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NON-COOPERATIVE BEHAVIOR AND EFFICIENT PROVISION OF PUBLIC GOODS*

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I. INTRODUCTION

It is well-known that the presence of a public good leads to serious problems for implementing an efficient decentralized or centralized allocation. A necessary condition for Pareto efficiency is that the sum of individuals' willingness to pay for the public good must equal its marginal cost in terms of some private numeraire good. One way to achieve this efficient solution is to assign each individual a personalized price, called a Lindahl price, which corresponds to his or her willingness to pay for the public good. At this price, everyone demands an identical and efficient level of public good. To implement this Lindahl allocation, one needs to know individuals' true willingness to pay and it is not at all clear that they have the correct incentives to reveal it. They generally choose to act as free riders and this results in an inefficient outcome.

In this note, we do not address the question of eliciting individuals' true preference for the public good. Rather, we show that on the basis of such a non-cooperative equilibrium a centralized agency may effectively carry out an efficient solution by an appropriate choice of subsidies to the individuals' contributions to the public good. We find, first, that there is a very simple characterization of the personal subsidies that would support an efficient allocation as a non-cooperative equilibrium. It is straightforward to ascertain whether a particular observed non-cooperative equilibrium is efficient or not, without having detailed information about the preferences of individual households. We also find that a non-cooperative equilibrium with efficient subsidies resembles the outcome that one would find with ideal benefit taxation or Lindahl pricing. Indeed, there is a 1-1 correspondence between efficient subsidy arrangements and systems of benefit taxes. Finally, we investigate the relationship between various efficient subsidy (or Lindahl pricing) schemes and individual welfare levels, which illuminates the distributional implications of different efficient allocations.

II. OPTIMAL SUBSIDIES: UNIFORM AND NON-UNIFORM

We start from a setting whereby the public good is financed by subscription (Malinvaud [1972]), with each individual making a contribution to finance the production of the public good.[1] When fixing the amount of his contribution, each individual is concerned only with the advantage that he personally will gain from the overall supply of the public good, irrespective of the gain to others. Yet, they are all linked by their joint consumption of the same good. In such a setting, one can expect individual contributions to be fixed at a too low and inefficient level. This arises because each individual is faced with a price for the public good which is equal to its cost whereas some of the benefits accrue to others.

The subsidy is assumed to be financed by lump-sum taxes. As we shall show, different efficient allocations will be characterized by different subsidy rates. However, the pattern of lump-sum financing of the subsidies is irrelevant, provided only that no one's lump-sum tax is large enough to drive his contribution to zero. Throughout this paper, it is assumed that all individuals will choose to make some positive contribution in any of the non-cooperative equilibria that we analyze.[2]

To see this in the simplest of settings, let there be n consumers and two goods, a private good x and a public good g both produced from a linear technology such that:

(1) $$\sum_i x_i + g = \sum_i \omega_i$$

where ω_i is consumer i's initial endowment and x_i is consumer i's consumption of the private good. Let s_i denote the ith consumer's voluntary contribution to the public good so that

(2) $$g = \sum_i s_i.$$

The ith consumer's utility function can be written with the usual assumptions as:

(3) $$u_i = u_i(x_i, g).$$

Given the contributions $\bar{s}_j$ of all consumers $j \neq i$, each consumer i is assumed to maximize u_i with respect to x_i and s_i subject to

(4) $$x_i + (1 - m)\, s_i = \omega_i - L_i$$

and

(5) $$g = s_i + \sum_{j \neq i} \bar{s}_j$$

where m is the matching subsidy on individuals' contributions, assumed initially to be the same for all, and L_i is a lump-sum tax. We shall assume that taxes and subsidies are chosen so that the government budget constraint

(6) $$mg = \sum_i L_i$$

is satisfied in equilibrium. Although a uniform subsidy rate is momentarily being

assumed for all individuals, there is no reason that their lump-sum tax payments would necessarily be identical. Even with identical individuals, it might be possible for lump-sum taxes to differ. All that is required is that they sum up to the total subsidies being paid out, and that they be consistent with positive contributions by all households.

Consumer i's utility maximization requires that

$$\frac{\partial u_i}{\partial g} \Big/ \frac{\partial u_i}{\partial x_i} \equiv MRS^i_{x,g} = 1 - m$$

whenever $s_i > 0$ (as we assume). Therefore, in any non-cooperative equilibrium, it will be true that

(7) $$\sum_i MRS^i_{x,g} = n(1 - m).$$

This condition can be contrasted with the well-known optimality condition

(8) $$\sum_i MRS^i_{x,g} = 1.$$

Comparison of (7) and (8) shows that in an economy where the public good is financed by non-cooperative contributions, the output of this good is likely to be too small in the absence of some subsidy. It also shows that there is a unique and easily determined subsidy rate which leads to an efficient provision of the public good in any non-cooperative equilibrium with positive contributions by all agents. That rate is simply[3]

(9) $$m = \frac{n - 1}{n}.$$

So far, we have described the optimal *uniform* subsidy rate m. However, there is of course a large number of efficient allocations of resources besides the one obtained with a constant subsidy rate. To attain these, we must allow subsidy rates to vary across individuals. Let m_i be the subsidy rate for individual i, and note that $MRS^i_{x,g} = 1 - m_i$ in equilibrium. Summing over i, we now have

(7′) $$\sum_i MRS^i_{x,g} = \sum_i (1 - m_i)$$

instead of (7). Therefore, an efficient set of subsidies will satisfy $\sum_i (1 - m_i) = 1$, i.e.,

(9′) $$\sum_i m_i = n - 1,$$

while the lump-sum taxes must be set to satisfy the feasibility condition

(6′) $$\sum_i m_i s_i = \sum_i L_i$$

Again, the sets of efficient subsidy rates are easy to compute. Using (9′), one can immediately verify whether a non-cooperative equilibrium with positive contributions is optimal or not.

Thus, there can be many efficient non-cooperative equilibria with different subsidy rates, all satisfying (9′). As we have shown elsewhere for the two-person case (Boadway et al. [1987]), the correspondence between optimal subsidies and efficient allocations has an odd feature: An increase in m_i offset by a decrease in m_j *raises* the welfare of j and *lowers* the welfare of i. We return to this point below.

III. OPTIMAL SUBSIDIES AND LINDAHL PRICES

According to the principle of benefit taxation, individuals should be charged for public goods and services according to the marginal benefits that they obtain from consuming them. Similarly, in a Lindahl equilibrium, individuals would face personalized Lindahl prices that would be equal to their marginal benefits from public good provision. In either case, there would be a system of prices $(\bar{p}_1, \ldots \bar{p}_n)$ that would be charged to each individual, and an efficient level of public good provision $\bar{g}$, such that $\bar{p}_i = MRS^i_{x,g}$ for all i evaluated at $\bar{g}$ and such that $\sum_i \bar{p}_i = 1$. This ideal benefit-taxation or Lindahl pricing outcome can be supported as a non-cooperative equilibrium in the following way. Define $m_i = 1 - \bar{p}_i$. Next, choose *any* vector of contributions $(s_1, \ldots, s_n)$ such that $s_i > 0$ for all i such that $\sum_i s_i = \bar{g}$. Then we define for each consumer a lump-sum tax:

$$L_i = p_i \sum_{j \neq i} s_j = (1 - m_i) \sum_{j \neq i} s_j$$

or

$$L_i = (1 - m_i)\, \bar{g} - (1 - m_i)\, s_i.$$

One can easily check that:

$$\sum_i L_i = \sum_i m_i s_i.$$

Thus, these L_i's satisfy the government budget constraint and the (m_i, L_i) so constructed can support one of our non-cooperative equilibria.

Not only can any ideal benefit or Lindahl pricing scheme be supported as a non-cooperative equilibrium, one can also establish a converse proposition: the allocation of any efficient non-cooperative equilibrium with positive contributions can be achieved as an ideal benefit or Lindahl pricing scheme given a suitable distribution of income.

The proof is straightforward. Let (m_i, L_i) be a subsidy and tax policy supporting an efficient non-cooperative equilibrium, with associated contributions s_i. Define personalized prices $p_i = 1 - m_i$, and let each household be given an income $y_i = \omega_i - L_i + (1 - m_i) \sum_{j \neq i} s_j$. Then it is easily checked that the personalized prices p_i will be Lindahl prices relative to this income distribution and will support the same efficient allocation of resources as achieved in the initial non-cooperative equilibrium.

Notice that these results only concern the formal equivalence between an ideal benefit or Lindahl pricing scheme and a non-cooperative equilibrium with matching subsidy. The mechanisms by which allocations are achieved differs in the two cases. We have discussed here the implementation of an efficient outcome within a non-cooperative setting; it requires an active government. As is well known, finding interesting decentralized mechanisms that yield benefit tax or Lindahl equilibria is very difficult, and we have not really addressed that question here.[4] In fact, it is not one that needs to be answered for our purposes. Our goal is simply to note that an idealized benefit pricing (or Lindahl pricing) scheme, if one could be directly determined in some way, would produce an outcome that matches up with one of the non-cooperative equilibria that are the focus of our analysis.

We noted earlier that one can attain different efficient non-cooperative equilibria with different individual subsidy rates satisfying (9′). We also noted that varying the individual subsidy rates by raising m_i and lowering m_j, at least in the two-person case, will entail an increase in j's welfare and a fall in the welfare of i. This may seem paradoxical. Moreover, the equivalence between our non-cooperative equilibria and Lindahl pricing schemes implies that the latter must have a similar property. That is, since an increase in m_i corresponds to a decrease in the personalized price p_i, a lower personalized Lindahl price must imply a *lower* level of real income and welfare for individual i.

Although this property of Lindahl pricing schemes may also seem odd, it can be verified fairly easily (independently of our previous results on non-cooperative equilibria). The proof is as follows.

Let $(\bar{p}_1, \bar{p}_2, \bar{g})$ be the Lindahl equilibrium prices and quantity in an initial equilibrium. Now suppose that income is transferred in lump-sum fashion from 1 to 2, and let (p'_1, p'_2, g') be a new Lindahl scheme. Note that $\bar{p}_1 + \bar{p}_2 = p'_1 + p'_2 = 1$. We wish to prove that $p'_1 < \bar{p}_1$. Therefore, suppose to the contrary that $p'_1 \geqslant \bar{p}_1$, so that $p'_2 \leqslant \bar{p}_2$. By normality, the fact that 1 faces a higher price for the public good with a lower income implies that $g' < \bar{g}$. But 2 faces a lower price with a higher income, and hence $\bar{g} < g'$, a contradiction. Thus, it must be the case that $p'_1 < \bar{p}_1$ and $p'_2 > \bar{p}_2$: the individual with increased income and welfare faces a higher personalized price, and conversely for the person with lower income and welfare.

If this result seems counter-intuitive, it is probably because one is accustomed to thinking of prices as parametric determinants of individual welfare — hence, lower prices should lead to *higher* welfare. If one inverts the intuition, however, the result seems more evident. Suppose one moves from one Lindahl equilibrium to another, reducing individual i's welfare in the process. In each case, the personalized price p_i will be equal to i's marginal valuation of the public good. If the public good is a normal good, then a reduction in welfare should lead to a reduction in p_i, which is what we have shown.

IV. CONCLUSION

In this paper, we have shown, first, that there is a straightforward characterization of the personalized, per unit, subsidies that support efficient non-cooperative Nash equilibria with positive contributions: the subsidies must sum to $n - 1$. In any situation where this equilibrium concept might be applicable, therefore, it is easy to check whether an equilibrium is efficient: one need only observe whether there are positive contributions and whether the subsidy rates sum to the correct total.

Secondly, we have established a correspondence between non-cooperative equilibria and Lindahl equilibria, and we have shown that Lindahl prices and individual welfare levels may be positively associated across Lindahl equilibria.

NOTES

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[1] See also Milleron [1972] who discusses this equilibrium with subscription. He labels this equilibrium "a kind of Cournot equilibrium" and compares it with Lindahl equilibrium.

[2] Non-cooperative equilibria with voluntary contributions for public goods have recently been analyzed by Warr [1982] and by Bergstrom et al. [1986]. The latter authors pay particular attention to "corner solution" cases in which some individuals choose not to contribute to the provision of the public good.

[3] Note that the extension to the case of variable producer prices is straightforward. If c is the relative producer price of the public good, the optimal subsidy rate in (9) is $m = (n - c)/n$.

[4] A discussion of that question may be found in Inman [1987].

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Summary: *Non-cooperative Behavior and Efficient Provision of Public Goods.* — We show that, on the basis of a non-cooperative equilibrium whereby pure public goods are financed by voluntary private contributions, a centralized agency may induce an efficient solution by an appropriate choice of subsidy on the individuals' contributions. We also show that a non-cooperative equilibrium with positive contributions is efficient if and only if the individual per unit subsidies sum to $n - 1$, where n is the number of individuals. We finally establish a correspondence between non-cooperative and Lindahl equilibria.

Résumé: *Comportement non-coopératif et provision efficiente de biens publics.* — Nous montrons, sur la base d'un équilibre non-coopératif par lequel les biens publics purs sont financés par des contributions privées volontaires, qu'une agence centralisée peut induire une solution efficiente par un choix approprié de subsidiation des contributions individuelles. Nous montrons aussi qu'un équilibre non-coopératif est efficient avec contributions positives si et seulement si les subsides unitaires s'additionnent à $n - 1$, où n est le nombre d'individus. Finalement nous établissons la correspondance entre un équilibre non-coopératif et celui de Lindahl.

Zusammenfassung: *Nicht-kooperatives Verhalten und effiziente Bereitstellung öffentlicher Güter.* — Der Artikel zeigt auf der Basis eines nicht-kooperativen Gleichgewichtes, in dem rein öffentliche Güter durch freiwillige private Beiträge finanziert werden, das eine zentrale Instanz mittels einer angemessenen Wahl von Subventionen für individuelle Leistungen eine effiziente Lösung erreichen kann. Auch ist ein nicht-kooperatives Gleichgewicht mit positiven Beiträgen dann und nur dann effizient, wenn die einzelnen Subventionen sich zu $n - 1$ summieren, wobei n für die Anzahl der Individuen steht. Der Artikel stellt schließlich noch einen Bezug zwischen nicht-kooperativen Gleichgewichten und Lindahl-Gleichgewichten her.