government*

The central government makes transfers of various kinds among localities. First, it may make a lump-sum transfer to locality i, denoted by L_i. (This may in fact be a tax, in which case $L_i<0$.) Second, the central government may subsidize local government contributions to the national public good via matching grants. Let m_i denote the subsidy paid to locality i per unit of contributions g_i. Third, the central government may collect an amount t_i per unit of the commodity y_i from locality i. If y_i is a private commodity, then t_i is simply a per unit tax or subsidy on this good (or factor). If y_i is a strictly local public good, then $-t_i$ corresponds to a matching grant or per unit subsidy. The central government is constrained to balance its budget, hence

$$\sum_i (L_i+m_i g_i-t_i y_i)=0. \tag{2}$$

[6]Allowing national lump-sum contributions to the financing of g would not affect our results.

2.4. Local optimizing behavior

Each locality faces a budget constraint:

$$x_i+(1+t_i)y_i+(1-m_i)g_i=w_i+L_i, \quad i=1,2, \tag{3}$$

and chooses (x_i, y_i, g_i) to maximize its utility subject to (3). In this and the next section, it is assumed that each locality i takes the relevant policy parameters of the central government (t_i, m_i, L_i) as exogenously given. It is also assumed that each locality takes as given the level of contributions of the other locality for the national public good. Thus, recalling from (1) that $g=g_1+g_2$, locality i solves the problem:

$$\max_{\langle x_i, y_i, g_i\rangle} \; u^i(x_i, y_i, g_i+\bar{g}_j), \quad i=1,2, \tag{P}$$

subject to (3), where $\bar{g}_j$ is the given level of contributions by locality $j\neq i$ toward the national public good.

One can derive the first-order conditions for this problem in the usual way. As discussed in detail by Bergstrom et al. (1986) a non-negativity constraint on contributions g_i may well be binding as a solution to (P) if $\bar{g}_j$ is sufficiently large. Most of the distinctive results that we wish to discuss in this paper arise because of the presence of a national public good g, contributions to which link the localities in an unusual way. These results break down or are weakened when contributions for some localities fall to zero. In order to present the results in the simplest way and in their strongest form, we assume henceforth that non-negativity constraints on individuals contributions are non-binding in equilibrium. (As remarked at the end of section 3, this assumption is actually relatively weak in some of the situations we analyze.)

Using the fact that $g_i=g-g_j$, it is now obvious that problem (P) is equivalent to

$$\max_{\langle x_i, y_i, g\rangle} \; u^i(x_i, y_i, g) \tag{P$'$}$$

subject to

$$x_i+(1+t_i)y_i+(1-m_i)g=w_i+L_i+(1-m_i)\bar{g}_j=I_i, \tag{3$'$}$$

where I_i denotes the effective income of locality i and is exogenous to it. It is equivalent in the sense that if (x_i, y_i, g_i) solves (P), then (x_i, y_i, g) with $g=g_i+\bar{g}_j$ solves (P′), and conversely. (See Bergstrom et al. for a similar transformation.) Formally, (P′) is just a standard consumer optimization problem in which a consumer chooses (x_i, y_i, g) given the prices $(1, 1+t_i, 1-m_i)$ for these commodities and given a fixed income I_i.

The solution to this problem yields demands as functions of these parameters. In particular let $\phi^i(1+t_i, 1-m_i, I_i)$ be locality i's demand function for g. The derivatives of ϕ^i with respect to each of its arguments will be denoted $\phi^i_y, \phi^i_g, \phi^i_I$, respectively. The own-substitution effect of a change in the relative price of g will be denoted s_i. From the Slutsky equation, $s_i = \phi^i_g - g\phi^i_I$. Throughout the remainder of the paper we shall assume that $0 < \phi^i_I < 1$ for $i = 1, 2$. This is guaranteed by (but of course does not require) normality of all goods. Finally, let $v^i(1+t_i, 1-m_i, I_i)$ denote the indirect utility function obtained as the solution to (P′).

2.5. Equilibrium

A *Nash equilibrium* in this model is a pair (g_1, g_2) such that

$$g_1 + g_2 = \phi^1(1+t_1, 1-m_1, I_1), \tag{4.1}$$

$$g_1 + g_2 = \phi^2(1+t_2, 1-m_2, I_2). \tag{4.2}$$

We shall assume the existence of a Nash equilibrium (with $g_i > 0$, $i = 1, 2$). One can show, under our assumptions, that the Nash equilibrium is unique.

3. The allocative and welfare effects of central government policy

3.1. Lump-sum redistribution

To begin with, let us investigate the effect of a change in lump-sum grants L_1 and L_2, with tax rates t_i and matching grant rates m_i held constant. The changes in L_1 and L_2 must be feasible, that is the central government budget constraint (2) must be satisfied both before and after the policy change. Thus, for a differential change in transfers $\mathrm{d}L_i$ and $\mathrm{d}L_j$,

$$\sum_i (\mathrm{d}L_i + m_i\,\mathrm{d}g_i - t_i\,\mathrm{d}y_i) = 0, \tag{5}$$

where $\mathrm{d}g_i$ and $\mathrm{d}y_i$ are the induced changes in the equilibrium national public good contributions and private good consumption, respectively.

From total differentiation of (4), the change in the Nash equilibrium values of g_1 and g_2 must satisfy:

$$\mathrm{d}g_1 + \mathrm{d}g_2 = \phi^1_I(\mathrm{d}L_1 + (1-m_1)\,\mathrm{d}g_2), \tag{6.1}$$

$$\mathrm{d}g_1 + \mathrm{d}g_2 = \phi^2_I(\mathrm{d}L_2 + (1-m_2)\,\mathrm{d}g_1). \tag{6.2}$$

We shall now show that, for any feasible change in central government policy $(\mathrm{d}L_1, \mathrm{d}L_2)$, the new Nash equilibrium for the economy satisfies:

$$\mathrm{d}g_i = -\frac{\mathrm{d}L_j}{1-m_j}, \quad i,j=1,2, \quad i \neq j, \tag{7.1}$$

$$\mathrm{d}g_1 + \mathrm{d}g_2 = 0, \tag{7.2}$$

$$\mathrm{d}x_i = \mathrm{d}y_i = 0, \qquad i=1,2. \tag{7.3}$$

To show that (7) describes the new Nash equilibrium, note that (7.1) implies that $\mathrm{d}I_i = 0$ for both i. Given that $\mathrm{d}t_i = \mathrm{d}m_i = 0$, this then implies (7.3). But, by direct substitution, one sees that (7) implies (5) and (6). Of course, given (7.2) and (7.3), it follows that $\mathrm{d}u_i = 0$ for both i. This result could also be derived for small changes using the comparative statics method used below.[7] Since the Nash equilibrium is unchanged for any differential changes in L_1 and L_2, so it will be for any discrete changes.[8] Thus:

Theorem 1. For any feasible change in lump-sum grants, the new Nash equilibrium leaves unchanged the level of provision of the national public good, and the private good consumption and welfare of each locality.

This neutrality result extends that of Warr and the other authors noted earlier. It shows that lump-sum redistribution is neutral even when there are 'distortions' in the economy in the form of taxes and subsidies on private goods or factors, strictly local public goods, or indeed on the national public good itself. These are significant generalizations of the neutrality proposition. It is also of interest to remark that Theorem 1 would be valid even if there were spillovers or externalities involving the y_i's, a generalization whose proof is straightforward.

The intuition behind this result is fundamentally the same as in the prior results which it generalizes. Note that in the special case where there are no matching subsidies, and where the only two commodities are the private good x and the public good g, Theorem 1 just reduces to the results already obtained by earlier writers. In this special case, eq. (7.1) just states that $\mathrm{d}g_i = \mathrm{d}L_i$ [after one takes into account the government budget constraint (5)]; that is, each agent's contribution adjusts so as to offset exactly the net incremental flow of resources that is received from the central government. This clearly reproduces the pre-transfer equilibrium, since the underlying real

[7]In particular, using (5) to eliminate $\mathrm{d}L_2$ from (6.2) and noting that $\mathrm{d}y_i = (\partial y_i/\partial I_i)\,\mathrm{d}I_i$, the solution of (6.1) and (6.2) using Cramer's rule immediately yields (7.1)–(7.3).

[8]Of course, for sufficiently large changes, the assumption of an interior solution with $g_i > 0$ for both i will become invalid, as discussed thoroughly by Bergstrom et al.

allocation of goods is unchanged and since, at this allocation, no agent has an incentive to contribute more or less to the public good.

In our more general model, the same general ideas apply. The main difference has to do with the way in which contributions adjust to offset the redistributive transfers. Clearly, if matching rates are zero, nothing changes: eqs. (7) imply perfect contribution offsets to lump-sum transfers. The presence of the taxed commodity y is irrelevant. However, if there are positive matching rates, one must take into account the fact that a dollar's worth of good x is no longer of equal value to a dollar's worth of the public good in the two jurisdictions. For locality j, one unit's worth of public good is worth only $(1-m_j)$ units of good x. Thus, suppose that $\mathrm{d}L_j=1$. If jurisdiction i reduces its contributions by \$1, this will leave jurisdiction j better off on balance, because the loss of the \$1 public good contribution by i is only worth a fraction $(1-m_j)$ of the transfer that it is receiving from the central government. It would be necessary for g_i to fall by $(1/(1-m_j)$ to offset fully the \$1 transfer received by j. When this adjustment occurs simultaneously in both jurisdictions, neither jurisdiction experiences a net increase or decrease in real income. Since the matching rates themselves also are fixed, there is no change in relative prices either. But with real incomes and relative prices unchanged, each jurisdiction will find it utility maximizing to choose the same bundle of private and public goods that it did before the transfers took place. Hence, Theorem 1.

3.2. Changes in matching grant rates

Now consider the effect of changes in the matching rates on contributions to the national public good, m_1 and m_2. In order to simplify the analysis, assume for the rest of this section that $t_i=0$, for $i=1,2$. This allows us to abstract from distortions caused by taxes and subsidies for private and strictly local public goods. We wish to ascertain whether matching grants, like lump-sum grants, are neutral, and if not, what their real effects are. Three different policy changes will be considered: an increase in m_1 financed by an increase in L_2, an increase in m_2 financed by an increase in L_2, and, starting with $m_1=m_2=m$, an increase in m (i.e. a simultaneous increase in m_1 and m_2) financed by an increase in L_2. Note, as an immediate consequence of Theorem 1, that use of L_2 rather than L_1 to finance the change in matching grants is of no consequence in the sense that the real impact of each policy change would be the same if L_1, or some combination of L_1 and L_2, were used to finance it instead. Thus, the results to follow refer to the effects of changes in matching grants financed by essentially any change in lump-sum grants.

We restrict our analysis to differential policy changes. The change in L_2 required to finance a change in m_1 and/or m_2 can be obtained from total differentiation of the central government budget constraint (2):

$$\mathrm{d}L_2 = -\sum_i (m_i \,\mathrm{d}g_i + g_i \,\mathrm{d}m_i). \tag{8}$$

To determine the comparative static adjustment of a Nash equilibrium to changes in the matching rates, totally differentiate (4):

$$\mathrm{d}g_1 + \mathrm{d}g_2 = -\phi_g^1 \,\mathrm{d}m_1 + \phi_I^1((1-m_1)\,\mathrm{d}g_2 - g_2 \,\mathrm{d}m_1), \tag{9.1}$$

$$\mathrm{d}g_1 + \mathrm{d}g_2 = -\phi_g^2 \,\mathrm{d}m_2 + \phi_I^2(\mathrm{d}L_2 + (1-m_2)\,\mathrm{d}g_1 - g_1 \,\mathrm{d}m_2). \tag{9.2}$$

Now substitute from (8) into (9.2) to eliminate $\mathrm{d}L_2$:

$$\begin{pmatrix} 1 & 1-\phi_I^1(1-m_1) \\ 1-\phi_I^2(1-m_1-m_2) & 1+\phi_I^2 m_2 \end{pmatrix} \begin{pmatrix} \mathrm{d}g_1 \\ \mathrm{d}g_2 \end{pmatrix} = \begin{pmatrix} a_1 & 0 \\ a_2 & b_2 \end{pmatrix} \begin{pmatrix} \mathrm{d}m_1 \\ \mathrm{d}m_2 \end{pmatrix}, \tag{10}$$

where

$$a_1 = -(\phi_g^1 + g_2\phi_I^1) = -(s_1 - g_1\phi_I^1) > 0, \tag{11.1}$$

$$a_2 = -g_1\phi_I^2 < 0, \tag{11.2}$$

$$b_2 = -(\phi_g^2 + g_1\phi_I^2 + g_2\phi_I^2) = -s_2 > 0. \tag{11.3}$$

The second equalities in (11.1) and (11.3) follow from the Slutsky equation and the inequalities in all three use the normality of public goods.

Let D denote the determinant of the left-hand-side matrix in (10). Direct computation yields:

$$D = (1-m_1)(\phi_I^1 + \phi_I^2 - (1-m_1-m_2)\phi_I^1\phi_I^2). \tag{12}$$

We assume henceforth that $m_i \geqq 0, i=1,2$ and $1-m_1-m_2 \leqq 1$. It then follows that $D>0$. Hence, one can solve the system (10) in the usual way to determine the derivatives of g_1 and g_2 with respect to the m_i's. Straightforward application of Cramer's rule yields:

$$\frac{\partial g_1}{\partial m_1} = -\frac{s_1(1+\phi_I^2 m_2)}{D} + \frac{g_1}{1-m_1} > 0, \tag{13.1}$$

$$\frac{\partial g_1}{\partial m_2} = \frac{s_2(1-\phi_I^1[1-m_1])}{D} < 0, \tag{13.2}$$

$$\frac{\partial g_2}{\partial m_1} = \frac{s_1(1-\phi_I^2[1-m_1-m_2])}{D} - \frac{g_1}{1-m_1} < 0, \tag{13.3}$$

$$\frac{\partial g_2}{\partial m_2} = -\frac{s_2}{D} > 0. \tag{13.4}$$

Thus, an increase in a matching rate has an unambiguously positive own-effect and an unambiguously negative cross-effect on the individual contribution levels. Note from (13.4) (taking $m=0$ if desired, for ease of interpretation) that matching grants stimulate local contributions more (per dollar transferred) than lump-sum grants, a standard result in the literature on grants [e.g. Wilde (1968), Oates (1972)]. Moreover, it follows directly from (13) that

$$\frac{\partial(g_1+g_2)}{\partial m_1} = -\frac{s_1\phi_I^2(1-m_1)}{D} > 0, \tag{14.1}$$

$$\frac{\partial(g_1+g_2)}{\partial m_2} = -\frac{s_2\phi_I^1(1-m_1)}{D} > 0. \tag{14.2}$$

Hence, matching grants are definitely *not* neutral under the behavioral assumptions of this section. They unambiguously increase the level of provision of the national public good.

To analyze the effect of matching grants on welfare, note that for locality i, an increase in m_i changes both the relative price of the national public good, giving rise to substitution and income effects, and the locality's 'full income' I_i, generating an additional income effect. By contrast, a change in m_j $(j \neq i)$ will only generate an income effect for i.

Let v_g^i and v_I^i denote partial derivatives of the indirect utility function v_i with respect to $1-m_i$ and I_i, respectively, and let $\mathrm{d}v^i/\mathrm{d}m_i$ and $\mathrm{d}v^i/\mathrm{d}m_j$ denote the change in the Nash equilibrium level of utility for i as m_i and m_j $(j \neq i)$ change. Recalling Roy's formula $(g = -v_g^i/v_I^i)$, and using (8), (12) and (13), we obtain (after some manipulations):

$$(v_I^1)^{-1}\frac{\mathrm{d}v^1}{\mathrm{d}m_1} = g - g_2 + (1-m_1)\frac{\partial g_2}{\partial m_1}$$

$$= \frac{(1-m_1)}{D}s_1(1-\phi_I^2[1-m_1-m_2]) < 0, \tag{15.1}$$

$$(v_I^1)^{-1}\frac{\mathrm{d}v^1}{\mathrm{d}m_2} = (1-m_1)\frac{\partial g_2}{\partial m_2} = -\frac{(1-m_1)s_2}{D} > 0, \tag{15.2}$$

$$(v_I^2)^{-1}\frac{\mathrm{d}v^2}{\mathrm{d}m_1}=-\left(\sum_i m_i\frac{\partial g_i}{\partial m_1}+g_1\right)+(1-m_2)\frac{\partial g_1}{\partial m_1}=-\frac{(1-m_1)s_1}{D}>0, \quad (15.3)$$

$$(v_I^2)^{-1}\frac{\mathrm{d}v^2}{\mathrm{d}m_2}=g-\left(\sum_i m_i\frac{\partial g_i}{\partial m_2}+g_2\right)-g_1+(1-m_2)\frac{\partial g_1}{\partial m_2}$$

$$=\frac{(1-m_1)s_2}{D}(1-\phi_I^1[1-m_1-m_2])<0. \quad (15.4)$$

To summarize these results:

Theorem 2. An increase in m_i financed by a change in lump-sum transfers (i) *increases the total level of provision of the national public good,* (ii) *lowers the welfare of i, and* (iii) *raises the welfare of j, where $j \neq i$.*

By property (i), we see that an increase in an individual matching grant is definitely not neutral: it unambiguously increases the equilibrium level of the national public good. (One can also easily show that changes in the t_i's are also non-neutral.) Much more striking is property (ii), which states that the *own*-welfare effect of an increase in a matching rate is negative, while the cross-effect is positive. This means, for example, that a matching subsidy to one jurisdiction, financed by a lump-sum tax on the residents of the other jurisdiction, will *lower* welfare in the jurisdiction receiving the subsidy, and *raise* welfare in the jurisdiction that pays the tax.

At first sight, this is a very counterintuitive result. To understand the intuition behind it, note than an increase in m_i 'distorts' the relative price of the national public good. Suppose that m_i is increased while L_i is adjusted to keep I_i constant. Then only a substitution effect of the change in m_i remains and it is well known that this sort of policy change would unambiguously reduce locality i's welfare.[9] This is exactly the policy change that we have analyzed, except that L_i does not adjust to keep I_i constant. However, by Theorem 1, we know that any such adjustments are irrelevant for welfare. Thus, in any case, an increase in m_i must lower welfare for i, regardless of the particular lump-sum financing method used.

Next, consider the case where $m_1=m_2=m$ and where m is increased. Formally, this amounts to a simultaneous increase in m_1 and m_2, and the previous analysis can be put directly to use here. It is easily seen from (14) that

$$\frac{\partial g}{\partial m}=\frac{\partial(g_1+g_2)}{\partial m_1}+\frac{\partial(g_1+g_2)}{\partial m_2}>0, \quad (16)$$

[9]See, for example, the classic analysis in Scott (1952).

as one would expect. By (15), the effect of changes in m on local welfare is given by:

$$(v_I^1)^{-1}\frac{dv^1}{dm}=(v_I^1)^{-1}\left(\frac{dv^1}{dm_1}+\frac{dv^1}{dm_2}\right)$$

$$=\frac{(1-m)}{D}([s_1-s_2]-s_1\phi_I^2[1-2m]), \tag{17.1}$$

$$(v_I^2)^{-1}\frac{dv^2}{dm}=(v_I^2)^{-1}\left(\frac{dv^2}{dm_1}+\frac{dv^2}{dm_2}\right)$$

$$=\frac{(1-m)}{D}([s_2-s_1]-s_2\phi_I^1[1-2m]), \tag{17.2}$$

which are ambiguous in general. However, if one assumes that both localities have identical preferences, the welfare effect for both will be positive if $m<\frac{1}{2}$ and will be zero if $m=\frac{1}{2}$. This is actually a local welfare maximum.[10] Summarizing:

Theorem 3. (i) *An increase in the common matching grant $m=m_1=m_2$ raises total contributions to the national public good.* (ii) *If both localities have identical preferences, the welfare of both localities is strictly increasing in m for $m<\frac{1}{2}$, and their welfare is (locally) maximized at $m=\frac{1}{2}$.*

Warr (1982) establishes Theorem 3, but only for the case of $m=0$ initially. Theorem 3 thus generalizes this result to the case $0<m\leqq\frac{1}{2}$, and establishes the local optimality of $m=\frac{1}{2}$. In view of the symmetry of the problem, it is quite a natural result.

3.3. Redistribution and the Pareto-efficiency frontier

The last result raises a broader question. What can one say about efficient policy in the more general case where households have diverse preferences?

To address this question, consider first the solution to the local government optimization problem (P′). At a solution, the first-order conditions,

$$MRS_{xy}^i=1, \tag{18.1}$$

[10] D remains positive for m slightly greater than $\frac{1}{2}$. It then follows immediately from (17) that $dv/dm=0$ at $m=\frac{1}{2}$.

$$MRS^i_{xg} = 1 - m_i, \quad i = 1, 2, \tag{18.2}$$

must hold, where $MRS^i_{xy} = u^i_y/u^i_x$ and $MRS^i_{xg} = u^i_g/u^i_x$. At any Nash equilibrium, the central government and household budget constraints (2) and (3) will hold. Summing these up and using (1) implies the economy-wide resource constraint:

$$\sum_i (x_i + y_i) + g = \sum_i w_i. \tag{19}$$

Next consider the set of Pareto-optimal allocations for this economy, that is, solutions to the problem:

$$\max_{\langle x_i, x_j, y_i, y_j, g \rangle} \quad u^i(x_i, y_i, g) \tag{S}$$

subject to

$$u^j(x_j, y_j, g) \geqq \bar{u}^j, \quad j \neq i,$$

and (19).

It is routine to show that necessary conditions for a solution to (S) are (18.1) and

$$\sum_i MRS^i_{xg} = 1. \tag{20}$$

Moreover, if an allocation (x_i, x_j, y_i, y_j, g) satisfies (18.1), (19) and (20), this allocation is Pareto optimal.

To see whether or when a Nash equilibrium will be Pareto efficient, sum (18.2) to get:[11]

$$\sum_i MRS^i_{xg} - 1 = 1 - \sum_i m_i. \tag{21}$$

Clearly, a Nash equilibrium will be Pareto efficient if and only if the matching rates sum to 1. That is, *any* m_1 and m_2 such that $m_1 + m_2 = 1$ produces a first-best Pareto-efficient allocation. Moreover, the welfare comparison of different efficient matching grant policies is known from Theorem 2. In particular, suppose $m^0_1 + m^0_2 = m^1_1 + m^1_2 = 1$ and that $m^1_1 > m^0_1$. Then, while both policies yield efficient allocations, the policy (m^1_1, m^1_2) is more favorable for locality 2, and less favorable for locality 1, than the policy (m^0_1, m^0_2). In fact, by varying m_1 subject to the constraint $m_1 + m_2 = 1$, one traces out the first-best Pareto frontier in utility space. Summarizing:[12]

[11] In the case where there are n localities, the right-hand side of (21) becomes $(n-1) - \sum_i m_i$.

[12] With n localities, this result goes through directly, except that efficiency requires $\sum_i m_i = n - 1$.

Theorem 4. There is a one-to-one mapping from first-best Pareto-efficient allocations to matching rates satisfying $m_1+m_2=1$. As m_1 rises, v^1 falls and v^2 rises.

Theorem 4 provides a striking contrast with the standard 'second theorem' of welfare economics. In that classical result, it is asserted that any Pareto-efficient allocation can be achieved as a competitive equilibrium, provided that the distribution of initial endowments is chosen properly. Here, however, Theorem 1 has already shown that one cannot move from one Pareto optimum to another by lump-sum redistribution of endowments, because these redistributions will leave consumption and welfare unchanged. Instead, it is through variations in individual corrective subsidy rates – keeping the sum of the subsidy rates fixed at the optimal level – that movements along the Pareto frontier are achieved. Furthermore, these movements display the rather counterintuitive property discussed in Theorem 3: an increase in m_i and a decrease in m_j *hurts* locality i and *benefits* locality j!

We conclude this section with two remarks. First, the assumption of an interior solution, in which each locality makes strictly positive contributions to the public good, is a relatively weak assumption when matching grants are at or near efficient rates. At any Pareto optimum the sum of individual contributions must equal the efficient level of g, which is strictly positive. There are obviously many combinations of the g_i's, and thus many choices of lump-sum transfers, that result in this level of g.

Second, note that an efficient Nash equilibrium is like a Lindahl equilibrium. Each agent i equates an individual marginal rate of substitution to a personalized 'price' $1-m_i$ for the public good, as shown in (18.2), and a Pareto optimum is achieved. Equivalently, we can say that every Lindahl equilibrium can be supported as a Nash equilibrium. Our analysis can then be interpreted as showing that lump-sum redistribution is welfare neutral across Lindahl equilibria with constant personalized prices, and that there is a continuum of Lindahl equilibria, with different personalized prices (summing to a constant), in which individual welfare levels and personalized prices vary inversely. This interpretation, of course, is just as valid when agents are individuals, and the public good in question could be any Samuelson public good.[13]

4. The possibility of general neutrality (seeing through the central government budget constraint)

The results of the previous section were obtained by analyzing policy-induced changes in the Nash equilibrium obtained when the local govern-

[13]The ideas in this paragraph are elaborated further in Boadway et al. (forthcoming).

ments behaved as if their actions had no influence on either the actions of the other local governments or the policy parameters chosen by the central government. Thus, in problem (P′), locality i takes as given both g_j and the policy parameters (t_i, m_i, L_i). This corresponds with the behavioral assumptions adopted by Warr (1982) and by Bergstrom, Blume and Varian (1986).[14] Recently, Bernheim (1986) has purported to have generalized the Warr and Bergstrom et al. neutrality result in the following significant way. Not only are lump-sum redistributions (or central government contributions to the public good) neutral, any policy changes involving lump-sum transfers, central government contributions, and distortionary tax finance are neutral. In the notation of our model, Bernheim claims that any combination of changes in (L_i, L_j, t_i, t_j) are neutral.[15]

It is clear that in the behavioral confines of sections 2 and 3, the Bernheim neutrality result cannot apply. The intent of this section is twofold. First, we wish to indicate the sorts of behavioral assumptions that will be sufficient to generate a neutrality result in the spirit of Bernheim. Second, we show that, given those behavioral assumptions, not only are distorting sources of tax finance neutral (t_i, t_j), but so are matching grants (m_i, m_j). Although our model is not identical with that used by Bernheim, the sorts of behavioral assumptions we adopt have direct parallels with those assumed by Bernheim, and so should assist in understanding what is driving this strong result. Indeed, one of the purposes of this section is to restate Bernheim's result in a manner which is compatible with our above model so that direct comparisons can be drawn and the precise differences in the requirements for the Bernheim versus the Warr results can be deduced.[16]

In particular, we show that the general neutrality will result if (i) both localities can 'see through' the central government budget constraint, in the sense that they recognize that central government policy parameters must be chosen to satisfy this constraint whatever decisions may be taken by local governments, and (ii) each locality adopts a particular assumption about

[14]The latter authors do, however, show that the neutrality of lump-sum redistributions in an otherwise undistorted world continues to hold if, in our notation, region i believes that changes in g_i will have a non-zero impact on g_j. They refer to this as a non-zero conjectural variation assumption.

[15]We need not consider central government contributions to the public good separately since they will be fully neutral even under the assumptions of section 2.

[16]Two key differences in our model are the following. First, Bernheim characterizes a government policy as a general function of the vector of labor supplies in the economy. In our model, central government policy is thought of as a particular choice of policy parameters, perfectly general except that they must satisfy the budget constraint. Second, Bernheim models the Nash equilibrium of the economy as a two-stage game in which households first choose labor supplies, and then choose contributions to the public good. The 'strategy space' in the second stage of the game can therefore be viewed as a choice of a function relating one's contribution to the vector of labor supplies, taking as given other agents' strategies. In our formulation, all decisions occur simultaneously. Given the essentially static nature of the problem, this should not be an important difference.

how the other behaves. This assumption will turn out to be consistent in the sense that under it each locality will, in fact, behave according to it.

Recall the decision problem (P′) of locality i. By substituting the central government's budget constraint (2) into the locality's budget constraint (3′), the problem (P′) can be rewritten:

$$\max_{\langle x_i, y_i, g\rangle} \; u^i(x_i, y_i, g) \tag{P''}$$

subject to

$$x_i + y_i + g = -L_j + t_j y_j - m_j g_j + g_j + w_i = T_j + g_j + w_i,$$

where T_j is the total (net) tax liability of locality j. A similar problem applies for locality j. A neutrality result similar to that proposed by Bernheim can now be stated:

Theorem 5. Let each locality assume that the other will respond to changes in central government policy by reducing its contribution to the public good g by an amount equal to the increase in its tax liabilities (i.e. $\mathrm{d}g_j = -\mathrm{d}T_j$). Then an optimal strategy for each locality in response to any change in taxes and transfers will be to leave its choice of x_i, y_i and g unchanged and to reduce its contribution to the public good g_i by the amount of the increase in its tax liabilities ($\mathrm{d}g_i = -\mathrm{d}T_i$). In this way, any policy changes will be fully neutralized and each locality's behavior will exactly conform with the conjecture of the other.

To show this result, note that neutrality is feasible from the point of view of each locality. Under the conjectured behavior, $T_j + g_j$ is constant in i's budget constraint, so an unchanged choice of (x_i, y_i, g) is feasible. Similarly for locality j. Furthermore, unchanged behavior is compatible with the central budget constraint since $(T_i + g_i) + (T_j + g_j)$, being constant, satisfies $(T_i + T_j) = \text{constant}$ if $(g_i + g_j)$ is constant. If each region treats $T_j + g_j$ as constant, it will in fact choose x_i, y_i and g unchanged by problem (P″). Finally, if $g = g_i + g_j$ is unchanged, and if $T_j + g_j$ is unchanged, so must $T_i + g_i$ by government budget balance. Therefore, the conjectured behavior is mutually consistent and the choices of x_i, x_j, y_i, y_j and g will be unchanged.[17]

[17]The neutrality result can easily be demonstrated more algebraically. Under the assumed conjectures, the 'income' term in the budget constraint of locality i can be treated by that locality as exogenous. Therefore, we may write the demand for g by locality i as $\phi^i(1, 1, T_j + g_j + w_i)$, where the 'prices' of x_i and y_i are unity. The same applies for j. The Nash equilibrium is then the solution to:

$$g_1 + g_2 = \phi^1(1, 1, T_2 + g_2 + w_1),$$
$$g_1 + g_2 = \phi^2(1, 1, T_1 + g_1 + w_2).$$

Using $\mathrm{d}T_2 = -\mathrm{d}T_1$, from the central government budget constraint, it is readily confirmed that, for any changes in central government policy, setting $\mathrm{d}g_i = -\mathrm{d}T_i$ $(i = 1, 2)$ satisfies these equations. By a similar analysis, it can be shown that $\mathrm{d}x_i = \mathrm{d}y_i = 0$ for $i = 1, 2$.

We would interpret this result as being analogous to the result obtained by Bernheim in his institutional setting.[18] Furthermore, this neutrality applies not only to distortionary taxes or subsidies on y_i (as in Bernheim), but also to matching subsidies to public goods contributions. This is a relevant extension of the general neutrality result since, in fact, governments seem to utilize such subsidies in the cases of private supply of public goods which are often cited (e.g. charitable donations, local government expenditures on national public goods).

The set of behavioral assumptions which generate the above neutrality result are obviously rather special in the sense that not only do they require that individual agents or localities see through the central budget, but also that they (correctly) conjecture the behavioral response of other agents. If these conditions do not apply, the neutrality result may not go through. On the other hand, there may well be other sorts of conjectures which, though not necessarily self-fulfilling, may nonetheless yield neutrality. For example, if each locality sees through the central government budget, takes the level of expenditures g_j of other localities as exogenous, and assumes that any revenue consequences of its own actions will affect only the lump-sum incomes of other regions, changes in government tax and subsidy rates will be neutral. This set of behavioral assumptions is not as attractive as those considered above (and by Bernheim) since they are not 'correct'.

5. Conclusions

This paper has analysed the effects of lump-sum and distortionary transfer policy in a model of non-cooperative private funding of pure public goods. It essentially rests on two assumptions. First, individuals (households or localities) do not care about the magnitude of their own contributions (assumed to be positive) but only about the aggregate level of funding of pure public goods. Second, they each behave non-cooperatively by taking as given the behavior of others.

Concerning the latter assumption, two alternative specifications are studied. In one of them, each individual takes as given the contributions by others. In the other specification, each individual can both 'see through' the government budget constraint and correctly conjecture the behavior of other individuals. Under both specifications, any policy consisting of lump-sum transfers has no effect on resource allocation. However, when considering policy consisting of distortionary financing or subsidy of the privately-

[18] In particular, it should be noted that the same crucial behavioral assumptions adopted here have their counterpart in Bernheim's analysis. He requires both that each agent conjecture that the other will behave so as to offset tax liabilities with changes in public goods contributions [his eq. (9)], and that agents see through the government budget constraint [his (13)]. Our analysis shows that these assumptions are enough to induce neutrality.

provided public good, then neutrality of resource allocation does result under the second specification, but does not under the first specification.

In fact, under the weaker specification of individuals just taking as given the others' contributions, we have shown that distortionary subsidizing of the public good is made at the expense of the apparent gainer; we have also shown that these distortionary instruments, if well chosen, can restore first best optimality.

Since we have presented our analysis in the context of local government behavior, let us conclude by mentioning some possible theoretical extensions and implications for empirical work in this area. First, we have assumed (in common with other contributions to the 'neutrality' literature) that expenditures on the public good by different jurisdictions are perfect substitutes for one another. (Thus, only total contributions by all enters the utility function.) The results must break down, strictly speaking, when this assumption is relaxed. It seems clear, but should be shown formally, that the results of the analysis will be approximately valid when the assumption of perfect separability is relaxed slightly.[19] Many localities are situated in large metropolitan areas in close proximity with a number of other jurisdictions, and many public services that they provide (water and sewage treatment, police and fire protection, perhaps education and transportation) generate significant external benefits for neighboring localities. If these services are reasonably good substitutes, then the scope for application of our analysis is wider than might at first appear to be the case.

Second, note that the theoretical analysis of intergovernmental grants [Oates (1972) is a standard reference] implies that matching grants stimulate local public spending more (per dollar of transfers) than lump-sum grants, and that lump-sum grants increase public spending at the same rate (per dollar) as private income. As noted above, our analysis carries the first of these implications. However, an increase in lump-sum transfers to a single locality would be expected to have a very large impact on local spending (e.g. dollar-for-dollar if the matching rates are zero), whereas a ceteris paribus increase in one locality's income would be expected to have a very much smaller effect.[20] Although we are unwilling to draw strong conclusions from this fact, we do note that it is consistent with the celebrated 'flypaper effect'.[21] More generally, our analysis suggests that empirical analysis of any one local government's response to grant policy may have to take into account the implications of simultaneous changes in the transfers received by

[19]If one parameterizes the preference structure to allow for various degrees of substitutability, then what would be required is that the Nash equilibrium vary continuously in the relevant parameter.

[20]Taking into account the reactions of other localities, a $1 increase in income for locality i will increase its equilibrium contribution by less than its ceteris paribus income derivative of demand for the public good.

[21]See Gramlich (1977) for the origin of the term.

other localities. Observations of public expenditure by local governments presumably reflect the fiscal interactions among localities simultaneously adjusting their expenditure levels, and these interactions need to be captured in empirical models if they are to be used to recover underlying structural parameters.

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